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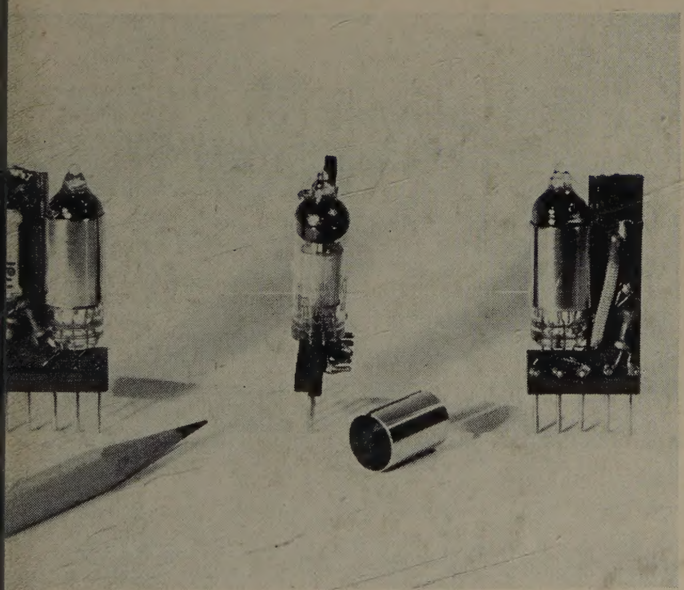

Sylvania Electric Products Inc.

Pre-Assembled Circuit

Subminiature tubes and associated circuit elements combine into compact plug-in units.

ollowing IRE Standards appear in this issue: Methods of Measurement me of Rise, Pulse Width, and Pulse Timing of Video Pulses in Television; Wave Propagation, Definitions of Terms.

PROCEEDINGS OF THE I.R.E.

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PROCEEDINGS OF THE I.R.E.

INSTITUTE NEWS AND RADIO NOTES SECTION

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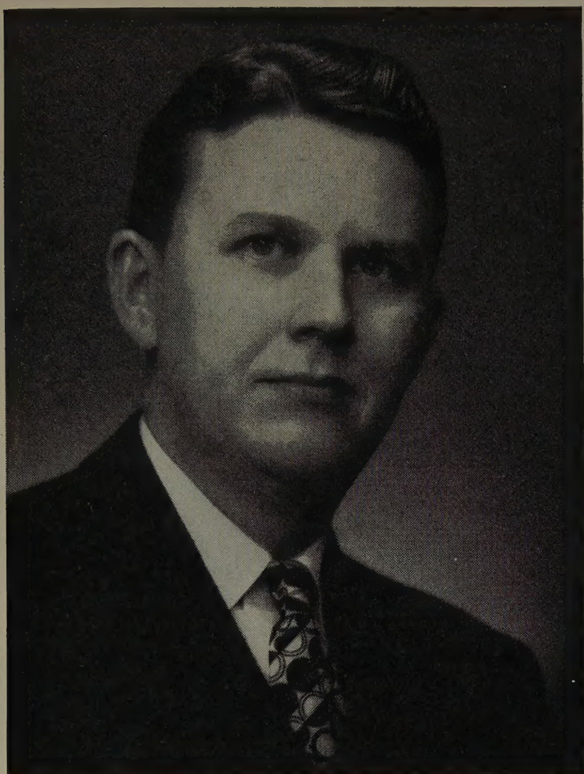
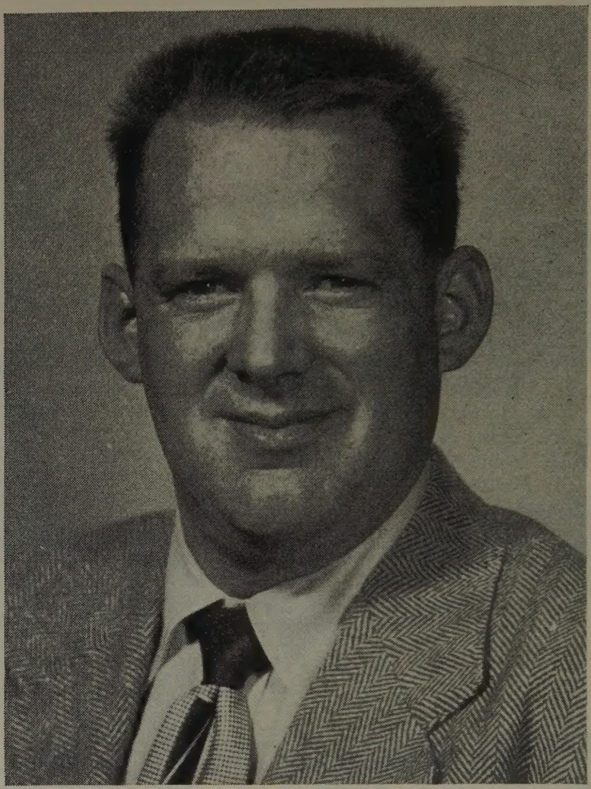
Chairmen of Recent IRE Sections



Harry L. Thorson

SCHENECTADY SECTION



Fred M. Ashbrook

INYOKERN SECTION

Harry L. Thorson, Chairman of the Schenectady Section, was born in Minneapolis, Minn., on September 28, 1908. Upon graduation from the University of Minnesota in 1931 with the B.S. degree in electrical engineering, he joined the General Electric Company test course at Schenectady. Mr. Thorson became associated in 1934 with the Vacuum Tube Engineering Department, which later became the Tube Division of the Electronics Department of that company, as a design engineer on the initial development of the metal receiving tube. Following this assignment he had design engineering responsibilities for television picture tubes, industrial power-control tubes, and transmitting tubes. In 1942 he was made assistant section engineer on transmitting tubes and in 1947 he was appointed section engineer, responsible for the design and application engineering of power control and rectifier tubes.

Mr. Thorson joined the Institute in 1942 as an Associate and became a Senior Member in 1950. He is Chairman of the IRE Gas Tube Subcommittee and is a member of the parent Committee on Electron Tubes and Solid-State Devices. In addition to his IRE activities, he has been active in several committees of the Joint Electron Tube Engineering Council of NEMA and RTMA. Active in the formation of the Schenectady Section, Mr. Thorson became the first Chairman in March, 1950, and will continue in office until June, 1951.

Fred M. Ashbrook, Chairman of Inyokern Section, was born in Alhambra, Calif., in 1918, and received the B.S. degree in electrical engineering from California Institute of Technology in June, 1942. Upon graduation he went directly to the Massachusetts Institute of Technology to join the staff of the Radiation Laboratory. While there he worked on the development of receiving equipment for microwave radars, and later joined a specialized group engaged in the research and development of antijamming techniques and equipment.

In January, 1946, Mr. Ashbrook transferred to the Naval Ordnance Test Station, Inyokern, Calif., to work on problems of electronic instrumentation for the rocket and missile ranges. At present he is head of the Missile Instrumentation Section at NOTS.

Mr. Ashbrook was active in the organization of the Inyokern Section of the IRE and was the first Vice-Chairman of the Section. He presented a paper on the "Problems of Rocket and Missile Range Instrumentation" at the 1950 National Convention of the IRE in New York, N. Y.

Mr. Ashbrook joined the Institute as a Student Member in 1941 and became an Associate in 1944. He was elevated to the rank of Member in 1945, and in 1949 advanced to the grade of Senior Member. He became Chairman of the Inyokern Section on January 26, 1950, and will hold office for a year.

The achievements of engineers stand in such high repute that they are often taken for granted by the public. Most products of engineering design generally function satisfactorily. As a result, it is usually assumed that engineers are competent and that the devices or mechanisms which they produce will function adequately.

The more complex—and costly—any product which is sold to the public, the more critical will be the purchasers. For this reason television receivers in the home represent a real responsibility imposed upon the engineer, the production experts, and the test personnel.

This subject has been treated in stimulating fashion by a pioneering television engineer who is as well an industrialist, a Fellow of The Institute of Radio Engineers, a past member of certain of the IRE Committees, and the recipient of awards from the Television Broadcasters Association and from the Veteran Wireless Operators Association.—*The Editor.*

Quality in Engineering

ALLEN B. DUMONT

As a manufacturer of cathode-ray tubes, oscillographs, and television equipment, I have from the very beginning insisted that quality should be the first consideration in all of the products bearing our company name. I have always believed that thorough inspection and testing were necessary to insure that material and workmanship were kept at high levels.

I would like to stress one fact which is commonly overlooked in connection with quality control. I find that many people, who are aware of the necessity for quality control, feel that because they have a staff of inspectors who are familiar with the latest sampling techniques, tables, and formulas, can talk knowingly of X-bars and A.Q.L., and utilize control charts and methods, they have done about all they can, so far as quality control goes.

I say to you that quality control can and must go far beyond that. In my plants, we have numerous placards posted which read "Quality cannot be inspected in, it must be built in." This is an excellent slogan, but it does not really get down to basic fact. I feel that quality *must be designed in!*

Every single component and piece of material used in the assembly may be thoroughly tested, every connection and solder joint may be perfect, every individual set may, on its completion, meet all of the operating specifications. However, if the basic design was not made with the idea of quality constantly in the mind of the engineer, the final result will be just another radio or television set.

Quality control must commence on the drawing board, not on the assembly line, and it is up to each engineer to recognize the need for quality. Don't leave it up to the assembly line inspectors. They can't produce a quality product no matter how hard they try, if the basic design is weak. Specifications should be made as complete as possible, and components should be included therein only after thorough investigation and test to determine their suitability for the work in hand. A television receiver, for example, can only be as good as its poorest component, and even the best components will fail if they are used in applications for which they were not intended.

I realize all too well that, in these days of competition in the industry, production costs are of vital importance and that, in many cases, top management has agreed to compromise in design or construction in order to shave a few cents here and there. However, too much of this type of economy will result in field failures or poor operation which will be reflected first, in excessive replacement costs under guarantee, and second, in the attitude of the buying public. In these early days of the giant new television industry, we should all as engineers and manufacturers strive constantly to bring the design of television equipment up to the highest standards possible.

In closing, I would like to quote a brief but eloquent thought on quality, the author of which is unknown to me, but which should be kept before us always:

> "Quality is never an accident. It is always the result of high intentions, sincere effort, intelligent direction, and skillful execution."

Management's Role in Research and Development of Electronic Systems*

RALPH I. COLE†, SENIOR MEMBER, IRE

Introduction

SYSTEMS ENGINEERING has many meanings to different engineering groups. To some, "systems" thinking involves mere assemblage and installation of a group of equipments which have been gathered together to work at a common site, followed by the preparation of an over-all instruction book as a guide in the operation. To others, it means the planning effort that goes into detailed study of the operation of a number of separate equipments, together with the design of whatever additional interconnection black boxes that are necessary. Other equally prevalent systems concepts involve methods and procedures of evaluating data channelled through a plurality of equipments, and being able thereby to express predicted operational characteristics from the engineering point of view. Not only do the divergent points of view of systems actively complicate the role of management in order to insure completeness of action, but equally important is the fact that it is very difficult clearly to distinguish, using concepts derived from "equipment" thinking, between any one system and an assemblage of integral systems. Generally speaking, the latter type or "*Comprehensive Electronic System*" may be regarded as made up of *minor systems* or *elements* to accomplished the following:

(a) Data gathering or data receiving element.
(b) Data processing, including such items as special treatment of data, storage of information as well as intercommunications with all directly related elements.
(c) Data utilization—final output information—display techniques.
(d) Data transfer—external communications.

While it is possible to catalogue most systems as to types using the above "Data" concepts, it will be shown that in most instances "systems engineering" thinking must be extremely broad and encompassing, possibly influencing the design of even minor components. Research and development management must ever be on the alert to keep the end objectives in the forefront to insure compatibility of the various elements of the systems and to achieve greatest efficiency in the use of men and facilities. Vagueness of systems concepts must be reduced if full advantage is to be taken of performance of individual elements.

"Systems" research and development of the type discussed herein is confined to that effort involved in the design, development, and evaluation of experimental systems ending with the preparation of procurement information for quantity production. This line of demarkation usually provides an acceptable basis for management, assisting in clarification of over-all responsibilities.

I. Generating and Directing "Systems Thinking"

Realizing that adequate electronics "systems" do not merely evolve from an assemblage of separate equipment concepts, it is well to consider in detail how management may plant the seeds that will create the atmosphere of systems thinking. Of initial importance is the interpretation of the desired end goals in terms of possible electronic performance. Having once set forth these engineering parameters required for the end objectives, it is logical to next consider the necessary basic electronic systems standards to be utilized, taking into account other established *systems* as well as *equipment standards* which have already been generally accepted. For example, the standards on electronic and electrical interference levels, power input voltage characteristics, ambient operating conditions, relay rack assemblages, and the like, as well as the influence of component standards back on the over-all system. In this latter regard, the interchangeability of electrical connectors and cables is a prime example of the need of possible continuous review by management of the over-all research and development program in order to insure that the various departments having cognizance over their own particular activity are able to produce devices to form integrated systems. Such thinking is necessary to create the atmosphere on the part of the design engineer wherein he becomes cognizant of the fact that he no longer may have free reign on the choice of his individual components, if by so doing he unnecessarily increases the number of types of parts or minor components without achieving material gain. Electronic systems engineering should therefore be accepted by management as a necessary and highly desirable specialization and given adequate support and backing.

II. Planning of Actual Specific Systems

While many electronic systems may initially evolve from an assemblage of already developed components, this is regarded as the exception, and not the type of operation that management can depend upon. Starting with the end objectives clearly enunciated, one of the first steps in the research and development of systems is the determination of the specific functional elements necessary to achieve the end results. In deciding upon these characteristics, adequate safety factors and tolerances must be added to each of the data processors in order that end results will not fall short of goals. At this stage, effective management insures that proper channels of co-ordination are set up between the equipment designers and the required systems engineers and should act in the role of arbitrators as difficulties and problems occur which cannot be readily adjudicated. As has been stated heretofore, the "system engineer" must have intimate knowledge of the performance capabilities of the elements chosen to make up the entire entity. The over-all systems engineer must insure the minimum of complexity and have the authority to make overriding decisions on the individual component characteristics. In merging the individual elements into an electronic system he must also take into account ancillary items such as test equipment, to insure that the final systems list does not include unnecessary duplication that would normally occur from merely adding together the test items considered necessary with each major equipment. This same philosophy must permeate into the systems engineering thinking to such other items as instruction books, spare parts, and the like, where undue cost will result if he and management do not play an active role in avoiding duplication. Furthermore, the electronic systems engineer must keep in mind that systems are often united with other systems, and hence their data output circuits and arrangements must be such as to provide for interconnection to form master operational entities.

III. Programming of the Assembly of Electronic Systems

Those engineers who are actively engaged in research and development of electronic components and/or equipments realize careful scheduling of the parts that are to make up the electronics systems is but one of the many items that management, in association with the systems project engineer, should give their utmost attention. This involves frequent reexamination of progress on the research and development of individual equipments and a great deal of flexibility in control of rate of completion of items. Furthermore, care must be exercised in avoiding premature assembly of the system prior to first having the opportunity of completely checking each major item to assure that when assembled into the final groupings the expected performance has a chance of being achieved. This pretesting phase should be carefully scheduled and due allowance made for a reasonable amount of replacement of such items as vacuum tubes, the life of which is not always correctly predictable.

IV. Evaluation of Systems

It is recognized that it is most difficult to give adjective ratings of the manner of performance of a complete electronic system, be it a minor one or an all-encompassing entity. This is due in large part to the inability to foresee the deleterious effect of one equipment upon another, as well as to have

* Decimal classification: R010. Original manuscript received by the Institute, May 17, 1950.
† Watson Laboratories, AMC, Red Bank, N. J.

foreseen the exact effect of individual tolerances upon the system objectives. Preparing a comprehensive systems testing program, setting forth clearly the individual objectives of each phase, should be considered by management to be as important as the actual research and development of the individual items. Co-ordination of this program prior to initiation of tests permits alerting of all separate departments who may be involved, as well as providing an excellent document for the briefing of all testing personnel on detailed procedures and techniques to be followed. Only after engineering management is satisfied that the evaluation program is the one which will have the greatest chance of yielding the most trustworthy appraisal should the actual taking of data begin. Furthermore, each step of the data taking process should be under careful supervision to insure that a sufficient quantity of information is recorded under controlled conditions in order to prevent unnecessary redoing of any phase of the program. This may mean that some tests may be temporarily delayed until it has been ascertained that the data recorded will suffice for the appraisal of the systems objective.

Of late there has come the realization that human engineering plays a large part in what may be called the efficiency of systems operations, and an excellent opportunity is afforded for appraising human engineering during the actual conducting of the test. In general, it may be well to request the systems engineer to carefully make comment upon the following items:

(a) Is the physical arrangement of equipments optimum?
(b) Have the environment factors, such as seating, lighting, operator fatigue, been taken into account?
(c) Processing of internal data through the system by the human operators involved. Stated in another manner, have the optimum time-motion factors been considered?
(d) Processing of the final output data. In this regard, is the output data in such form as to require the minimum of operator processing time?

As has been previously mentioned, engineering appraisal must be made from time to time of all data as it is taken and analyzed. The final formal engineering report, however, should present only that information which is vital to the rating of the system and clearly delineate engineering conclusions based upon thorough reliable information. The defects as well as the good points should be discussed, and management's role is to determine whether or not the system will satisfactorily meet the objectives. Furthermore, this group of reports can well serve as the background for preparation of the instructional literature which will later be issued to the operating personnel when and if quantity production is undertaken.

V. Preparing of Procurement Specifications for Quantity Production

The problem of preparing adequate procurement specifications for systems requires an appreciation on the part of management and systems engineers that the reproducibility of a complete system involves far more than production of individual elements. Particular attention must be paid to setting of the performance requirements below that actually obtained on the full engineering model in order to allow for tolerances beyond the control of either the *Single System* or the "*Comprehensive*" *Systems Engineer*. Furthermore, test procedures that can be accomplished at the source of manufacture must be analyzed to permit, insofar as possible, synthesis of the actual test that may have been accomplished in the field under controls not now available. Analysis of test procedures used on other possibly similar types of systems usually provide guide lines to be followed. Reports from the field on operation of other items of similar nature may also prove of great value.

Another phase of procurement information preparation concerns requirements for test equipments of the systems type. This must not be overlooked, since the systems performance may well deteriorate far below safe values and will therefore require readjustment at sometime in the future. The systems engineer must also be concerned with the spare parts requirement, and should carefully appraise this factor and take it into account. The requirements for instructional literature of the systems type must also be very carefully scrutinized to insure that the operating personnel would be able to achieve the best possible results from the assemblage of equipments making up the individual systems. The tying together of all of the above factors can best be insured by the use of check lists applicable to the system under consideration which lists all the separate items making up the system, plus their individual identifiable separate components.

VI. Conclusions

It is recognized that there are other aspects of systems engineering, such as "Installation," which may be as important as any discussed herein. However, research and development personnel are usually not directly involved. The phases of systems engineering which have been discussed point up the importance of this effort and indicate that management must play a vital role in view of the wide scope of this type of activity involving many diverse types of engineering talent. The success of systems engineering of the type discussed will be in a large measure governed by the keenness of engineering management to appreciate the many problems involved.

Statistical Methods in Research and Development*

L. LUTZKER†, ASSOCIATE, IRE

Manufacture or construction, in the early stages of their development, are conducted vigorously but in somewhat haphazard fashion. As these processes evolve, orderly controls are essential to determine statistically the quality of the product or service at any time. Thus waste, defective output, and resulting disorganization are largely avoided.

The following paper on this subject has received the approval of the IRE Professional Group on Quality Control, and is published through the good offices of that Group.—*The Editor.*

Summary—**Many engineers have heard of the recent advances in the field of applied statistics. It is the purpose of this paper to introduce and demonstrate one of the many statistical tools that can be of considerable aid in reducing trial and error experimentation for the design, development, or research engineer.**

Some variability enters into every experiment that may be undertaken. Some of the variability is due to chance errors of measurement and some is due to definitely assignable causes operating on the factors producing the data. Separation of the various assignable causes entering the experiment and their comparison with experimental error is known as the "analysis of variance."

* Decimal classification: R010. Original manuscript received by the Institute, May 8, 1950. Presented, 1950 IRE National Convention, New York, N. Y., March 6, 1950.
† Allen B. DuMont Laboratories, Inc., Passaic, N. J.

In order to most efficiently determine the significance and magnitude of the assignable variabilities, experimental error must be minimized. Methods of accomplishing this end are discussed. Following a discussion of some basic concepts, a computational example of an analysis is shown. The paper concludes with a summary of steps used in carrying a development problem to completion.

MANY ENGINEERS have probably heard of the recent advances made in the field of applied statistics and have wondered how these advances could be of use to the design, development, or research engineer. It is the purpose of this paper to answer this question in part by introducing and demonstrating one of the many statistical tools that the engineer can use. By using this tool, many needless hours of trial-and-error experimentation can often be eliminated, in addition to making more information obtainable from the data than is usually possible with conventional methods of analysis. The underlying theory of this tool will not be treated as it may be found in the literature listed in the bibliography. However, several important terms will be defined and discussed to help you understand the tool.

All are aware of the fact that some degree of experimental error is attendant upon every observation that we make, whether it be in a laboratory or on the production line. In order to determine the reliability and value of a given set of data, it is therefore necessary to compare the data with an estimate of the chance errors inherent in the data. This comparison is called a "test of significance" by the statistician.

In performing an experiment, the engineer always seeks to minimize the magnitude of error in order that any significant details in the results may show up clearly and not be masked by the errors involved in the experiment. Minimization of errors may be accomplished in a number of ways. One is the use of precision measuring apparatus. Another is to replicate or repeat the experiment a number of times and average the results. Still another method is to control all factors except the particular one in which the experimenter is interested. In designing the experiment to yield the most effective data, all the factors in the experiment that cannot be kept constant must be allocated at random throughout the entire experiment. The reason for this statement will become clear very shortly.

The development of the technique under consideration, hereafter called "analysis of variance," may be followed most easily by considering a hypothetical experiment and noting how the problems arising in connection with it are solved. Let us assume that a company wishes to develop an ultra-high-frequency tube. Four men are assigned to the project to work independently for a specified period of time. At the end of this period, each one has developed a tube. The company now wishes to know which, if any, of the four tubes will perform most satisfactorily.

The variables that enter in the experiment are conditions of construction, materials used, tools used, and technique of the technicians building the tubes. There are undoubtedly many others but, for our purposes, let us assume that the above-mentioned are the only ones. Since one of the methods of minimizing experimental error is control of the variables, the tubes will be built under the same conditions in so far as possible. In addition, the materials used will be drawn from batches of material known to be homogeneous. To speed completion of the project, four technicians and four machines (tools) are assigned. The explanation will be simplified by referring to the tube developments as I, II, III, and IV; to technicians as A, B, C, and D; and to machines as 1, 2, 3, and 4. As the experiment can be carried out in a number of ways, several of these will be considered and the best determined.

The first method that suggests itself is to assign one tube development and one machine to each technician, letting each one build eight tubes (replication or repetition of each tube development so that some of the error is averaged out). The performance data would then be be recorded as shown in Table I. Each letter-number combination represents the technician-machine combination that built the tube development models indicated at the top of the column. In this form, the averages or totals for each column may be compared to determine whether or not they are essentially alike. By examining this array of data, it is immediately obvious that any significant differences[1] among the four columns may be attributed to differences among tube developments, technicians, machines, or some combination of these factors. Since the experimenter is interested only in differences that may exist among tube developments, it becomes necessary for him to assign the technicians and machines differently in order that significant differences among columns be due only to tube developments. As was suggested earlier, technicians and machines are assigned at random for the construction of each development. Randomization is carried out in such a manner that each of the four levels of a given variable appears in each column once and only once. Thus, each column will contain the same factors an equal number of times and permit the averaging out of error due to these factors.

TABLE I

Tube Developments			
I	II	III	IV
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4
A 1	B 2	C 3	D 4

This step results in the layout shown in Table II.

[1] See Appendix I for a discussion of significant differences.

Each row now contains each technician and each machine only once. Thus any real differences among technicians or machines are averaged out. Any real differences that now exist among the columns can be due only to differences among tube developments. It can now be stated that the reason for allocating all factors, except the ones with which the experiment is concerned, at random is to average out the effects of the variables that could not be controlled.

TABLE II

Tube Developments			
I	II	III	IV
C 2 C 2	A 4 A 4	B 3 B 3	C 1 C 1
A 1 A 1	D 1 D 1	C 4 C 4	B 3 B 3
B 4 B 4	C 3 C 3	A 2 A 2	D 2 D 2
D 3 D 3	B 2 B 2	D 1 D 1	A 4 A 4

Table III represents a set of hypothetical data collected in the form indicated by Table II. The analysis of variance consists of breaking the total variance in the data into its component parts. In the experiment at hand, these are the variance among tube developments (among-column variance) and the error variance (within-column variance).

TABLE III

Tube Developments			
I	II	III	IV
144 147	147 143	160 156	141 146
154 156	141 148	154 159	148 142
153 145	154 145	150 155	144 153
149 156	155 157	158 155	153 143

The computational arithmetic, though essentially of a very elementary nature, may be simplified by reducing the magnitude of the numbers dealt with. This is done by shifting the mean of the data so that the converted data has a mean of approximately zero. In this case, subtracting 150 from each observation accomplishes this purpose. Shifting the mean is permissible since the variance of the data is not affected.

Table IV shows the data after the mean has been shifted. The figures in parenthesis are the squares of each observation to the left of the parenthesis. Before undertaking the formal analysis of variance,[2] the classical theory requires the assumption that the samples come from normal populations of the same variance be verified. If the variances were not the same but were significantly different from each other, it would not be possible to ascertain whether the source of the significant differences in the data was the differences among means or the differences among the variances. The above stated assumption is tested by calculating

$$L_1 = \left(\frac{s_1^2 \cdot s_2^2 \cdots s_k^2}{s_a^2 \cdot s_a^2 \cdots s_a^2}\right)^{1/k}$$

and finding the level of significance at which the value of L_1 falls. In the above equation for L_1

$$ns_1^2 = \sum_1^n (x - \bar{x}_1)^2 \qquad ns_2^2 = \sum_1^n (x - \bar{x}_2)^2, \cdots$$

$$Ns_a^2 = n \sum_1^k (s_i^2).$$

TABLE IV

Tube Developments			
I	II	III	IV
−6 (36) −3 (9)	− 3 (9) − 7 (49)	+10 (100) + 6 (36)	− 9 (81) − 4 (16)
+4 (16) +6 (36)	− 9 (81) − 2 (4)	+ 4 (16) + 8 (64)	− 2 (4) − 8 (64)
+3 (9) −5 (25)	+ 4 (16) − 5 (25)	+ 0 (0) + 5 (25)	− 6 (36) + 3 (9)
−1 (1) +6 (36)	+ 5 (25) + 7 (49)	+ 8 (64) + 5 (25)	+ 3 (9) − 7 (49)
(168) +4 (16)	(258) −10 (100)	(330) +46 (2,116)	(268) −30 (900)

The level of significance is determined by looking up the value of L_1 in a table of L_1 values.[3] The data shown in Table IV yield a value of 0.903 for L_1. Since this does not reach the 5-per cent level of significance, the hypothesis that the populations from which the data originate are normal and have no significant differences among their variances is verified. The analysis of variance may therefore continue. The arithmetical computations follow.

Square the individual observations and add.

$$\sum X^2 = 168 + 258 + 330 + 268 = 1{,}024.$$

Square each column total, add, and divide by the number of observations in each column.

$$\sum X_c^2/n_c = (16 + 100 + 2{,}116 + 900)/8 = 391.5. \quad (2)$$

[2] This is testing the hypothesis that the observed sample means come from normal populations whose means are essentially the same and whose variances are assumed the same.

[3] H. A. Freeman, "Industrial Statistics," John Wiley and Sons, Inc., New York, N. Y., Table X, p. 175; 1942.

Square the grand total, and divide by the total number of observations in the data.

$$(\sum X)^2/N = (10)^2/32 = 3.125. \tag{3}$$

Table V is the table summarizing the analysis of variance.

TABLE V

Source of Variance	Sum of Squares	Degrees of Freedom	Mean Squares
Among tube developments (among columns)	(2)–(3) = 391.5 −3.125 = 388.375	$(c-1)$ $4-1=3$	388.375/3 = 129.46
Error (within columns)	(1)–(2) = 1,024 −391.5 = 632.5	$c(n_c-1)$ $4(8-1)=28$	632.5/28 = 22.59
Total	(1)–(3) = 1,024 −3.125 = 1,020.875		

Before continuing the discussion, it would be well to interject a few words about degrees of freedom. The degrees of freedom that a given statistic has is usually "the number of ways that a group concerned can be arbitrarily filled in if the total is determined. . . ."[4] Thus, when $c-1$ column totals have been filled and the grand total is known, the last column total is uniquely determined. For within column degrees of freedom, each column can be filled (n_c-1) ways; since there are c columns, there are $c(n_c-1)$ degrees of freedom for the within-column variance. Finally, the total degrees of freedom is equal to the sum of the components, which is $c-1+c(n_c-1)$ or cn_c-1. Having determined the degrees of freedom associated with each source of variance, the mean squares are found by dividing each sum of squares by its respective degrees of freedom.

The last step in the analysis of variance is to compare the various sources of variance with the error variance for significance. This is done by means of the variance ratio or F test which consists of finding the ratio source variance/error variance and determining the level of significance at which the ratio falls, from a table of variance ratios.[5] Table V shows that the source (among columns) variance or mean square is 129.46 while the error mean square is 22.59. The variance ratio, 129.46/22.59 = 5.73, is significant below the 0.01- or 1-per cent level of probability but not quite at the 0.001- or 0.1-per cent level. Thus, there are grounds for believing that the tube developments are not alike in their performance.

[4] K. A. Brownlee, "Industrial Experimentation," Chemical Publishing Company, New York, N. Y., p. 50; 1948.
[5] See Table VIII, pp. 170–173, of footnote reference 3..

This being the case, which development or developments are best? If the larger observation implies better performance, Development III would appear to be the best. This hypothesis may be tested with a second table of variances. If tube I, II, and IV are alike, the resulting variance ratio should not reach the 0.05 level of significance. Table VI is the table for this analysis. The calculations necessary for this table are

$$\sum X^2 = 168 + 258 + 268 = 674 \tag{4}$$

$$\sum X_c^2 = (16 + 100 + 900)/8$$

$$= 1{,}016/8 = 127 \tag{5}$$

$$(\sum X)^2/N_{24} = (-36)^2/24 = 54. \tag{6}$$

$$F = 36.5/26/05 = 1.41.$$

TABLE VI

Source of Variance	Sums of Squares	Degrees of Freedom	Mean Squares
Among columns I, II, and IV	(5)–(6) = 127.0 − 54.0 = 73.0	$(c-1)$ $3-1=2$	73.0/2 = 36.5
Within columns I, II, and IV (error)	(4)–(5) = 674.0 −127.0 = 547.0	$c(n_c-1)$ $3(8-1)=21$	547.0/21 = 26.05
Total	(4)–(6) = 674.0 − 54.0 = 620.0		

Since an F ratio of 3.47 is required for the 0.05 level of significance, the hypothesis that the tubes I, II, and IV are alike is true.

If, on the other hand, a small observation had implied better performance, the company would have had to choose among three developments to determine the one to use. In this case, economy and ease of manufacture would probably have been the deciding factors since the the data did not support any particular one of the three developments.

In the same manner that I, II, and IV were hypothesized to be alike, any other combination might have been tested. For example, if the variance ratio from Table VI had actually been significant, one of the three developments could have been assumed superior or inferior to the remaining two, and the latter tested for differences. The analysis of any given set of data can be carried on until no further conclusions can be drawn from the data.

To summarize, the steps followed in analyzing a development or research problem are:

1. Several designs are developed and it is desired to find out which, if any, are superior to the rest.

2. An experiment is designed to yield data in systematic groupings to permit the analysis of the varia-

tions in the data into its components. All factors that are not being studied are controlled at the same level. If this is not practical, these variables are allocated at random.

3. Replication or repetition of the experiment is used to reduce the estimate of experimental error.

4. The data are analyzed by comparing the source variances with the error variance.

5. If the resulting ratio is significant at or below the 0.01 level of significance, a second analysis is made to determine which groups are alike and which are responsible for the significance in the data.

6. If the resulting ratio is significant between the 0.05 and 0.01 levels, additional replications are desirable. This will reduce the estimate of error and the level of significance if significant differences in the data really exist.

7. If the resulting ratio is not significant at or below the 0.05 level, there are insufficient grounds for believing that any differences exist.

The advantages gained from a systematic analysis such as described herein are:

1. Elimination of trial-and-error experimentation to a great extent, thereby obtaining accurate information from fewer data.

2. Attainment of conclusions based upon objective tests.

Appendix I

Significant Differences

One of the important concepts that one deals with in statistical work is that of significant differences. If one were to build a large number of tubes of design I, using any particular combination of technician, machine, material, and the like, the measurements of a given characteristic would be found to array themselves in the form of a frequency distribution popularly known as "the normal curve of distribution." Let this curve, arising from design or development I, be called distribution I. Similarly, a large number of measurements of the same characteristic on development II would form a normal curve which would be called distribution II.

Suppose now that both developments were exactly alike with regard to the performance characteristic being studied. It could be expected that distribution I could be superimposed upon distribution II and there would be no differences between the two .

If the parent distributions are alike, it is natural to expect that samples drawn from these distributions should also be alike. In a statistical sense, they are alike. However, the means and variances (measure of variation of the data about the mean) of successive samples differ. When the means of a large number of successive samples, taken from the same or like parent distributions, are arranged in the form of a frequency distribution, it is found that this also approximates the normal curve. The distribution of sample means is similar to the parent distribution in that the central tendencies or means of the two curves are the same. The difference is that the variance of the parent distribution is larger than the variance of the curve of smple means. To be exact, the variance of the parent curve is n times as large as the variance of the curve of sample means, where n is the sample size. Thus, for each value of the variable, there will be a certain probability that a sample mean will occur at that value when the sample was drawn from the assumed parent distribution.

In order to determine whether a given sample came from the assumed distribution, the statistician sets up the hypothesis that the sample actually did come from the assumed population. The probability that the hypothesis is correct is the probability figure discussed at the end of the last paragraph. If the probability is high, the hypothesis is considered to have been borne out. If, on the other hand, the probability is low, doubt is cast upon the hypothesis since it informs the statistician that it is improbable that the sample came from the assumed population.

The probability figure that is found is termed the "level of significance." As a rule, 0.05 is used as the level below which the difference between the sample and the population is said to be significant. In order words, the probability is 1 in 20 that the sample did come from the assumed population. Since the probability that the sample did not come from the assumed distribution is 19 in 20, it may be stated that the odds are 19 to 1 against the occurrence of a sample which appears at the 0.05 level of significance. In some cases, the experimenter may wish to be more certain that the differences exist and consequently will adopt a lower figure such as 0.01 probability as the level below which the difference is considered significant. In this case the odds would be 99 to 1 against the hypothesis that the observed sample came from the assumed distribution or was like the sample with which it was compared.

The best practical compromise is to consider all results at or near the 0.05- or 5-per cent level as worth following up and all results at or below the 0.01 or 1-per cent level indicative of significant differences. If the test for significance yields a result at or near the 5-per cent level and the significant difference really exists, gathering more data will usually result in a lowering of the level of significance toward or below 1 per cent.

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Standards on Television: Methods of Measurement of Time of Rise, Pulse Width, and Pulse Timing of Video Pulses in Television, 1950*

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1. INTRODUCTION

1.1 General Description

Fig. 1 portrays a typical composite picture signal wave form. The timing, the pulse widths or durations, and the time of rise or decay of various pulses are specified within definite limits. In order to insure conformance to these specifications, a measurement method must be available. This technique is applicable to the measurement of other television pulses.

The tolerances prescribed for the steepness of wave fronts, as well as the duration and timing of the various pulses that make up the composite synchronizing signal, require measuring methods capable of determining fractional microsecond time intervals. Along with the required high precision, it is necessary that the methods be as simple and easy to use as possible under operating conditions.

1.2 Definitions

1.2.1 Time of Rise (Decay) of Video Pulses in Television

The duration of the rising (decaying) portion of a pulse measured between specified levels.

1.2.2 Pulse Width of Video Pulses in Television

The duration of a pulse measured at a specified level.

1.2.3 Pulse Timing of Video Pulses in Television

The determination of an occurrence of a pulse or a specified portion thereof at a particular time.

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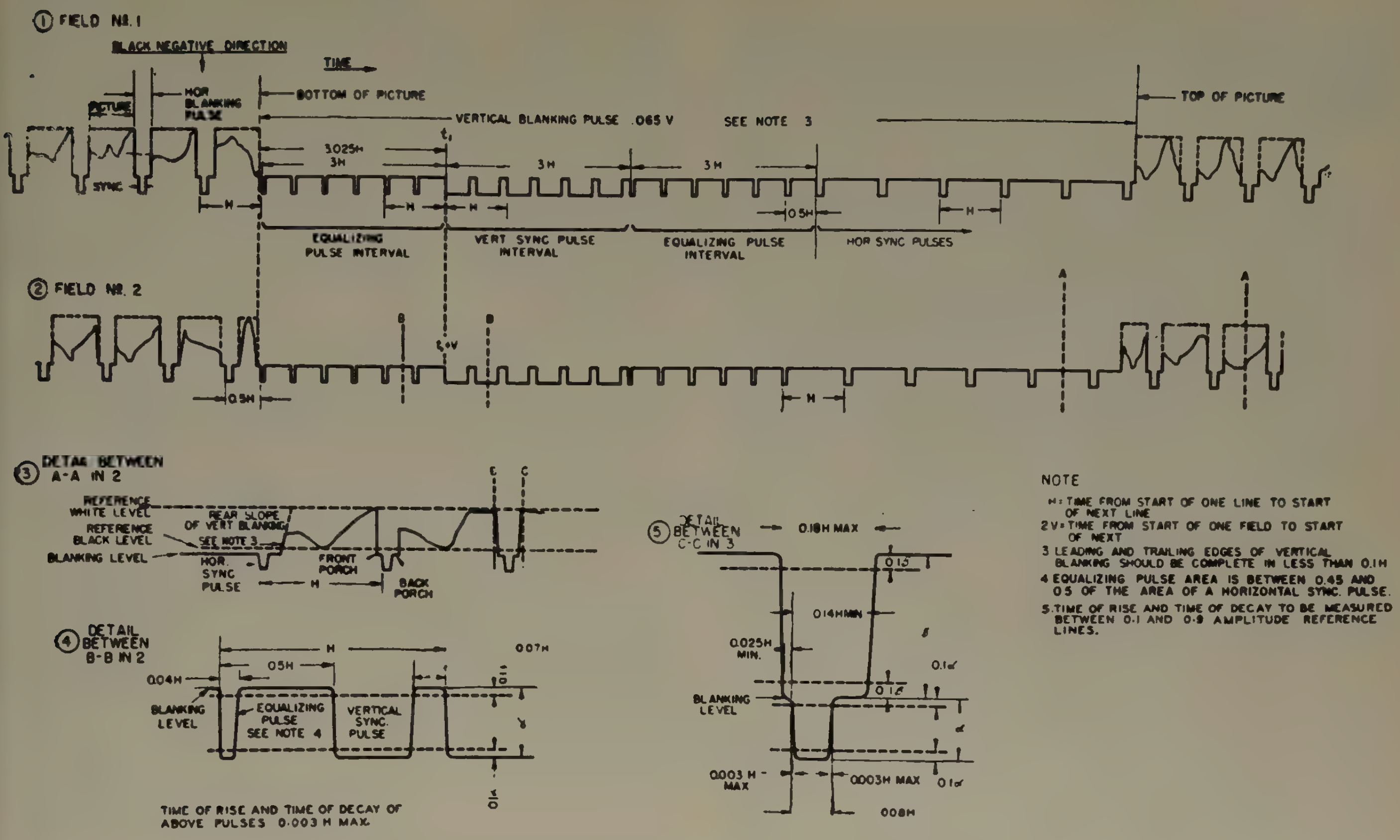

Fig. 1—A typical composite picture signal wave form. The pulse dimensions and notes 1 to 5 are given merely to portray the scope of measurement required.

2. APPARATUS AND CIRCUITS

2.1 Basic Method for Measurement of Time of Rise, Pulse Width, and Pulse Timing of Synchronizing Pulses

2.1.1 Time of Rise, Pulse Width, and Pulse Timing by Means of an Expanded-Sweep Method

A convenient method of measuring the small time intervals involved in the rise and decay of line synchronizing pulses, as well as the larger time intervals in pulse duration and timing measurements, is the use of an oscilloscope incorporating some form of expanded sweep. The measurement is made by varying the phase of the sweep a known amount or by noting the relationship of the pulse portions to timing markers.[1–3]

[1] R. P. Abbehouse, "A cathode-ray oscillograph for precision time measurement," Presented, IRE National Convention, New York, N. Y., March 25, 1948.

[2] T. Soller, M. Starr, and G. Valley, "Cathode Ray Tube Displays," *MIT Rad. Lab. Series*, McGraw-Hill Pub. Co., Chap. 7; pp. 251–302.

[3] IRE Standards on Television: Methods of Testing Television Transmitters, 1947.

2.1.2 Time of Rise, Pulse Width, and Pulse Timing by Means of a Phased Sine-Wave Sweep Method

In this method, a sine-wave sweep, synchronized to the pulses it is desired to measure, is so phased that the proper pulse section appears in the center or the fastest portion of the sweep. The portion of the center of the sine wave sweep that the pulse section occupies bears a known relationship to the sweep period. The measurement may then be obtained by calculation or by reference to a nomographic chart.[3,4]

2.1.3 Time of Rise, Pulse Width, and Pulse Timing by Means of a Sine-Wave Sweep and a Phased Marker

A sine-wave sweep is synchronized to the pulses it is desired to measure. A blanking marker signal synchronized to the sweep frequency, the delay of which is capa-

[4] R. A. Montfort and F. J. Somers, "Measurement of the slope and duration of television synchronizing impulses," *RCA Rev.*, vol. 6, pp. 370–389; January, 1942.

ble of being continuously varied with respect to the sweep, is used to determine the time relationship between pulse sections. Calibration of the marker phasing control permits direct determination of the time involved.[5]

2.1.4 Area of Equalizing Pulses by an Integration Method

The requirement that the equalizing pulse area be between 0.45 and 0.50 of the line synchronizing pulse area is not necessarily satisfied by a pulse width measurement because of the slope of the leading and trailing edges of pulses. The synchronizing signal is applied to an integrating circuit and the output is viewed on an oscilloscope. The integrated amplitude during the equalizing pulse region will be the same as during the line pulse region if the equalizing pulse area is one half the line pulse area.[3,6]

2.1.5 Timing by Means of the "Pulse-Cross" Method

The number of equalizing pulses, the number of vertical pulses, and an approximate measurement of the timing of these pulses and other synchronizing and blanking pulses can be determined by the "pulse-cross" method. In this method, a picture monitor is used to portray the composite signal in such a way that the vertical blanking bar appears horizontally near the center of the picture, and the horizontal blanking bar appears vertically through the center of the picture.[3,6,7]

2.2 Practical Measuring Circuits

2.2.1 Time of Rise, Pulse Width, and Pulse Timing by an Expanded-Sweep Method

An oscilloscope having an expanded sweep of several different lengths, the phase of which may be varied, so as to view the desired portions of the cycle, is useful in measuring time of rise, pulse width and pulse timing directly. Such an expanded sweep may be obtained in any of the following forms:

2.2.1.1 Line. A saw-tooth sweep of variable duration, synchronous with the line frequency and capable of being delayed with respect to the line-synchronizing pulses, may be used in conjunction with a brightening pulse for application to the cathode-ray-tube grid. This pulse should be synchronous with field frequency, with a duration equal to that of the sweep, and be capable of being adjusted in phase to any portion of the field period. If the composite video signal is then applied to the vertical-deflection circuit of the oscilloscope, any portion of the signal may be examined by varying the phase of the brightening pulse. Direct measurements may be made if the sweep delay is variable and calibrated. Alternatively, timing markers of the order of 0.3 per cent of line period may be used. It is suggested that these timing markers be initiated by the start of the sweep.

2.2.1.2 Field. A saw-tooth sweep of variable duration, synchronous with field frequency and capable of being adjusted in phase to any portion of the field period, may be used in conjunction with a brightening pulse having the same duration as the sweep and occurring at the same time for application to the cathode-ray-tube grid. If the composite video signal is then applied to the vertical-deflection circuit of the oscilloscope, any portion of the signal may be examined by varying the phase of the sweep and brightening pulse. Direct measurements may be made if the sweep delay is variable and calibrated. Alternatively, timing markers of the order of 0.3 per cent of line period may be used. It is suggested that these timing markers be initiated by the start of the sweep.

2.2.1.3 Frame. A saw-tooth sweep of variable duration, synchronous with frame frequency and capable of being adjusted in phase to any portion of the frame period, may be used in conjunction with a brightening pulse having the same duration as the sweep and occurring at the same time for application to the cathode-ray-tube grid. If the composite video signal is then applied to the vertical-deflection circuit of the oscilloscope, any portion of the signal may be examined by varying the phase of the sweep and brightening pulse. Direct measurements may be made if the sweep delay is variable and calibrated. Alternatively, timing markers of the order of 0.3 per cent of line period may be used.

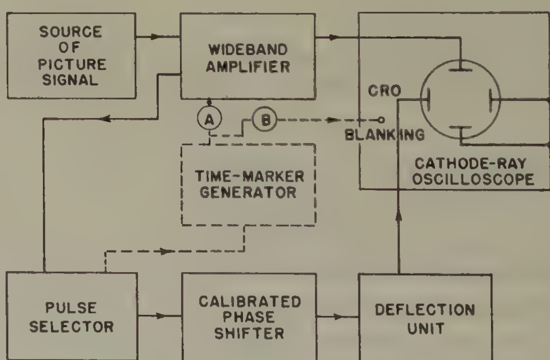

Fig. 2—Connection of apparatus for measurements by the expanded sweep method.

[5] H. L. Morrison, "Precision device for measurement of pulse width and pulse slope," *RCA Rev.*, vol. 8, pp. 276–288; June, 1947.
[6] A. V. Loughren and W. F. Bailey, "Special oscilloscope tests for television waveforms," Presented, Rochester Fall Meeting, Rochester N. Y., November 13, 1940.
[7] R. Page Burr, "The pulse cross generator applied to television receiver production test equipment," *Tele-Tech.*, pp. 36–39; April, 1949.

It is suggested that these timing markers be initiated by the start of the saw-tooth sweep.

Fig. 2 shows a typical connection of equipment.

2.2.2 Time of Rise, Pulse Width, and Pulse Timing by a Phased Sine-Wave Sweep Method

Fig. 3 shows the connection of equipment for measuring line frequency (or multiple of line frequency) pulses.

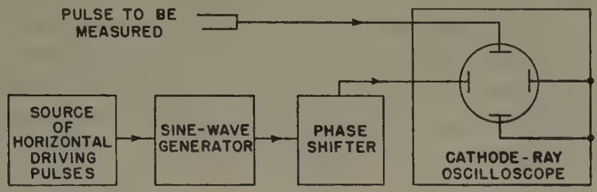

Fig. 3—Connection of equipment for measurements by the phased sine wave sweep method.

For greater accuracy in the measurement of the time of rise of pulses, it is helpful to use a harmonic generator developing a sine-wave time base which is the 10th harmonic of the line frequency, Measurement of field pulses is made on a sine-wave time base obtained directly, with provision for phase control, from the alternating-current power line.

2.2.3 Time of Rise, Pulse Width, and Pulse Timing by Means of a Sine-Wave Sweep and a Phased Marker

The time base for the oscilloscope is a sine wave obtained from a line-frequency generator locked to the line-frequency driving pulse of the synchronizing generator. Fig. 4 shows the connection of equipment. In the

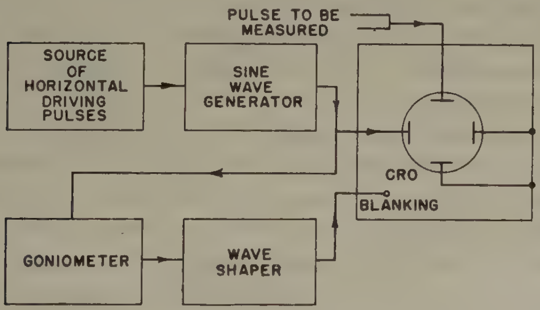

Fig. 4—Connection of equipment for measurements by means of a sine wave sweep and a phased marker.

auxiliary marker circuits, a goniometer is used to obtain another line frequency sine wave synchronized to the time base signal and capable of being precisely shifted in phase over a maximum continuously variable range of 360 degrees. By means of amplifier and limiter circuits, a narrow pulse at line frequency is obtained which may be applied to the oscilloscope blanking amplifier.

2.2.4 Area of Equalizing Pulses by an Integration Method

The synchronizing signal is applied to a single section resistance-capacitance circuit having a time constant equal to 5 to 10 times the line period, the output of which is viewed on an oscilloscope having an expanded sweep.

2.3 Requirements and Characteristics of Measuring Equipment

2.3.1 Cathode-Ray Oscilloscope

In all of the methods requiring an oscilloscope in order to make accurate measurements, the instrument must reproduce sync pulses satisfactorily. This requires that the oscilloscope have a good transient characteristic. The oscilloscope vertical-amplifier rise-time response to a unit step input should be no greater than 0.1 per cent H for an error of 5 per cent on a measurement of a sync pulse having a time rise of 0.3 per cent H, where H is the line period. Contribution of the amplifier to overshoot and tilt should be sufficiently small to prevent interference with the accuracy of measurement. In addition, it is required that the vertical amplifier be free from compression or nonlinearity over the range of amplitudes used in making measurements. For accuracy in all measurements, it is desirable that the oscilloscope maximum peak-to-peak deflection prior to any appreciable limiting be at least 1.5 inches. Where it is necessary to interpolate with the use of stationary timing markers, it is desirable that the horizontal amplifier be free from nonlinearity. When used with intensity markers, the blanking amplifier should have a bandwidth of at least 4 megacycles to produce narrow dots.

2.3.2 Sine Wave Generator

In both methods requiring a sine-wave time base, error will be minimized by care in holding the total harmonic content of the sine wave to a minimum.

2.3.3 Monitor for "Pulse-Cross" Method

The monitor must have provision for shifting the phase of the scanning oscillators approximately one-half period from the phase position normally used. Alternatively, the monitor sweep frequencies may be reduced to half line rate for the horizontal and to frame rate for the vertical scans. It is desirable, although not absolutely necessary, that provision exist for applying a negative picture signal (black positive) to the cathode-ray-tube grid. This produces white areas for the synchronizing signals, gray for the blanking and black level regions, and black for picture highlights. Provision for expanding the central portion of the field or frame scan to three or four times normal scanning amplitude enables the determination of the number of equalizing and vertical pulses.

3. PROCEDURE

3.1 Measurement Technique

3.1.1 Time of Rise, Pulse Width, and Pulse Timing by an Expanded-Sweep Method

With the synchronizing signal applied to the oscilloscope vertical amplifier and the gain adjusted for a wave form amplitude convenient for measurement, the sweep length should be adjusted to permit a spread on the screen of the signal portions to be measured. The measurements may then be made by means of the electronic time markers or by advancing the pulse sections past a fixed scale reference point by means of the calibrated phase shifter or sweep delay control.

3.1.2 Time of Rise, Pulse Width, and Pulse Timing by Means of a Phased Sine-Wave Sweep Method

3.1.2.1 Pulse Width. The pulse to be measured is applied to the vertical-deflection amplifier, and the sinusoidal time base is applied to the horizontal-deflection plates. By means of the variable phase-shifting device, the pulse is shifted to the center of the horizontal trace. With a transparent millimeter scale, the width of the pulse is measured at the 10 per cent amplitude point (Fig. 5). This width in per cent of sweep frequency time can be found by means of the following equation:

$$\text{per cent} = \frac{\sin^{-1}(C/D)}{1.8}$$

where C is the measured width of the pulse and D is the width of sinusoidal trace on the oscilloscope screen. The angle is expressed in degrees.

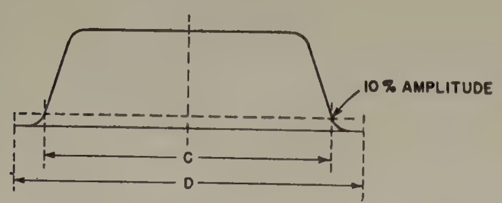

Fig. 5—Measurement points on cathode-ray oscilloscope trace.

In order to minimize the amount of calculation, a nomographic chart is provided in Fig. 6.

3.1.2.2 Pulse Timing. The above method may be used to determine the timing of pulses. Another method makes use of a calibrated phase shifter. The first part of the wave is lined up with a reference point or cross hair on the oscilloscope, the setting of the calibrated phase shifter being noted. The phase is then shifted to bring the second part of the wave back to the reference point. Knowing the frequency of the sine wave and the phase shift in fraction of a cycle, the timing in microseconds is found by dividing this phase shift by the frequency in megacycles per second.

3.1.2.3 Time of Rise (Decay). A harmonic generator developing a sine-wave time base which is the 10th harmonic of the line frequency is used to increase the accuracy of measurement of the time of rise (decay) of pulses. The pulse edge is centered by means of a phase-shift control and measurements are made as above with the transparent scale. The values obtained from the nomographic chart or by substitution in the equation are divided by 10 to give the duration of the time of rise in per cent of the line period.

3.1.3 Time of Rise, Pulse Width, and Pulse Timing by Means of a Sine-Wave Sweep and a Phased Marker

In this method, the pulse to be measured is applied to the oscilloscope vertical-deflection amplifier and the sine-wave time base to the horizontal amplifier. By means of the marker variable phase-shifting control, the marker is aligned with the beginning of the interval to be measured, and dial reading noted. The control is then adjusted for alignment of the marker with the end of the interval, and a second dial reading taken. The difference in dial readings corresponds to a definite time interval in per cent of the horizontal period or in microseconds, obtained from a calibration chart.

3.1.4 Area of Equalizing Pulses by an Integration Method

After being fed through the RC circuit, the integrated synchronizing signal is viewed on an oscilloscope with an expanded sweep phased so as to view the field retrace period of the signal. A sharp rise representing the integrated vertical pulses will then be observed; preceding this will be the equalizing pulse period and the horizontal pulse period. The integrated amplitude during the equalizing-pulse region will be the same as during the horizontal-pulse region if the equalizing-pulse area is one half the horizontal pulse area. An increase or decrease in equalizing-pulse area will cause the integrated output to rise or fall during the equalizing-pulse region. Correct adjustment, therefore, is obtained when the integrated amplitude decreases slightly or does not change during the equalizing-pulse region.

3.1.5 Pulse Timing by the "Pulse-Cross" Method

The display on the monitor picture tube is in the form of a "pulse cross." This is a useful operational test by which it is possible to count the number of equalizing pulses and vertical pulses, and obtain a rough check on the width of the front porch and back porch in relation to the width of horizontal sync. For observation of the field synchronizing region, the central portion of the field sweep should be expanded to three or four times normal scanning amplitude.

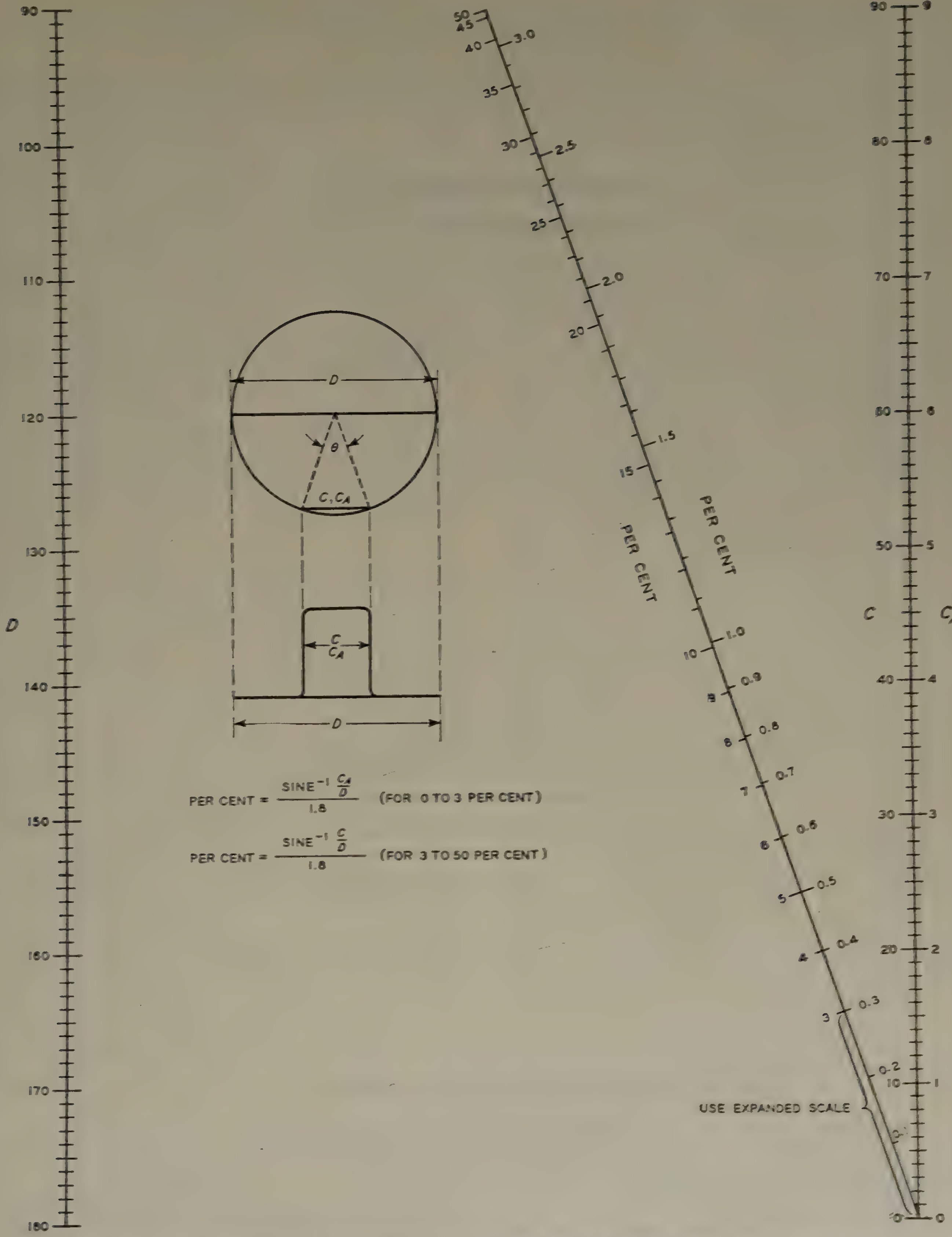

Fig. 6—Impulse-measurement chart. For measuring impulse widths in per cent, using sine-wave horizontal deflection on the cathode-ray oscillograph.

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Absorption. The irreversible conversion of the energy of a radio wave into other forms of energy as a result of its interaction with matter.

Angular Frequency. The frequency expressed in radians per second. It is equal to the frequency in cycles per second multiplied by 2π.

Atmospheric Duct. An almost horizontal layer in the troposphere, extending from the level of a local minimum of the modified refractive index as a function of height, down to the level where the minimum value is again encountered, or, down to the earth's surface if the minimum value is not again encountered.

Atmospheric Radio Wave. A radio wave that is propagated by reflections in the atmosphere. It may include either or both of the components, ionospheric wave and tropospheric wave.

Attenuation. Of a quantity associated with a traveling wave in a homogeneous medium, the decrease with distance in the direction of propagation. (Note—In a diverging wave, attenuation includes the effect of divergence.)

Attenuation Constant. For a traveling plane wave at a given frequency, the rate of exponential decrease of the amplitude of a field component (or of the voltage or current) in the direction of propagation, in nepers or decibels per unit length.

Attenuation Ratio. The magnitude of the propagation ratio.

Circularly Polarized Wave. An electromagnetic wave for which the electric and/or the magnetic field vector at a point describes a circle. (Note—This term is usually applied to transverse waves.)

Critical Frequency. The limiting frequency below which a magneto-ionic wave component is reflected by, and above which it penetrates through, an ionospheric layer at vertical incidence.

Cylindrical Wave. A wave whose equiphase surfaces form a family of coaxial or confocal cylinders.

D Region. The region of the ionosphere up to about 90 kilometers above the earth's surface.

Direct Wave. A wave that is propagated directly through space.

Direction of Polarization. For a linearly polarized wave, the direction of the electric vector.

Direction of Propagation. At any point in a homogeneous, isotropic medium, the direction of time average energy flow.

E Layer. An ionized layer in the *E* region.

E Region. The region of the ionosphere between about 90 and 160 kilometers above the earth's surface.

Effective Radius of the Earth. An effective value for the radius of the earth, which is used in place of the geometrical radius to correct for atmospheric refraction when the index of refraction in the atmosphere changes linearly with height. (Note—Under conditions of Standard Refraction the effective radius of the earth is 8.5×10^6 meters, or 4/3 the geometrical radius.)

Electric Displacement Density. Electric Flux Density.

Electric Field. A state of the medium in which stationary electrified bodies are subject to forces by virtue of their electrifications.

Electric Field Strength. The magnitude of the electric field vector (Note—This term is sometimes called the electric field intensity, but such use of the word intensity is deprecated in favor of field strength, since intensity connotes power in optics and radiation.)

Electric Field Vector. At a point in an electric field, the force on a stationary positive charge per unit charge. (Note—This may be measured either in newtons per coulomb or in volts per meter. This term is sometimes called the *electric field intensity* but such use of the word intensity is deprecated in favor of field strength since intensity connotes power in optics and radiation.)

Electric Flux Density. At a point, the vector whose magnitude is equal to the charge per unit area which would appear on one face of a thin metal plate introduced in the electric field at the point and so oriented that this charge is a maximum. The vector is normal to the plate from the negative to the positive face. (Note—The term *electric displacement density* or *electric displacement* is also in use for this term.)

Electric Vector. *See:* Electric Field Vector.

Electrical Length. The physical length expressed in wavelengths, radians or degrees.

Electromagnetic Wave. A wave characterized by variations of electric and magnetic fields. (Note—Electromagnetic waves are known as radio waves, heat waves, light waves, etc., depending on the frequency.)

Elliptically Polarized Wave. An electromagnetic wave for which the electric and/or the magnetic field vector at a point describes an ellipse.

Envelope Delay. The time of propagation, between two points, of the envelope of a wave. It is equal to the rate of change with angular frequency of the difference in phase between these two points. It has significance over the band of frequencies occupied by the wave only if this rate is approximately constant over that band.

Equiphase Surface. Any surface in a wave over which the field vectors at the same instant are in the same phase or 180° out of phase.

Extraordinary-Wave Component. The magneto-ionic wave component in which the electric vector rotates in the opposite sense to that for the ordinary-wave component. (*See:* Ordinary-Wave Component.)

***F* Region.** The region of the ionosphere above the *E* region.

F_1 Layer. The lower of the two ionized layers normally existing in the *F* region in the day hemisphere.

F_2 Layer. The single ionized layer normally existing in the *F* region in the night hemisphere and the higher of the two layers normally existing in the *F* region in the day hemisphere.

Fading. The variation of radio field strength caused by changes in the transmission medium with time.

Ground Wave. A radio wave that is propagated over the earth and is ordinarily affected by the presence of the ground and the troposphere. The ground wave includes all components of a radio wave over the earth except ionospheric and tropospheric waves. (Note—The ground wave is refracted because of variations in the dielectric constant of the troposphere including the condition known as a surface duct.)

Group Velocity. Of a traveling plane wave, the velocity of propagation of the envelope of a wave occupying a frequency band over which the envelope delay is approximately constant. It is equal to the reciprocal of the rate of change of phase constant with angular frequency. (Note—Group velocity differs from phase velocity in a medium in which the phase velocity varies with frequency.)

Guided Wave. A wave whose energy is concentrated near a boundary, or between substantially parallel boundaries, separating materials of different properties, and whose direction of propagation is effectively parallel to these boundaries.

Horizontally Polarized Wave. A linearly polarized wave whose electric field vector is horizontal.

Incident Wave. In a medium of certain propagation characteristics, a wave which impinges on a discontinuity or a medium of different propagation characteristics.

Incoherent Scattering. When radio waves encounter matter, a disordered change in the direction of propagation of the waves.

Ionosphere. The part of the earth's outer atmosphere where ions and electrons are present in quantities sufficient to affect the propagation of radio waves. (Note—According to current opinion, the lowest level is approximately 50 kilometers above the earth's surface.)

Ionospheric Wave. A radio wave that is propagated by way of the ionosphere. (Note—This is sometimes called a *sky wave.*)

Left-Handed (Counter Clockwise) Polarized Wave. An elliptically polarized transverse electromagnetic wave in which the rotation of the electric field vector is counter clockwise for an observer looking in the direction of propagation.

Linearly Polarized Wave. At a point in a homogeneous, isotropic medium, a transverse electromagnetic wave whose electric field vector at all times lies along a fixed line.

Lowest Useful High Frequency. The lowest high frequency effective at a specified time for ionospheric propagation of radio waves between two specified points. (Note—This is determined by factors such as absorption, transmitter power, antenna gain, receiver characteristics, type of service, and noise conditions.)

Magnetic Field. A state of the medium in which moving electrified bodies are subject to forces by virtue of both their electrifications and motion.

Magneto-Ionic Wave Component. Either of the two elliptically polarized wave components into which a linearly polarized wave incident on the ionosphere is separated because of the earth's magnetic field.

Maximum Usable Frequency. The upper limit of the frequencies that can be used at a specified time for radio transmission between two points and involving propagation by reflection from the regular ionized layers of the ionosphere. (Note—Higher frequencies may be transmitted by sporadic and scattered reflections.)

Modified Index of Refraction. In the troposphere, the index of refraction at any height increased by h/a, where h is the height above sea level and a is the mean geometrical radius of the earth. When the index of refraction in the troposphere is horizontally stratified, propagation over a hypothetical flat earth through an atmosphere with the modified index of refraction is substantially equivalent to propagation over a curved earth through the real atmosphere.

***O* Wave.** Ordinary-Wave Component.

Optimum Working Frequency. The most effective frequency at a specified time for ionospheric propagation of radio waves between two specified points. (Note—In predictions of useful frequencies the optimum working frequency is commonly taken as 15 per cent below the monthly median value of the maximum usable frequency, for the specified time and path.)

Ordinary-Wave Component. That magneto-ionic wave component deviating the least, in most of its propagation characteristics, relative to those expected for a wave in the absence of the earth's magnetic field. More exactly, if at fixed electron density, the direction of the earth's magnetic field were rotated until its direction is transverse to the direction of phase propagation, the wave component whose propagation is then independent of the magnitude of the earth's magnetic field.

Penetration Frequency. *See*: Critical Frequency.

Periodic Electromagnetic Wave. A wave in which the electric field vector is repeated in detail in either of two ways: (1) At a fixed point, after the lapse of a time known as the period, (2) At a fixed time, after the addition of a distance known as the wavelength.

Phase Constant. For a traveling plane wave at a given frequency, the rate of linear increase of phase lag of a field component (for the voltage or current) in the direction of propagation, in radians per unit length.

Phase-Propagation Ratio. The propagation ratio divided by its magnitude.

Phase Velocity. Of a traveling plane wave at a single frequency, the velocity of an equiphase surface along the wave normal.

Plane Earth Factor. The ratio of the electric field strength that would result from propagation over an imperfectly conducting plane earth to that which would result from propagation over a perfectly conducting plane.

Plane of Polarization. For a plane polarized wave, the plane containing the electric field vector and the direction of propagation.

Plane Polarized Wave. At a point in a homogeneous isotropic medium, an electromagnetic wave whose electric field vector at all times lies in a fixed plane which contains the direction of propagation.

Plane Wave. A wave whose equiphase surfaces form a family of parallel planes.

Propagation Constant. For a traveling plane wave at a given frequency, the complex quantity whose real part is the attenuation constant in nepers per unit length and whose imaginary part is the phase constant in radians per unit length.

Propagation Factor. *See:* Propagation Ratio.

Propagation Ratio. For a wave propagating from one point to another, the ratio of the complex electric field strength at the second point to that at the first point.

Radio Field Strength. The electric or magnetic field strength at a given location resulting from the passage of radio waves. In the case of a sinusoidal wave, the root-mean-square value is commonly used. Unless otherwise stated, it is taken in the direction of maximum.

Radio Frequency. A frequency at which coherent electromagnetic radiation of energy is useful for communication purposes.

Radio Horizon. The locus of points at which direct rays from the transmitter become tangential to the earth's surface. (Note—On a spherical surface the horizon is a circle. The distance to the horizon is affected by atmospheric refraction.)

Radio Wave Propagation. The transfer of energy by electromagnetic radiation at frequencies lower than about 3×10^{12} cycles per second.

Refracted Wave. That part of an incident wave which travels from one medium into a second medium.

Refractive Index. Of a wave transmission medium, the ratio of the phase velocity in free space to that in the medium.

Refractive Modulus. In the troposphere, the excess over unity of the modified index of refraction, expressed in millionths. It is represented by M and is given by the equation

$$M = (n + h/a - 1)10^6,$$

where n is the index of refraction at a height h above sea level, and a is the radius of the earth.

Relative Refractive Index. Of two media, the ratio of their refractive indices.

Right-Handed (Clockwise) Polarized Wave. An elliptically polarized transverse electromagnetic wave in which the rotation of the electric field vector is clockwise for an observer looking in the direction of propagation.

Selective Fading. Fading which is different at different frequencies in a frequency band occupied by a modulated wave.

Shadow Factor. The ratio of the electric field strength which would result from propagation over a sphere to that which would result from propagation over a plane, other factors being the same.

Sinusoidal Electromagnetic Wave. In a homogeneous medium a wave whose electric field strength is proportional to the sine (or cosine) of an angle that is a linear function of time, or a distance, or of both.

Sky Wave. *See:* Ionospheric Wave.

Spherical-Earth Factor. The ratio of the electric field strength that would result from propagation over an imperfectly conducting spherical earth to that which would result from propagation over a perfectly conducting plane.

Spherical Wave. A wave whose equiphase surfaces form a family of concentric spheres.

Standard Propagation. The propagation of radio waves over a smooth spherical earth of uniform dielectric constant and conductivity, under conditions of standard refraction in the atmosphere.

Standard Refraction. The refraction which would occur in an idealized atmosphere in which the index of refraction decreases uniformly with height at the rate of 39 $\times 10^{-6}$ per kilometer. (Note—*Standard refraction* may be included in ground wave calculations by use of an effec-

tive earth radius of 8.5×10^6 meters, or 4/3 the geometrical radius of the earth.)

Standing Wave. A wave in which, for any component of the field, the ratio of its instantaneous value at one point to that at any other point does not vary with time.

Surface Duct. An atmospheric duct for which the lower boundary is the surface of the earth.

Tangential Wave Path. In radio wave propagation over the earth, a path of propagation of a direct wave, which is tangential to the surface of the earth. The tangential wave path is curved by atmospheric refraction.

Transmitted Wave. *See:* Refracted Wave.

Transverse Electric Wave. In a homogeneous isotropic medium, an electromagnetic wave in which the electric field vector is everywhere perpendicular to the direction of propagation. (Note—This is abbreviated "*TE* Wave".)

Transverse Electromagnetic Wave. In a homogeneous isotropic medium, an electromagnetic wave in which both the electric and magnetic field vectors are everywhere perpendicular to the direction of propagation. (Note—This is abbreviated "*TEM* Wave.")

Transverse Magnetic Wave. In a homogeneous isotropic medium, an electromagnetic wave in which the magnetic field vector is everywhere perpendicular to the direction of propagation. (Note—This is abbreviated "*TM* Wave.")

Traveling Plane Wave. A plane wave each of whose frequency components has an exponential variation of amplitude and a linear variation of phase in the direction of propagation.

Troposphere. That part of the earth's atmosphere in which temperature generally decreases with altitude, clouds form, and convection is active. (Note—Experiments indicate that the troposphere occupies the space above the earth's surface to a height of about 10 kilometers.)

Tropospheric Wave. A radio wave that is propagated by reflection from a place of abrupt change in the dielectric constant or its gradient in the troposphere. (Note—In some cases the ground wave may be so altered that new components appear to arise from reflections in regions of rapidly changing dielectric constants; when these components are distinguishable from the other components, they are called tropospheric waves.)

Uniform Plane Wave. A plane wave in which the electric and magnetic field vectors have constant amplitude over the equiphase surfaces. (Note—Such a wave can only be found in free space at an infinite distance from the source.)

Vertically Polarized Wave. A linearly polarized wave whose magnetic field vector is horizontal.

Virtual Height. The apparent height of an ionized layer determined from the time interval between the transmitted signal and the ionospheric echo at vertical incidence, assuming that the velocity of propagation is the velocity of light in a vacuum over the entire path.

Wave. A physical activity in a medium such that at any point in the medium some of the associated quantities vary with time, while at any instant of time, they vary with position.

Waveguide. A system of material boundaries capable of guiding waves.

Wave Interference. The variation of wave amplitude with distance or time, caused by the superposition of two or more waves. (Note—As most commonly used, the term refers to the interference of waves of the same or nearly the same frequency.)

Wavelength. In a periodic wave, the distance between points of corresponding phase of two consecutive cycles. The wavelength λ is related to the phase velocity, v, and the frequency, f, by $\lambda = v/f$.

Wave Normal. A unit vector normal to an equiphase surface with its positive direction taken on the same side of the surface as the direction of propagation. In isotropic media, the wave normal is in the direction of propagation.

***X* Wave.** *See:* Extraordinary-Wave Component.

Quality Rating of Television Images*

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***Summary*—Two methods of evaluating impairments in television images are described. Both employ observers and, therefore, yield subjective evaluations. The first is an extension of Baldwin's in which observers vote a preference between pictures with different impairments; one of the pictures is optically projected somewhat out of focus and is used as a reference. In the second method, the impairment is rated by observers in terms of pre-worded comments which are numbered and form a rating scale. Both methods permit an evaluation in terms of liminal increments as computed from the distribution of votes of the observers. These methods have been used to evaluate the impairing effects of echoes and noise in television pictures, and also to relate picture sharpness to other quality parameters.**

Introduction

Television images are subject to various degrees of impairment from any one or more of numerous causes broadly classified as distortion or interference. Regardless of the source of the distortion or interference, whether in the camera chain, the transmission path, or in the receiver, the impairing effect on the final image is only as slight or as serious as the viewer judges it to be. Impairment is, therefore, a subjective quantity and is measurable in terms of the reaction of observers. For engineering purposes, it is desirable that evaluations by observers be expressed in numbers which apply equally well to all kinds of impairments, and which serve as a scale of quality of television images.

This paper reports the results of experiments in which observers were used to evaluate the quality of pictures. Two techniques, a comparison and a direct method, were used to determine the impairing effects of noise and echoes in television images. The comparison method was also used to obtain some rough estimates of the relative importance of sharpness, contrast, and brightness in determining the quality of pictures.

I. Recapitulation

Two methods of rating impairments of pictures were investigated. The first method establishes the equivalence of a pair of pictures having different impairments. For example, one picture, a lantern slide projection, is defocused until its quality, in the judgment of observers, equals that of another impaired picture of the same subject. The second method makes use of pre-worded comments in terms of which observers rate various impairments.

The ratings obtained by these two methods can be expressed in three ways: (1) in terms of the defocusing of the reference picture, as determined directly from the first method; (2) in terms of pre-worded comments, as determined directly from the second method; and (3) in terms of liminal units, as derived from the distribution of votes in either method. (One liminal unit indicates a 75 per cent vote preference for one picture condition over another.)

The ratings obtained by the two methods can be compared. The data show that the two methods agree reasonably well.

As part of this investigation, data were gathered on the impairment to television pictures caused by single echoes and noise. These data, which are summarized in this paper, have already been found useful in the study of transmission requirements and tolerances.

Initial data are also presented on the relative importance of sharpness, contrast ratio, and highlight brightness as parameters of the quality of pictures.

A calibration of the defocused projections used in the comparison method was made in terms of resolvable test lines and equivalent television bandwidth. The results of this supplementary study are given in the Appendix.

II. Comparison Technique

In this procedure, observers were asked to indicate their preference for one of a pair of pictures derived from duplicate lantern slides and viewed side by side as illustrated in Fig. 1. One of the pictures (a) was a tele-

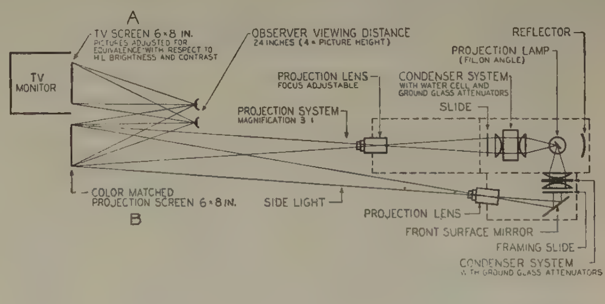

Fig. 1—Arrangement of apparatus for comparison tests.

vision image with controlled amounts of echo. The other picture (b) was optically projected with controlled amounts of defocusing. In all other respects, including size, contrast, brightness, and color temperature, the two pictures were made as nearly identical as practicable. The projection optics are similar in principle to those described by Baldwin.[1] In order to match contrast, an adjustable fraction of the light from the side of the projection lamp was directed, by reflection, approximately uniformly over the area of the screen.

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[1] M. W. Baldwin, Jr., "The subjective sharpness of simulated television images," *Bell Sys. Tech. Jour.*, vol. 19, pp. 563–587; October, 1940.

By this procedure the impairing effect of a known amount of echo in a television picture is compared to that of a sharpness degradation in the projected picture. An illustration of such data is given in Fig. 2. The

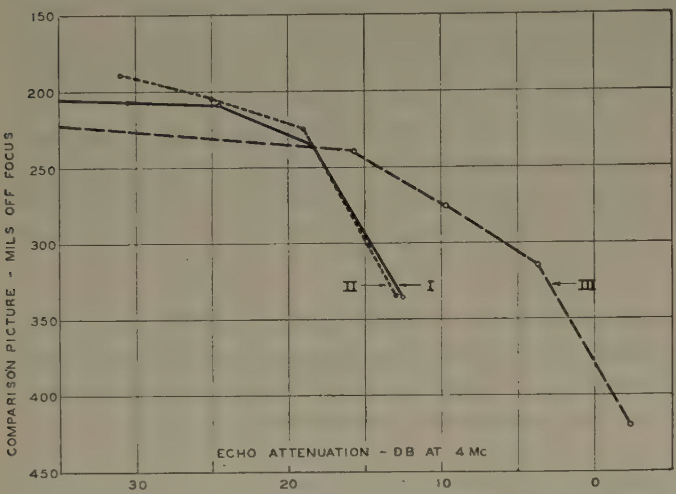


Fig. 2—Measure of echo impairment by defocusing of comparison picture, "Teacup Lady" slide. I—Undistorted echo, delayed 2 microseconds. II—Undistorted echo, delayed 12 microseconds. III—"Partially differentiated" echo, delayed 2 microseconds.

points plotted are those for which 50 per cent of the observations favored the television picture with its echo, and 50 per cent favored the optical picture out of focus by the amount indicated. The abscissae, "Echo Attenuation—DB," denote the number of decibels by which the 4-Mc picture signal intensity from the echo path was lower than the picture signal intensity from the main transmission path. The attenuation frequency distortions in the echo path for the indicated test conditions are shown in Fig. 3. The ordinates of Fig. 2 de-

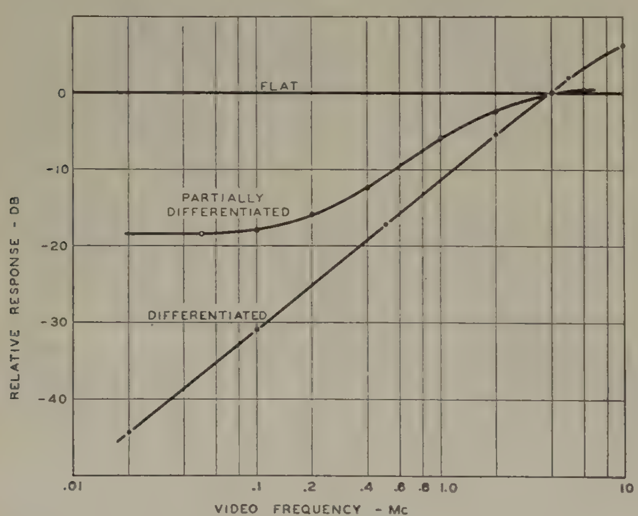


Fig. 3—Attenuation-frequency distortion of echoes used.

note the distance in thousandths of an inch, or mils, by which the projection lens was off focus, and indicate the degradation in sharpness of the projected picture.

Data were obtained for both "positive" and "negative" echoes. Positive echoes are such that a white picture feature leads to a white echo, and a black picture feature to a black echo. Negative echoes are the reverse. The data for positive and negative echoes show no marked differences, and, therefore, have been pooled in all of the experiments described here.

By analyzing the distribution of the vote resulting in the median curves of Fig. 2, it is possible to evaluate how much the preference amounts to for any given comparison. This follows a method used by the psychologists and applied by Baldwin[1] to measure the subjective appreciation of sharpness in pictures. It can also be used as the basis of a system of rating picture quality.

The vote analysis, in brief, consists merely of setting as one "limen" the difference between two pictures where 75 per cent of the observers prefer the one to the other. The vote distribution is found in practice to follow approximately the normal error law, so that the difference becomes two limens where the preference vote is about 91.1 per cent, and three limens where it is about 97.8 per cent. The difference in quality between two pictures of a pair in this system of rating is measured by the number of "liminal units" computed from the preference vote.

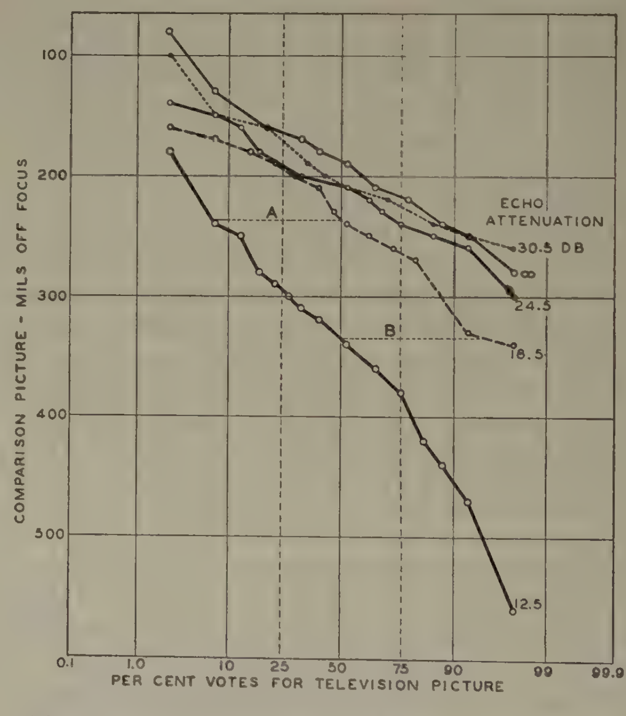


Fig. 4—Distribution of preferences between television and comparison pictures—Case I of Fig. 2.

The distributions of votes from which the median points were taken for curve I in Fig. 2 are shown in Fig. 4. Each curve in Fig. 4 represents the distribution for a given attenuation of echo with respect to the main picture. This plot is used to compute the differences in liminal unit rating between two television pictures having differing echo amplitudes. At A, television pictures with echoes respectively 12.5 and 18.5 db down were

both compared with projected pictures 235 mils off focus. The preference vote in the first case was 7 per cent, and in the second case 50 per cent. Assuming a normal law, this gives a difference of 2.2 liminal units. The same television pictures were both compared at B with projected pictures 334 mils off focus, with preference votes respectively of 50 and 95 per cent, giving a difference of 2.47 liminal units. An average difference of $2\frac{1}{3}$ liminal units is therefore deduced between the impairment for an echo 12.5 db down and one 18.5 db down.

When the difference in impairment is large, e.g., between an unimpaired picture (infinite attenuation of echo) and one impaired by an echo only 12.5 db below the main picture, it is preferable to take the sum of the successive differences between intermediate impairments, rather than the total difference in a single step. This was done using the distribution curves of Fig. 4 for both increasing and decreasing impairments, as outlined in the previous paragraph. The resulting two sets of values of liminal units are plotted as small circles in Fig. 5. The averages of the two sets of values are represented by the heavy solid line (curve I). Similarly, the results for the other echo conditions are shown as heavy curves (II and III) in the same figure.

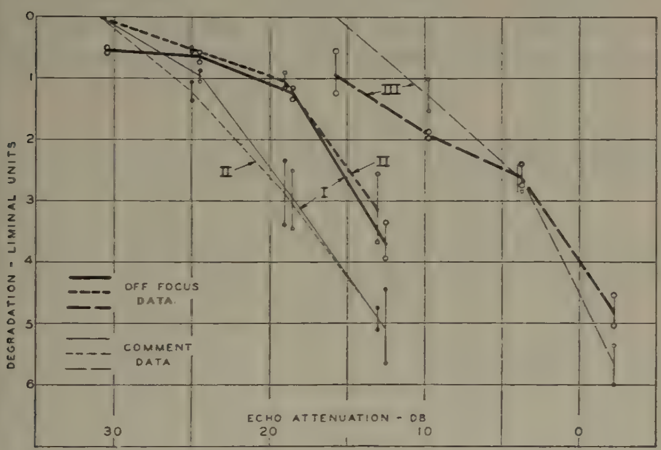


Fig. 5—Measure of echo impairment in liminal units. Data of Figs. 2 and 6.

III. Comment Technique

The second technique which has been explored was also derived from the psychologists, though some changes (which will be discussed below) have been made for its application here. It consists in presenting to the observer a picture affected by differing and controlled amounts of the given impairment, in irregular sequence. The observer is given a list of comments, and asked to specify which comment most nearly characterizes his judgment regarding the impairment to the picture. The comments which have been used are

1. Not perceptible
2. Just perceptible
3. Definitely perceptible, but only slight impairment to picture
4. Impairment to picture but not objectionable
5. Somewhat objectionable
6. Definitely objectionable
7. Not usable.

This technique has been applied to rating the seriousness of the effect of a greater variety of impairments than the first, or comparison technique. To permit of a ready comparison between the two techniques, there will first be described the results of a series of experiments performed at the same time, and on the same impairments, as those used to illustrate the first technique. These results, for the same three echo cases as before, are plotted as a set of curves in Fig. 6. The curves, as before, represent median results. A typical distribution for Case I (paralleling that for mils off focus of Fig. 4) is shown in Fig. 7. From this distribution it is also possible to evaluate the ratings in liminal units, exactly as was done for Fig. 4. This was carried out for the three

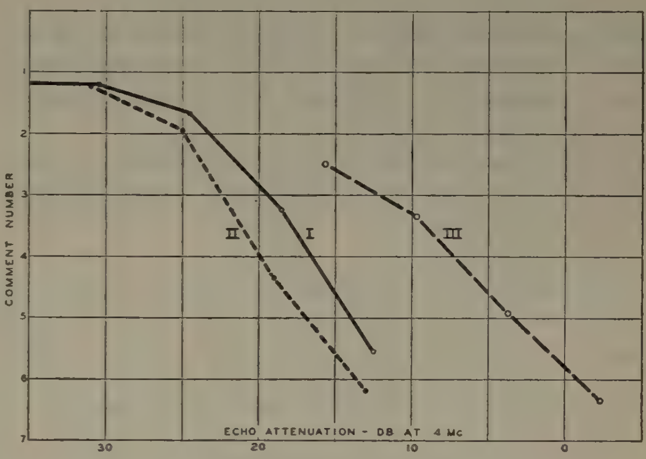


Fig. 6—Measure of echo impairment by comment number. Same subject matter as Fig. 2.

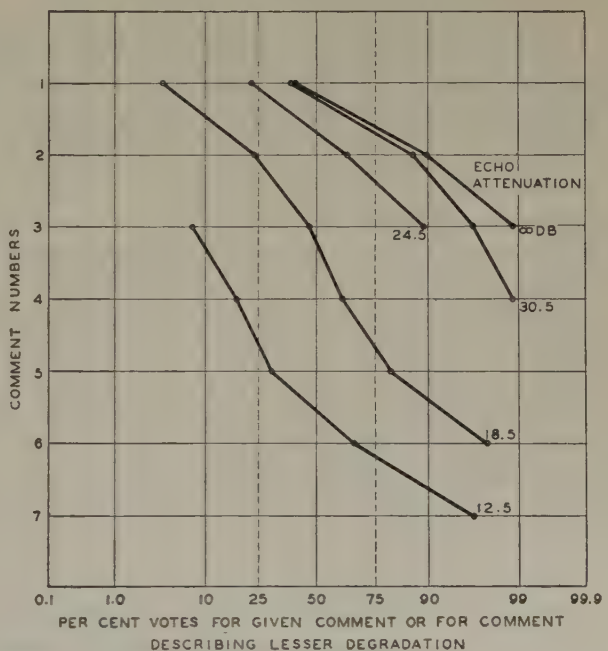


Fig. 7—Distribution of comment votes—Case I of Fig. 6.

cases, and the cumulative impairments obtained thereby have been plotted as the fine lines in Fig. 5. The comparison between the ratings determined by these various procedures will be discussed in the next section.

In addition to the above, a considerable number of other data were taken on different subject material. The data on five motion picture film subjects have been consolidated, and are depicted in a somewhat different manner in Fig. 8. The co-ordinates here are like those of Fig. 6, but the points shown represent the original observations, without any averaging or other processing. The area of each point, as designated in the legend, is used to indicate the number of individual multiplicate observations falling at the same location.

If the observations are cumulated, from those indicating the minimum perception of degradation to those indicating the maximum perception of degradation, a solid figure is built upon the co-ordinates of Fig. 8. A perspective sketch of the nature of this solid figure when smoothed, is shown in Fig. 9. Contour lines are dotted in to indicate various cumulated proportions of observations in the progression from lenient to exacting demands.

In the process of smoothing the data of Fig. 8 auxiliary plots are made, shown in Figs. 10 and 11. These,

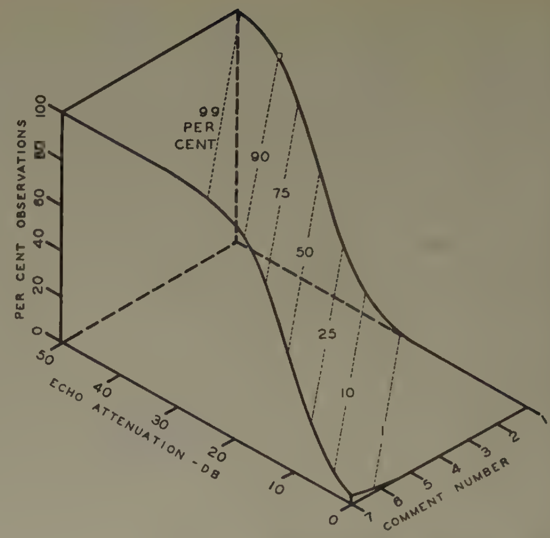


Fig. 9—Three-dimensional schematic plot of array of Fig. 8.

when individually smoothed, can then be conceived of as sections of Fig. 9 parallel to the comment number axis or to the echo attenuation axis. The first are shown in Fig. 10 and the second in Fig. 11. The common experience is that averaging by either set of sections alone leads to lines of regression, which are in general different

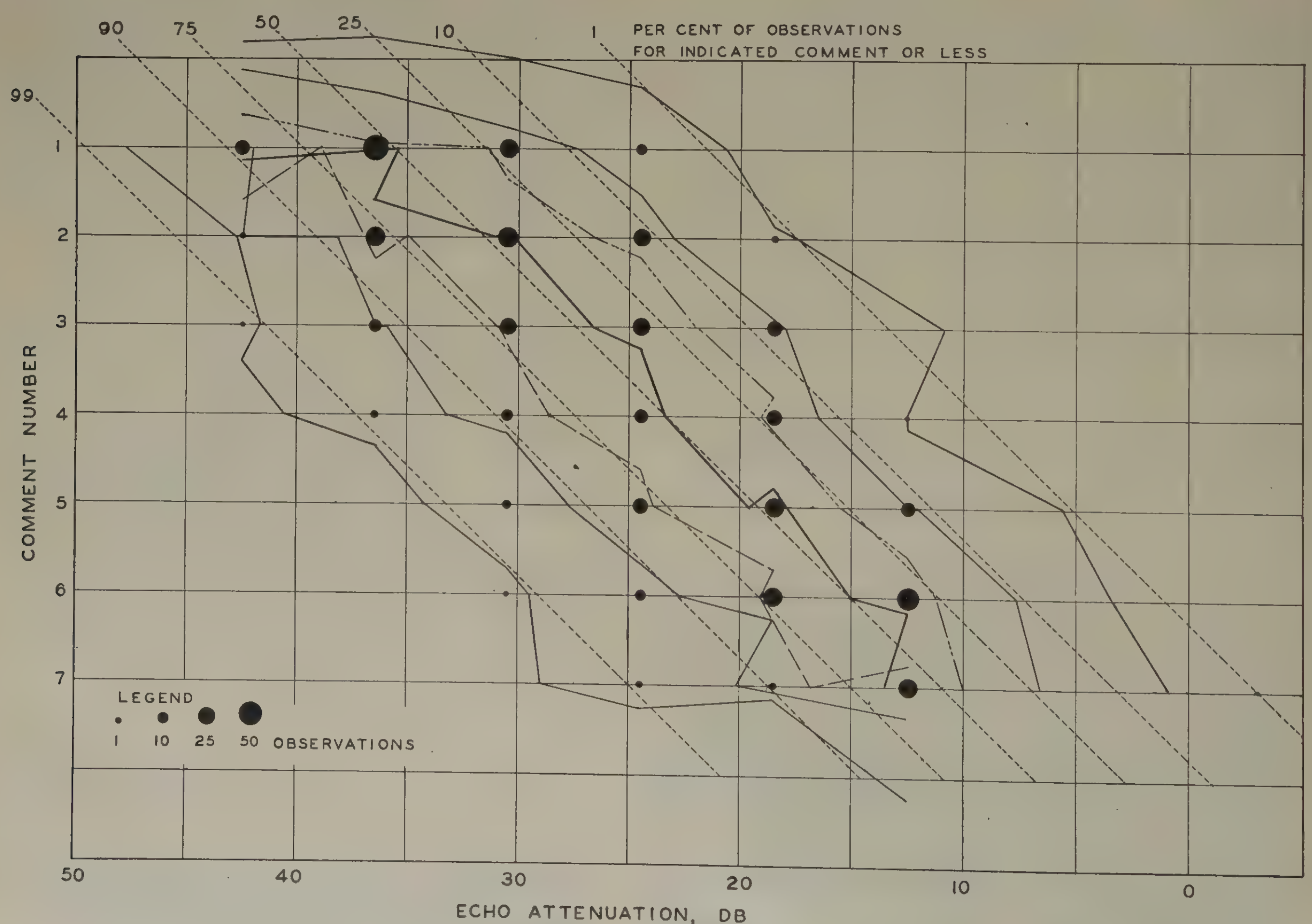


Fig. 8—Array of comment observations. Pool of 5 film subjects listed in Fig. 12. Undistorted echo delayed 2 microseconds.

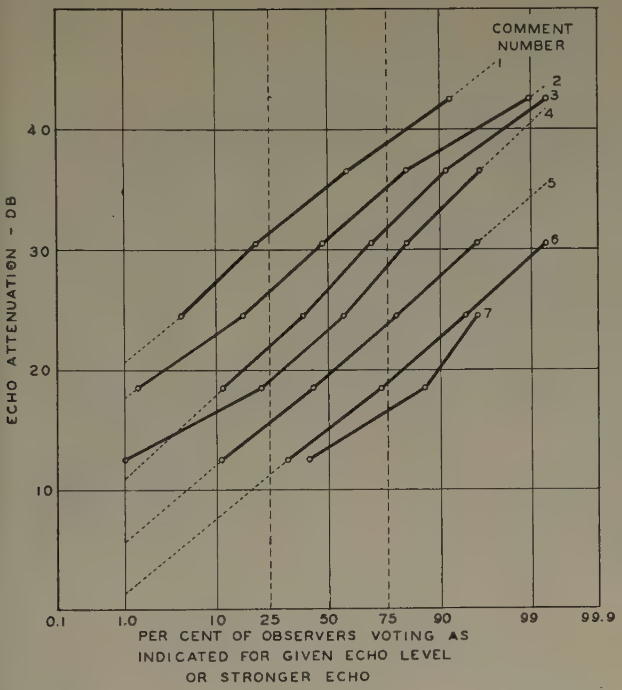


Fig. 10—Cross section of array of Fig. 8 at comment number contours.

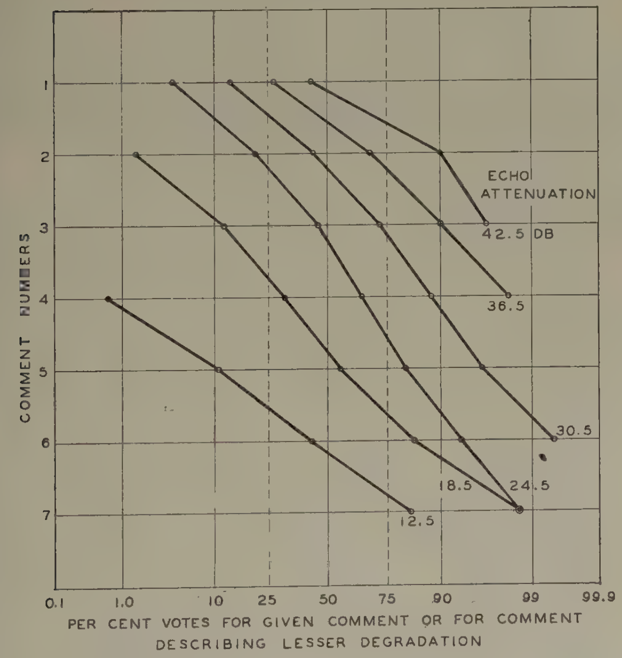


Fig. 11—Cross section of array of Fig. 8 at echo level contours.

for the two.[2] To avoid a bias it is therefore desirable to smooth in both ways and use both sets of results in plotting the final curve. This has been done in plotting the broken lines in Fig. 8, which indicate the contour lines corresponding to the various cumulated percentages of observations. These broken curves are not reliable, of course, near the extreme ends of the comment number scale, because no better comment rating can be given than "not perceptible," and none was supplied worse than "not usable." The dotted lines in Fig. 8 have then been drawn in as expressing about the best trend of the broken lines, except near the ends of the comment number scale. These represent the projections upon the horizontal plane (comment number versus echo attenuation) of the corresponding dotted contour line, of Fig. 9.

As stated above, the data of Figs. 8, 10, and 11 are taken from consolidated tests on five motion pictures. The smoothed 50 per cent curves for the individual film subjects (whose general nature is indicated by the captions) are shown in Fig. 12, and there compared with the curve for the "Teacup Lady" lantern slide subject which was plotted as curve I in Fig. 6. It will be noted from Fig. 12 that there is a range of over 10 db in the susceptibilities of the various subjects to echo, for a given rating of impairment, and that this range is somewhat greater for slight than for serious impairments. The consolidated (or "average") curve is not far from the center of the range, i.e., about 5 db more lenient than for the most susceptible picture. It is also to be observed that the "Teacup Lady" slide, which was used for the common data to compare the two broad procedures, turns out to be a subject which is about the least susceptible to echo impairment of the whole group. The curves are shown here as curves and not as their best straight-line approximations, as was done in Fig. 8.

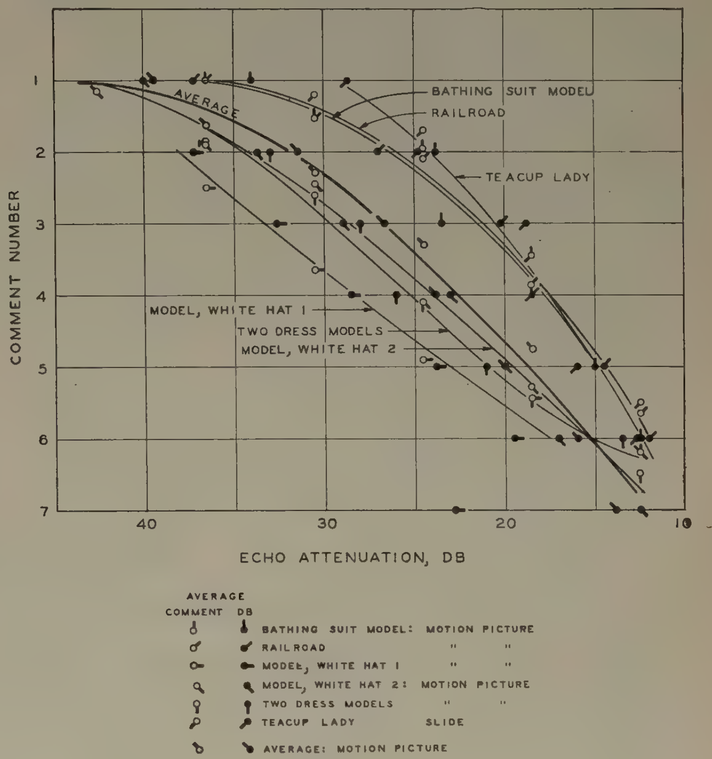


Fig. 12—Median distributions of individual subjects and average of Fig. 8, compared with same for "Teacup Lady" slide. Undistorted echo delayed 2 microseconds from cross-sectional plots like Figs. 10 and 11.

[2] H. L. Rietz, "Handbook of Mathematical Statistics," p. 126, Houghton Mifflin Company, New York, N. Y.; 1924.

Data, similar to that of Figs. 8 and 12, have also been obtained for an undistorted echo delayed 12 microseconds, and are shown in Figs. 13 and 14. In Fig. 13 it is observed that the impairment rating follows the same course as in Fig. 8, but that echo amplitudes for the same rating are about 5 db lower (more severe judgment). This is shown again in the comparison between Figs. 14 and 12, where it is also seen that the spread between picture subjects is larger (namely, about 15 db) than for the echoes delayed 2 microseconds.

Data on additional types of echoes were taken for two subject pictures, and are illustrated in Figs. 15 and 16, respectively. The first is with the "Teacup Lady" lantern slide subject used for Fig. 6, and the second with one of the more sensitive film subjects of Figs. 12 and 14 ("Model with White Hat 2").

As was the case with the echoes discussed in connection with Fig. 2, the distortion of the echo picture corresponds to the transmission of its signal through a path having one or the other of the attenuation characteristics illustrated in Fig. 3. The "differentiated" echo is one equivalent to passing the signal through a path having a response rising with frequency at the rate of 6 db per octave over the major part of the transmitted band. For "partial differentiation" the rise is somewhat less fast than this, and not uniform.

It is to be observed from Figs. 15 and 16, first, that the impairing effect of the undistorted ("flat") echo increases with its delay (as has already been noted);

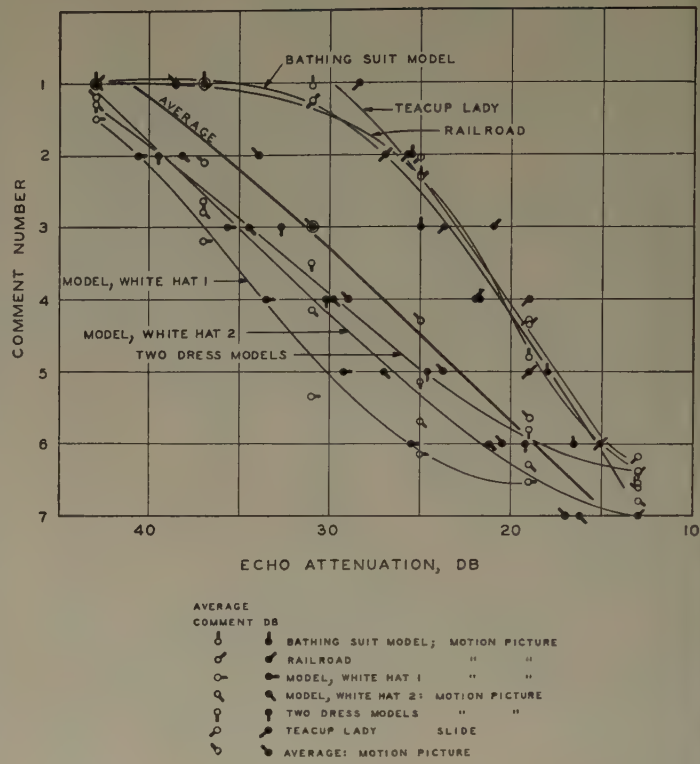


Fig. 14—Median distributions of individual subjects and average of Fig. 13, compared with same for "Teacup Lady" slide. Undistorted echo delayed 12 microseconds. From cross-sectional plots like Figs. 10 and 11.

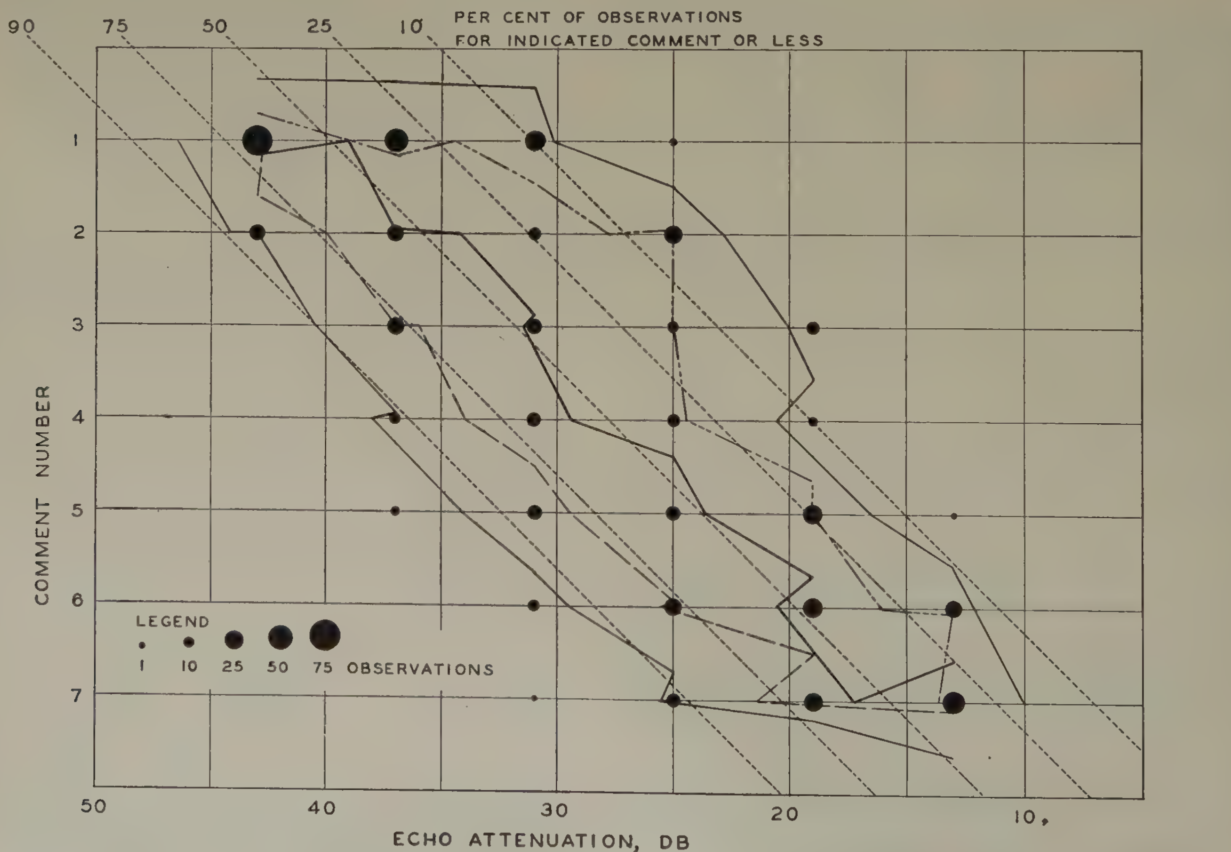


Fig. 13—Array of comment observations. Pool of 5 film subjects listed in Fig. 14. Undistorted echo delayed 12 microseconds.

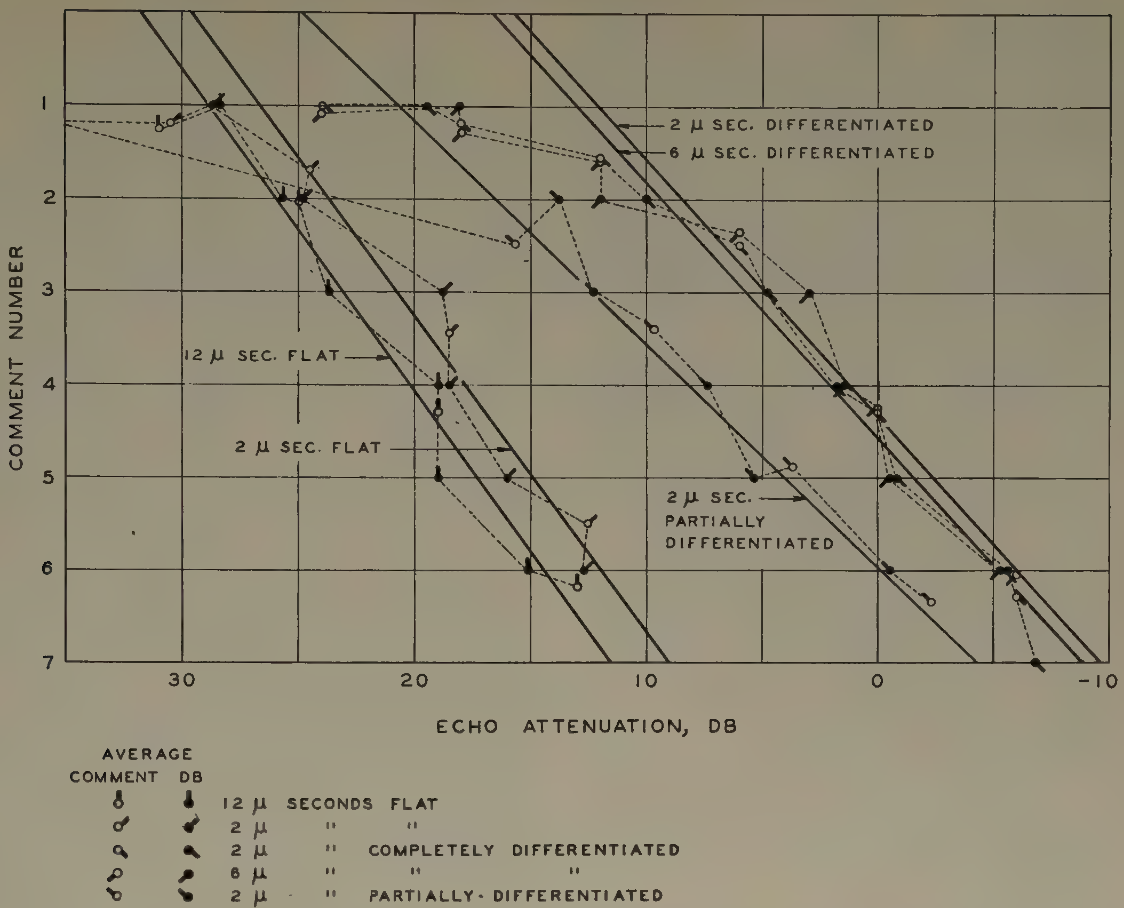


Fig. 15—Median distributions on "Teacup Lady" slide, variety of echoes as shown, from cross-sectional plots like Figs. 10 and 11.

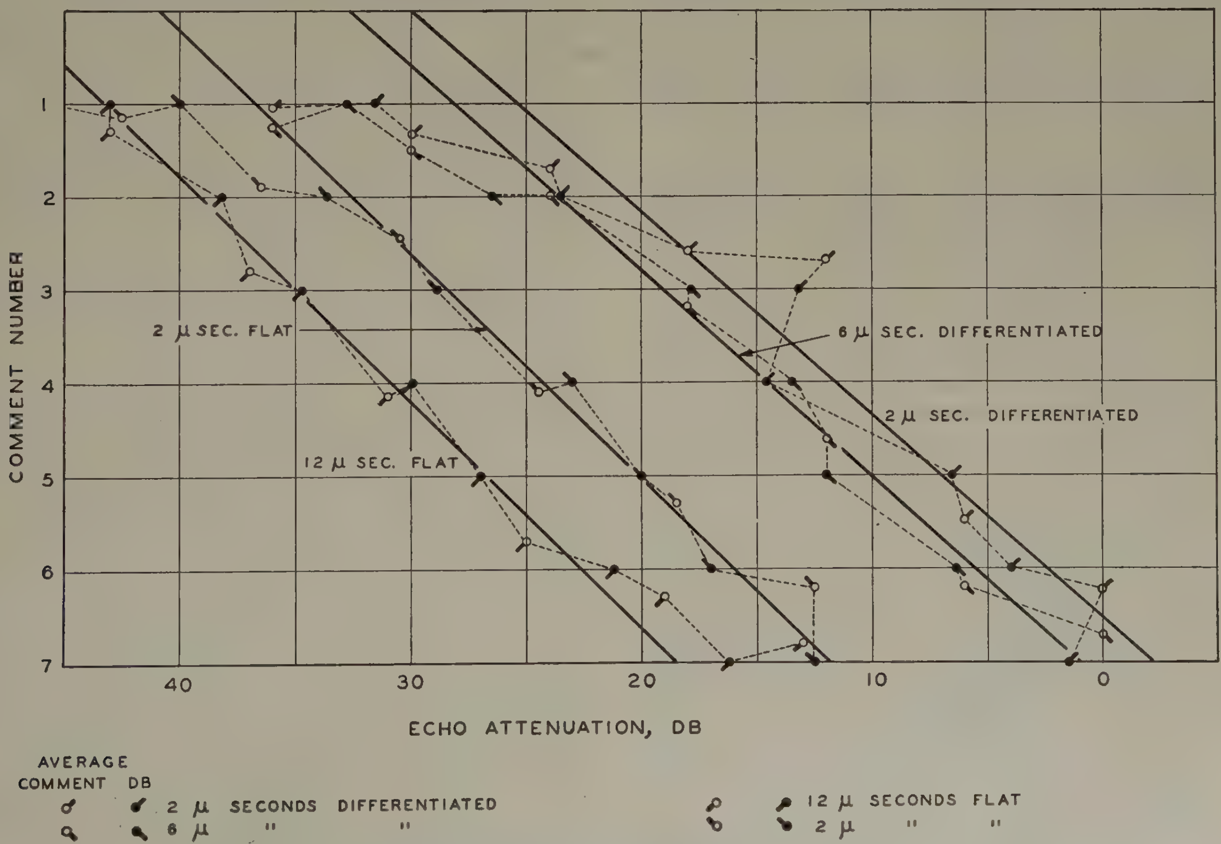


Fig. 16—Median distributions on "Model with White Hat, series 2" film, variety of echoes as shown, from cross-sectional plots like Figs. 10 and 11.

second, that distorting the echo reduces the impairing effect (by some 15 to 20 db for "differentiation," and somewhat less than this for "partial differentiation"); and third, that the effect of differences in delay is less clear for the distorted echoes.

For engineering use of the data, given for example in Fig. 8, it is preferable to simplify this plot and present it on "probability" paper instead of drawing in the normal distribution scale. For that purpose Fig. 8 would be converted to the plot of Fig. 17.

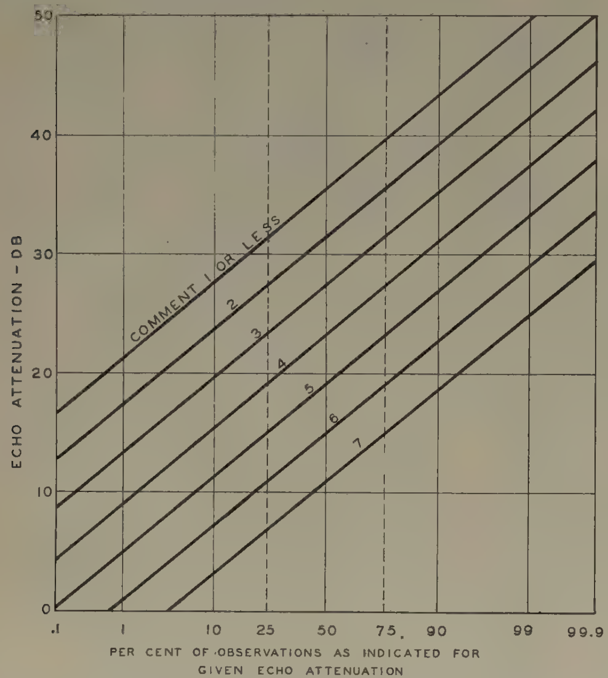


Fig. 17—Characterization of 2-microsecond delayed echoes on pool of 5 film subjects. Processed data of Fig. 8 presented for engineering use.

The technique making use of pre-worded comments has also been applied to rating pictures impaired by random noise having a particular power frequency distribution. The distribution studied is one which is expected in some hypothetical coaxial cable system designs, and is plotted as "unweighted noise" in Fig. 18. The scale of ordinates in that figure represents specifically the ratio, expressed in decibels, of the watts per 100 kc to the watts in the entire band, in the electrical signal. It has been shown[3,4] that in the perception of such noise the eye is less sensitive to the higher frequencies in this noise than it is to the lower frequencies. A weighting function mentioned in the literature,[3] corresponding to the viewing of a standard television picture from a distance equal to 4 times its height, has been assumed in evaluating the electrical noise. It, and the distribution of the noise after this weighting, are also shown in Fig. 18. The weighted noise represents a power 6.9 db below the unweighted noise. For a "flat" distribution of noise

[3] P. Mertz, "Perception of television random noise," *Jour. Soc. Mot. Pic. and Telev. Eng.*, vol. 54, pp. 8–34; January, 1950.

[4] O. H. Schade, "Electro-optical characteristics of television," *RCA Rev.*, vol. 9, Parts I–IV, pp. 5, 245, 490 and 653; March, June, September, and December, 1948.

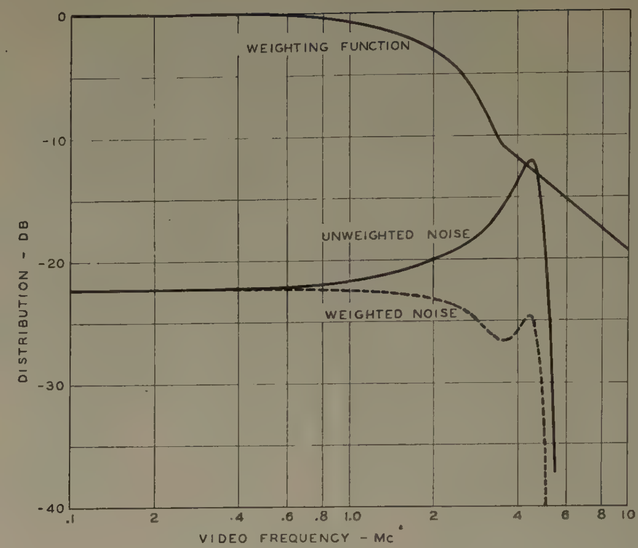


Fig. 18—Random noise distributions and weighting function used for tests.

the weighting would represent a power reduction of 2.2 db from the unweighted noise.

The data on impairments caused by noise have been plotted, in a manner similar to Figs. 8 and 13, in Fig. 19. These cover the consolidated results on three slides (the "Teacup Lady," a white vase and a two-character scene from a play). The abscissae represent the ratio of the peak-to-peak amplitude of a standard video signal including synchronizing pulses to the rms amplitude of the weighted noise, as measured in decibels.

Observation of the broken lines in Fig. 19 shows the usual erratic behavior at the extreme comment numbers, but aside from this there is a distinct curvature downwards towards low values of noise. It had been obvious in setting up the pictures for these tests that there was a low but visible random noise on the pictures before any external noise was applied. If such a constant noise were assumed, then to the nominal noise indicated by the abscissae would be added (on an rss basis) a fixed noise having dominant weight where the nominal noise is low but negligible weight where it is high. This would increase the comment numbers in the first case, but leave them unchanged in the second, and is therefore in the direction indicated by the curvature of the lines. The dotted lines have been drawn in the figure first by assuming straight lines asymptotic to the trend of the broken lines for the higher comment numbers. Then the straight lines have been distorted by finding, for each value of abscissa, a new nominal value which yields the old value when an estimated magnitude of fixed noise is added; and plotting the curves through these points. The fixed noise was adjusted to give the best fit for the 50 per cent curve, and the same value then used for the other percentage curves. The fit is seen to be good enough for the present exploratory work, though there is enough deviation for the extreme percentage curves to warrant reconsideration in a more definitive study.

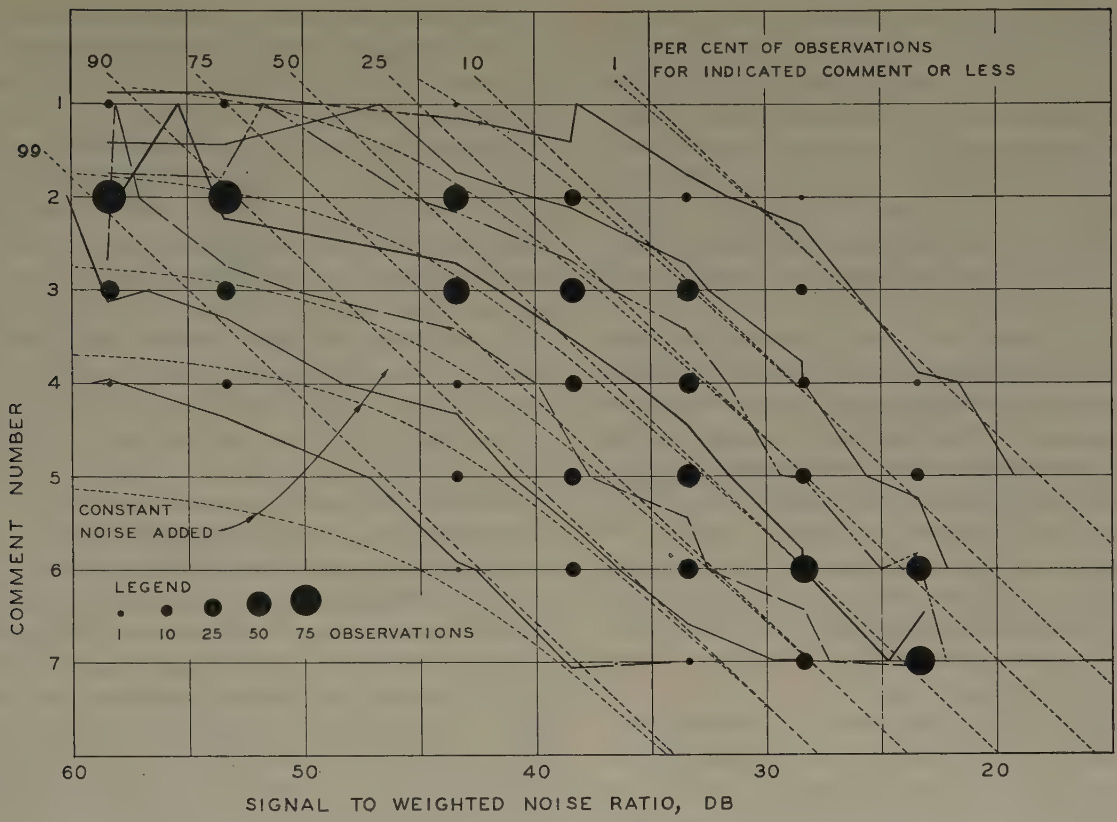


Fig. 19—Array of comment observations. Pool of 3 slide subjects. Random noise distribution of Fig. 18.

IV. Comparison Between Techniques

If systems of rating are to be realistic it is necessary, of course, that they lead approximately to the same rating, irrespective of the system or technique used. It is to check this that a portion of the experiments were performed using the two techniques substantially at the same time. From the plots of Figs. 2 and 6 it is possible to show the relationship between impairments resulting from echoes as measured in mils off focus of a comparison picture, and in comment numbers. This relationship has been plotted in Fig. 20. The curve expressing this relationship should of course be independent of the cases I, II, and III, particularly for comment number 1, for which there is no picture impairment. The curves of Fig. 20, however, do show some variation in this relationship, enough to warrant an explanation. The tests, from which the points were plotted, occupied a period of 10 days. The preliminary adjustments of both the television picture and the projected picture were made with great care to get the first as good as possible, and the second to match the first as nearly as possible. The judgments of several observers were used to set the adjustments. In the subsequent adjustments during the tests, however, only the highlight luminance and contrast ratio were measured several times each day. The collective judgments were not repeated.

It appears in retrospect that this technique was not sufficient to prevent the television picture from becoming degraded over such a period of time. The projected picture, however, retained its original quality. Thus a separate smoothed curve, indicated by the dotted parabolas, has been plotted for each case. If the television picture had been readjusted at frequent enough intervals, the curves would obviously have coincided at comment number 1. The plot of the smoothed curves shows that there would probably have then been reasonably good coincidence for the other comment numbers.

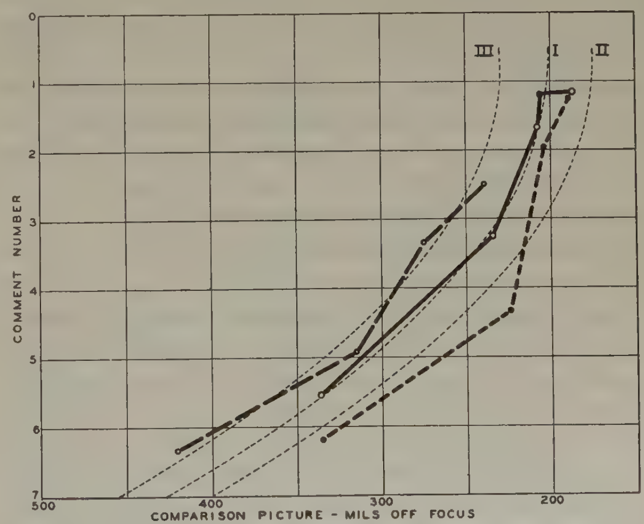


Fig. 20—Relation between defocusing and comment numbers. Subject and cases of Figs. 2 and 6.

The correlation of the image ratings among the various techniques and methods of deriving the results, as exemplified by Figs. 2, 5, and 6, is seen to run from fair to extremely good. Some of the uncertainties in the correlation, for example in the evaluation of the relative susceptibilities to echoes in cases I, II, and III, show evidence of the effect of the limited number of observations. Where more observations were taken, as illustrated in Figs. 15 and 16, the relative susceptibilities appear much more distinctly.

In a relative evaluation of the techniques, the comparison method is found to be more difficult for the experimenter to set up, and it requires more observations to obtain a significant result than the comment method. However, the observations are much simpler and easier to make, requiring only an indication of picture preference, instead of an evaluation in terms of words. The observer judgments are more nearly independent of previous training. They are also more absolute, as was indicated in the discussion of Fig. 20, because a projected picture is more reproducible, in the present state of the art, than a television picture.

At this point it is appropriate to consider some criticism which has been made of the language of the comments used in the second technique. In the usual technique employed by the psychologists, some aspect or dimension of the picture quality is specified, and the observer is asked to report in terms of predetermined steps or categories of the dimension. In the present case there are four dimensions, i.e., perceptibility, impairment, objectionableness, and utility. The first is given three steps, the second two, the third three, and the fourth one, and some of the steps of one dimension are identified with other steps of another. The objection is made that the observer may be confused by this, and rank the picture in a single dimension of his own and following the ordered comment numbers.

There is some validity to such an objection, but it is necessary to note that the information on picture impairments is sought over a great range of these, and the observer's interest centers upon the successive aspects of the impairment over the course of this range. The prime objective of the technique is to catch the successive points at which these aspects reach importance, and the few steps are all that is necessary for each aspect for such a determination. The aspects themselves were chosen to reach importance successively.

The criticisms were voiced in general form before the experiments reported were carried out. In view of the exploratory character of the experiments, it was decided to carry them out with the comment wordings as prepared. To thwart the observer as far as possible in setting up an arbitrary numerical rating of his own, which might differ from observer to observer, the successive objective degradations in the picture were presented in irregular order, and sometimes two different types of degradations were interleaved in irregular order.

The general experience with the comment wordings chosen, during the course of the experiments, was quite favorable. There seemed to be little if any confusion in the observer's minds as to the meaning to them of the comments, and it is guessed that there was far more uniformity in their interpretations than is usually attached to the words "just tolerable," which have been conventional in the past. There was still some suspicion that the observers were constructing scales of their own, but there was repeated evidence that the great majority of the observers were using the comment scale, in that the observers made numerous references to the sheet of paper on which the comments were written and frequently would audibly read the words of two or more of the numbered comments before deciding which comment suited the particular impairment.

When the comment wordings were chosen, it was not known whether they would be spaced uniformly on a general quality scale, nor was this an objective, although it has desirable aspects. The spacing of the comments in liminal units can be determined directly from the vote distributions on the comment numbers. For example, in Fig. 7 it is possible to measure off the number of comment spacings between the 25 and 75 per cent votes (corresponding to 2 liminal units), divide this by 2, and plot the result against the comment number at the 50 per cent vote. This has been done for the cases I, II, and III that were plotted in Fig. 6. and the results are

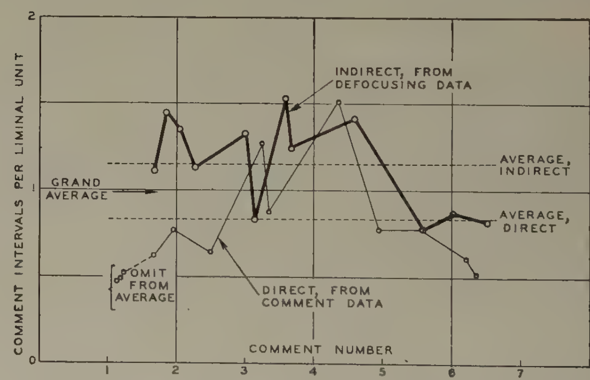


Fig. 21—Liminal differences measured in comment intervals from data on subject and cases of Figs. 2 and 6.

shown by the fine lines in Fig. 21. A few values very close to comment 1 undoubtedly are affected by the saturation near the extremes of the comment scale, and have therefore been omitted from an average of the data.

Similarly the mils off focus spacings as recorded in Fig. 4 between the 25 and 75 per cent votes on the defocused picture have been computed. These have been converted to comment number spacings per liminal unit by using the smoothed curves correlating the two in Fig. 20, and divided by two. The results form the indirect data connected by the heavy lines in Fig. 21, and these were also averaged.

Examination of Fig. 21 shows that the comment interval spacings per liminal unit are systematically some-

what greater for the indirect than for the direct data. It also shows that while the data are a bit scant, there is no strong evidence for variation in the spacing as a function of comment level except near the very ends of the comment number scale.

Omitting the points near comment number 1, the grand average of Fig. 21 indicates a comment number spacing of about one liminal unit. The additional data, reported on Figs. 8, 13, and 19, as smoothed, indicate a constant averaged value for each figure. The comparison of all these is about as follows:

Fig. 2 (Cases I, II, and II, indirect)	1.15
Fig. 6 (Cases I, II, and II, direct)	.83
Fig. 8	1.00
Fig. 13	1.155
Fig. 19	1.00
Average	1.03

The broad conclusion, therefore, is that the comment spacings, at least as used by the observers, are uniform, and are about one liminal unit.

V. Comparisons Between Picture Quality Parameters

The comparison technique has also been used to compare sharpness or definition as a quality parameter of the picture with the other quality parameters.

Two pictures have been exhibited to the observer side by side, say, A and B. Picture A has a given definition and contrast ratio. Picture B has, say, a lower contrast range and its definition is varied over a range of values, all higher than for A. The observer is asked, for each value, presented in irregular sequence, to choose his preference for A or B. For some value of definition the vote is 50-50. Picture B at this sharpness and its contrast is then equally chosen with picture A at a higher contrast but lower definition. A line can then be drawn joining these two points on a graph, as in Fig. 22. This then can represent a segment of a curve of presumed equal quality, which could be plotted on the graph. By taking other pairs of pictures, other such segments can be drawn. This would permit sketching in the graph a succession of contours of presumed equal quality. From the 25 and 75 per cent votes the quality spacing between the contours can be estimated in liminal units. These are indicated in the figure by light lines. The contours have not, however, been drawn in the plot. In plotting the curves the mils off focus have been squared to give a quantity approximately following frequency bandwidth. They have also been plotted from right to left, so that increasing sharpness, as bandwidth, goes from left to right.

In Fig. 23 a similar comparison has been plotted between contrast and highlight luminance, and in Fig. 24 between sharpness and highlight luminance.

The three quality parameters can be presented in a unified composite graph having three dimensions. The

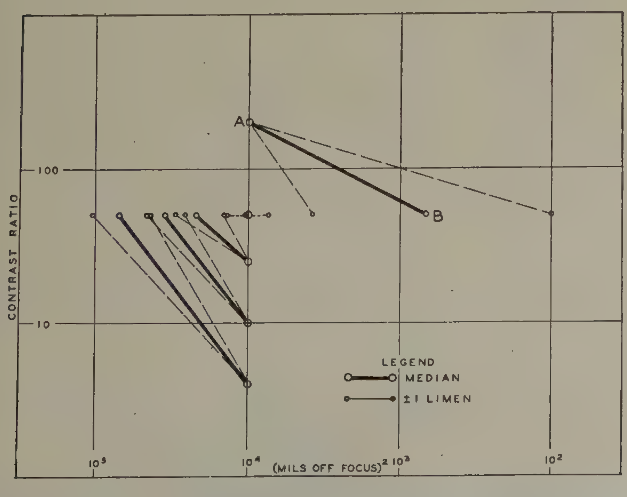


Fig. 22—Contrast versus definition. "Swan" slide at 27 millilamberts highlight luminance.

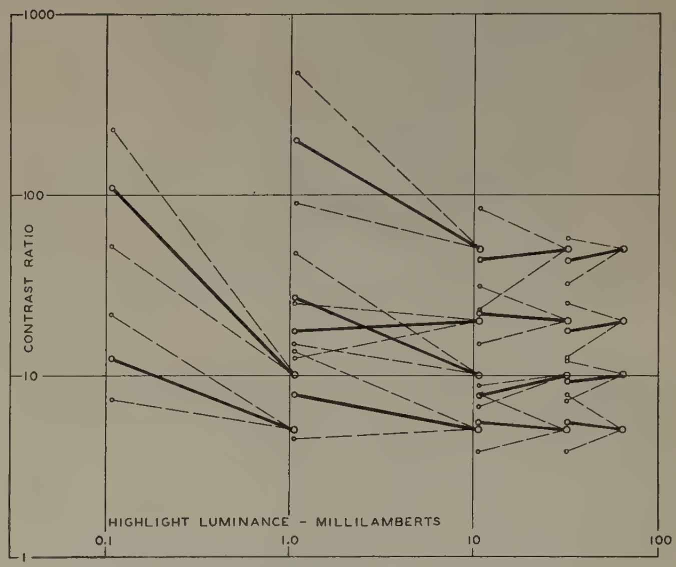


Fig. 23—Contrast versus highlight luminance. "Swan" slide, 80 mils off focus.

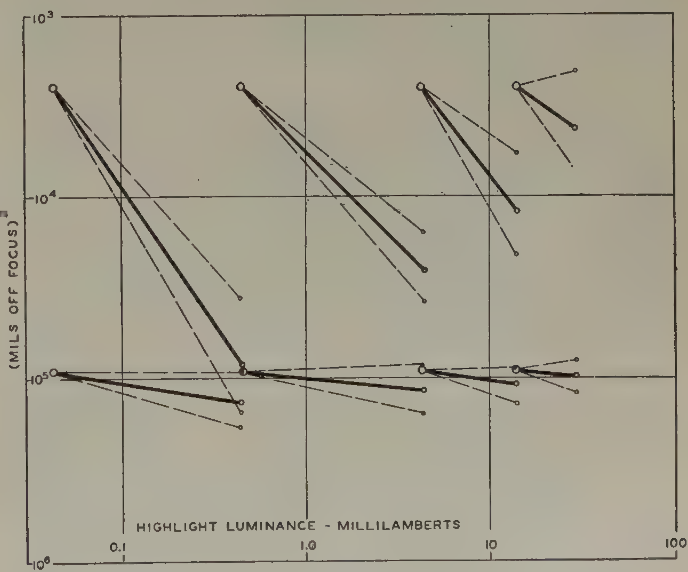


Fig. 24—Sharpness versus highlight luminance. "Dinner table" slide at 50:1 contrast ratio.

three planes of Figs. 22, 23, and 24 have been shown as intersecting to form this graph in Fig. 25. In such a figure the locus of pictures of presumed equal quality is a curved surface, and its intersection with the three planes forms a contour curve on each of the planes. In Fig. 25 one such contour curve has been sketched in for each of the three planes, and some idea of the curved surface may be derived from the perspective drawing. Other curved surfaces may be sketched, differing from that shown by any given number of liminal units. These have not been shown, however, in order to avoid complicating the drawing.

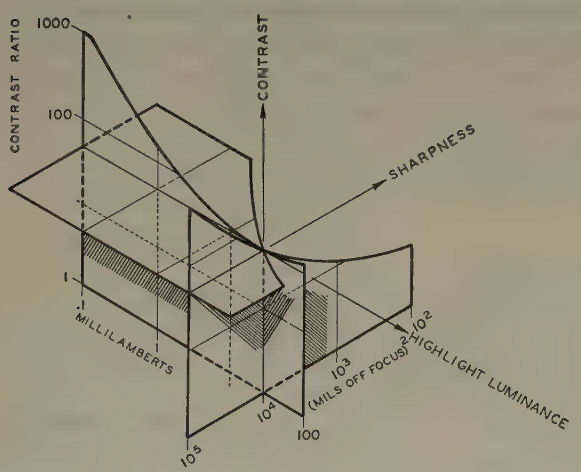


Fig. 25—Solid of picture quality parameters.

VI. Appendix

The defocused projected picture has been used extensively in these tests as an illustration of a picture of reduced sharpness that permits comparison with a television picture of limited frequency bandwidth. As part of the test, a calibration of the bandwidth corresponding to the defocused picture was carried out, as a function of the defocusing.

The calibration could of course have been carried out by comparing an actual television picture of known frequency bandwidth with the defocused picture. It was desired, however, to project the comparison to a picture of more perfect scanning spot structure than at present available on television screens. The telephotograph system[5] developed some years ago gives an excellent and almost perfect structure of conventional type. Consequently several pictures of various subject material were made into projection lantern slides and also transmitted locally over a telephotograph system of known band-pass characteristics (see Fig. 26—the influence of the transmitting and receiving scanning apertures has been estimated) and made into positive transparencies. By

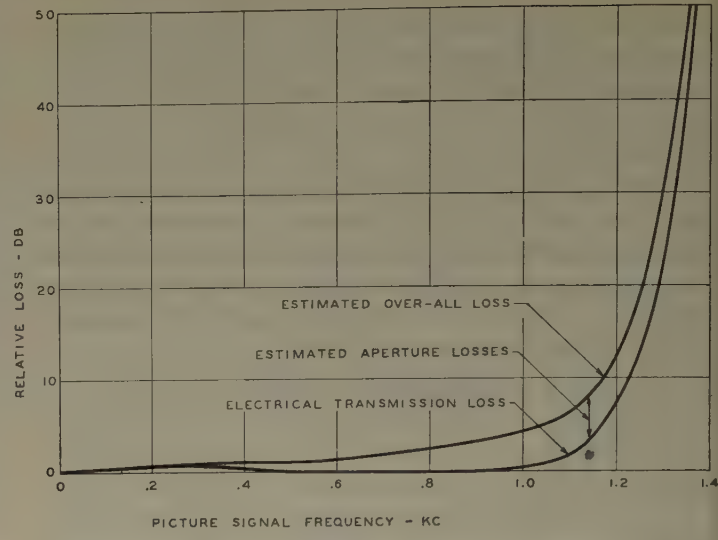


Fig. 26—Over-all band-pass characteristics, telephotograph system.

transmitting the pictures at varying amounts of magnification varying numbers of scanning lines could be obtained.

The transparencies (suitably illuminated from the rear) replaced the television picture in Fig. 1 and were compared with the defocused projections. The results of these comparisons are shown in Fig. 27. The ordinates here are expressed in terms of a normalized variable αg which characterizes defocusing under varied projection conditions. The quantity g is the actual lens displacement off focus in mils. The factor α is defined by

$$\alpha = \pi(m - 1)/(2FH) \tag{1}$$

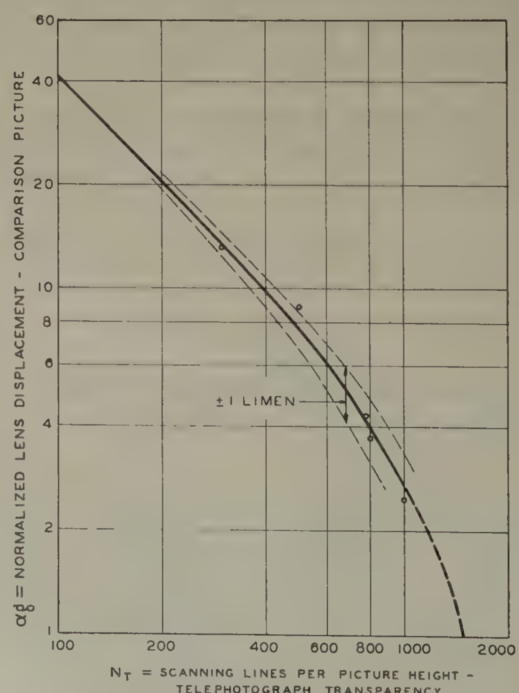


Fig. 27—Subjective correlation of telephotographed transparencies with off-focus projections.

[5] F. W. Reynolds, "A new telephotograph system," *Bell Sys. Tech. Jour.*, vol. 15, p. 549; October, 1936.

where

m = in-focus magnification

F = ratio of focal length to diameter of aperture of projection lens

H = height of picture in inches.

With α as above, and g in mils, the variable αg comes out in convenient numbers, which are 1,000 times what they would be if g were expressed in inches.

The points shown represent the averages of the observations. A smoothed curve has been drawn, with a doubtful dotted extrapolation. The 25 and 75 per cent votes have also been represented by smoothed dotted lines.

The frequency conversion from the telephotograph to the television bands is accomplished by multiplying the frequencies in Fig. 26 by the ratio of the scanning speeds for the assumed television systems to that for the telephotograph systems. Each telephotograph transparency has a number of scanning lines in height, designated as N_T. The scanning is done on the machine itself at 100 lines to the inch, and at 20 inches per second, consequently covers a horizontal distance along a scanning line equal to N_T scanning line widths in 0.0005 N_T seconds. In the corresponding television systems it is assumed that all of them have a frame frequency of 30 per second, and horizontal and vertical blanking intervals of 17.25 and 6.5 per cent, respectively (present standards). Then the time required to cover N_T scanning line widths, in a horizontal direction, is $0.774/(40N_T)$ seconds. The ratio of the two speeds is consequently $0.02584N_T^2$.

The "bandwidth" of a transmitted signal is, of course, an elastic concept, depending upon the purposes for which the quantity is to be used. For the present it will be assumed as the frequency band up to the 45-db cutoff, including electrical and aperture losses. This gives, in Fig. 26, a "bandwidth" of 1.345 kc, and, for varying values of N_T, the equivalent bandwidths indicated in Fig. 28. Translating the scanning line numbers of Fig. 27 into frequency bands gives Fig. 29, which gives the calibration of the mils off focus used in Figs. 2 to 4 and 20, for which $\alpha = 0.0654$.

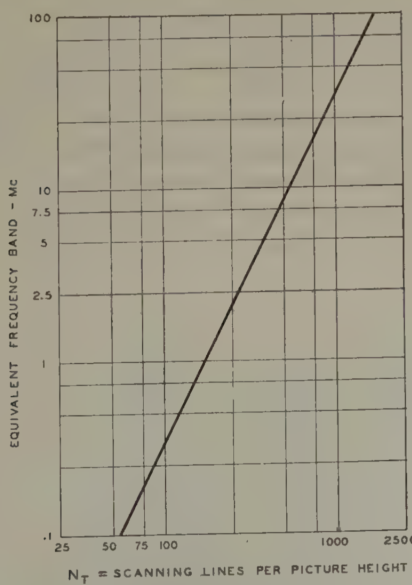


Fig. 28—Equivalent frequency bands for telephotographed transparencies.

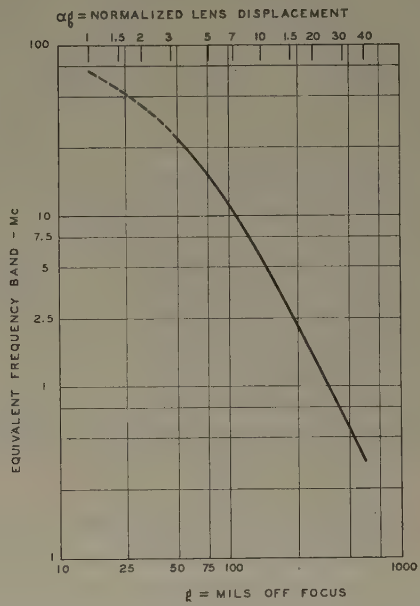


Fig. 29—Calibration of off-focus displacement in terms of equivalent frequency band. $\alpha = 0.0654$.

For many purposes it is desirable that the calibration shall be given in terms of resolving power. This is done by viewing a parallel line test chart and measured in terms of lines just resolved (counting black lines and white lines) in a picture height. For this test it turned out to be more convenient to measure just beyond this, i.e., lines just not resolved. The experimental indications were that the difference in lens displacements satisfying these two criteria is probably less than 2 per cent. These resolving powers for the various lens displacements, for three different projection setups all viewed at 4 times picture height are plotted as the various points in Fig. 30.

A theoretical curve has been derived for the data shown in Fig. 30, briefly as follows. The influence upon resolution of bar patterns of eye limitations, lens limitations, and defocusing is considered as an attenuation or loss varying with the closeness of the lines.[1,3,4] For convenience the losses are measured logarithmically, as $20 \log_{10}\rho$, where ρ is the luminance ratio. Then

$$L(N, \alpha g) = L_1(N) + L_2(N) + L_3(N, \alpha g), \qquad (2)$$

where

L = over-all loss

L_1 = loss due to eye limitation

L_2 = loss due to lens limitation

L_3 = loss due to lens defocusing

N = number of lines in picture height

αg = normalized lens displacement.

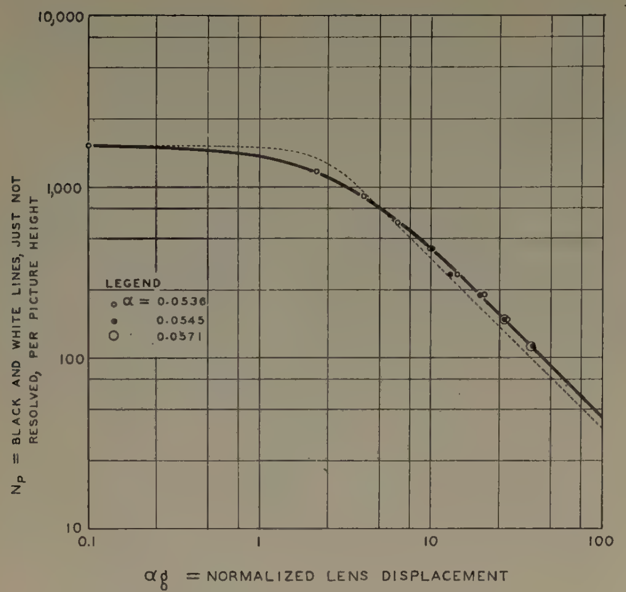


Fig. 30—Subjective correlation of off-focus displacement with test lines just not resolved.

The eye and lens limitations are independent of defocusing, as noted. The over-all loss, near threshold, can be taken as a constant figure (say L_0), which establishes then an implicit functional relation between N and αg determined by (2).

The eye and lens losses are in general approximately parabolic[6] near the origin, but they are exactly parabolic if the distribution in their figures of confusion is Gaussian. This distribution is indicated by experiment to be approximately the case for most lenses. I.e.,

$$L_1(N) + L_2(N) = k_1N^2 + k_2N^2 = k_3N^2. \qquad (3)$$

The proportionality constant can be measured from the resolution at sharp focus N_0 for which $L_3 = 0$.

Hence

$$k_3 = L_0/N_0^2. \qquad (4)$$

The defocusing loss is[6]

$$L_3(N, \alpha g) = 20 \log_{10} [(\alpha g N/1{,}000)/2J_1(\alpha g N/1{,}000)], \qquad (5)$$

where

$J_1(\ )$ = Bessel function of first kind and order 1.

The factor of 1/1,000 in the argument is needed to offset the measure of g in mils as explained under (1).

Thus the implicit equation between N and αg is

$$20 \log_{10} [(\alpha g N/1{,}000)/2J_1(\alpha g N/1{,}000)] = L_0[1 - (N/N_0)^2]. \qquad (6)$$

This is the equation shown by the fine dotted line in Fig. 30, for $L_0 = 45$, and $N_0 = 1{,}750$. The Bessel function is used for arguments between zero (for zero value of g) and its first root (for small values of N). As N approaches zero, the right-hand member of the equation approaches constancy, and therefore the argument of the Bessel function also approaches constancy. Thus g becomes nearly inversely proportional to N. This corresponds to the portion of the curve toward the right of Fig. 30. It is to be noted that this portion is displaced somewhat from the actual points. The exact reason for this displacement is not clear, but it has been noted before[1] that the effective size of an optical figure of confusion tends to run to about 85 per cent of its calculated value from geometrical optics.

The experimental points in Fig. 30 group themselves about a quite smooth curve, indicating the general validity of the normalized variable which has been derived from (1). Because of this the smoothed empirical curve, shown as a solid line in Fig. 30, has been used rather than the theoretical curve, for the application to calibration of the line resolution in effective frequency bandwidth.

This calibration is effected by using the relationship determined in Fig. 29 between the normalized lens displacement and frequency bandwidth to substitute the latter for the former in Fig. 30. This then gives Fig. 31, which is the calibration desired.

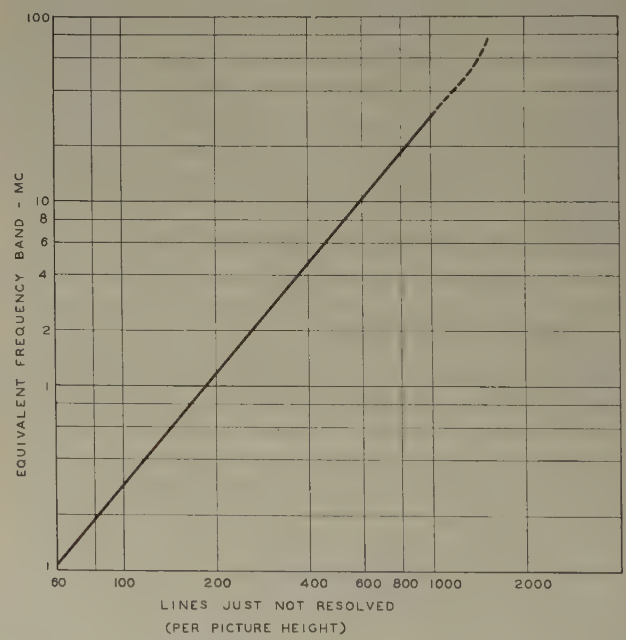


Fig. 31—Calibration of lines just not resolved, in terms of equivalent frequency band.

This calibration, it will be noted, permits an independent determination of the factor[7] which has been referred to as the "Kell factor." This, in a conventionally used television system, is the ratio of the vertical definition, in resolved lines, to the actual number of scanning lines. Calling the first N_p, and the second N_v, (it will not do simply to use N_T from Fig. 27, because this was determined from over-all subjective evaluation

[6] P. Mertz and F. Gray, "Theory of scanning," *Bell Sys. Tech. Jour.*, vol. 13, p. 464; July, 1934.

[7] R. D. Kell, A. V. Bedford, and M. A. Trainer, "An experimental television system," Proc. I.R.E., vol. 22, pp. 1246–1266; November, 1934.

with a different horizontal resolution), the ratio gives the relation

$$N_p = kN_v. \tag{7}$$

The frequency bandwidth required is

$$F = (1/2) \times M \times B \times N_v \times (4/3)N_p, \tag{8}$$

where M is the frame frequency, B is the allowance required for horizontal and vertical blanking (equal to 1/0.774 or 1.292 in present standards, and already made in Fig. 28), and it is assumed that the horizontal resolution is N_p (measured along a length equal to picture height, and aspect ratio 4 to 3). Thus with the vertical resolution also N_p

$$F = (2/3)MBN_p^2/k \tag{9}$$

or

$$k = (2MBN_p^2)/3F. \tag{10}$$

The value of k as determined from substituting F and N_p from the straight line part of Fig. 31 is 0.89. This is somewhat higher than the figures which have up to the present been given.[1] In part of course it is due to the measurement in Fig. 30 of "lines just not resolved" instead of "lines just resolved." The figure is also however very sensitive to the smooth curve plotted through the rather irregular points of Fig. 27. It is to be hoped that further measurements can some time be made either with telephotograph transmitted transparencies or good quality television pictures to permit a more precise plot in this figure.

The viewing conditions for the major part of the work have been sketched in Fig. 1. They were chosen largely for simplicity and ease of reproduction, and to permit of a generally critical evaluation, but do not pretend to be a standard. For the data in Figs. 22 to 25, inclusive, a somewhat larger picture, namely 15 by 20 inches, was projected on the screen, and the value of α for the lens was 0.0852. The viewing was in as complete darkness as could easily be obtained. It was realized that this was an artificial condition, but for exploratory purposes it was deemed not too far off, it was easily reproducible, and it did not interfere with the high picture contrast possibilities which were desired. To secure relaxed vision at the comparatively close viewing distances, lens pairs were made available to the observers to hook on over their own spectacles (or over empty frames) if they wished. These moved back the virtual plane of the viewed screens, by an amount depending upon their focal lengths. The lenses were available in a variety of focal lengths.

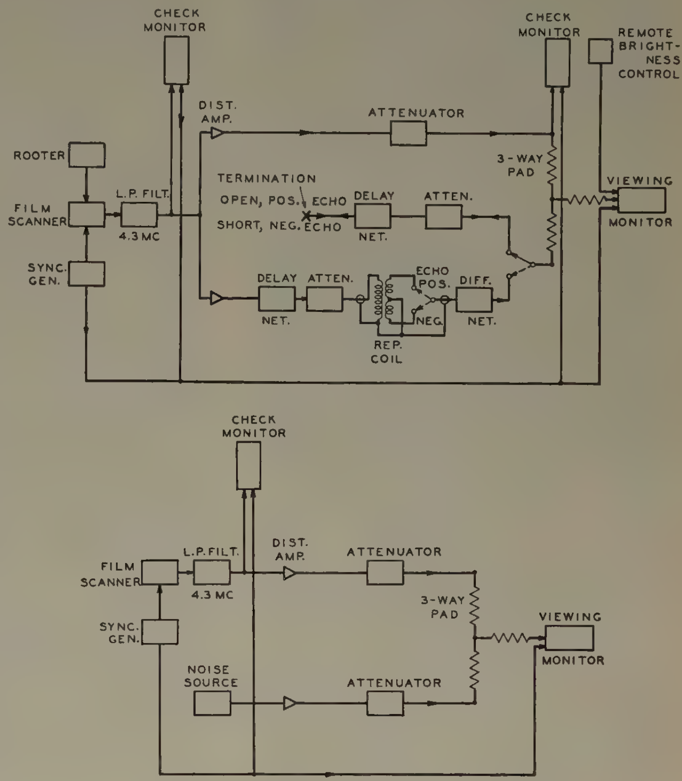


Fig. 32—Circuits for echo and noise tests.

Block schematics of the circuits used for the echo and noise tests are illustrated in Fig. 32. These are conventional in character and self-explanatory. The networks used to obtain differentiation and partial differentiation were of the minimum phase type. As picture source a film scanner of a general type which has already been described[8] was used. An arrangement on it permitted scanning still slides.

[8] A. G. Jensen, "Film scanner for use in television transmission tests," Proc. I.R.E., vol. 29, pp. 243–250; May, 1941.

CORRECTION

J. M. Pettit, author of the paper, "Ultra-High-Frequency Triode Oscillator Using a Series-Tuned Circuit," which appeared on pages 633–635 of the June, 1950, issue of the Proceedings of the I.R.E., and F. J. Kamphoefner, author of the paper, "Feedback in Very-High-Frequency and Ultra-High-Frequency Oscillators," which appeared on pages 630–632 of the same issue, have brought the following omission to the attention of the editors:

The authors wish to acknowledge the support of the Office of Naval Research in the work reported in their respective papers.

Tone Rendition in Photography*

W. T. WINTRINGHAM†, SENIOR MEMBER, IRE

Summary—The photographic field is reviewed to find whether the tone rendition of a good picture can be predicted. The television engineer can find no solace in the fact that good photographs were made before measurements were made of the photographic media. He will be thwarted further when he learns that the best print is the result of experienced criticism of a work print. However, the experience of the photographer in obtaining pleasing results in spite of the limitations and distortions of the photographic process should be useful to the television engineer.

Introduction

TONE RENDITION is one of the important factors which make up a pleasing picture. However, it would appear to be poorly understood, even in the century-old photographic field. Nevertheless, what little data we have was collected in that field, where some work has been attempted in correlating sensitometry and the end result—a pleasing picture. First, let us understand the fact which should be obvious, that the most usual task for any reproduction system (including television systems) is the production of a pleasing picture with the appearance of realism.

Lest this statement of the aim of reproducing processes be considered as biased in favor of artists and pictorialists, let us examine the technical photographic literature. Mees[1] points out that the problem of tone rendition may be considered from two points of view; first, the objective one where brightness in the original and in the reproduction are compared; and second, the subjective, where the appearance of the reproduction conveys an impression of realism. Similarly Neblette[2] and Miller[3] preface their discussions of tone rendition with statements that the end result of the photographic process is the impression conveyed to the viewer. This same thought is found in as unemotional a source as a manual prepared for the use of technicians in motion picture laboratories.[4]

Is it so surprising that level-headed scientists should write in this vein? If it were otherwise, good pictures could have been only fortunate accidents before the practice of sensitometry. And as you know, good pictures were the rule during a part of the half century that preceded even the first experiments in sensitometry. Of course it is not stated that daguerreotypes showed good tone values—quite the contrary was the fact. But Brady's wet-plate pictures of the Civil War era were good. Examine his portraits, say, of Lincoln, and you will see realistic and pleasing pictures seldom equaled today. Similarly, Jackson's pictures of the West were marvelous. The photographers of that time used color-blind negative materials and printing-out papers. Sensitometry, being unknown, contributed not at all to their results. In fact, sensitometric descriptions of the materials they used are almost nonexistent. So sensitometry can not be said to have contributed to good still photography, except in that it may have made the process somewhat easier.

But, you say, how about the movies? Compare, if you will, a clean print from one of Strauss' best negatives with any modern picture you choose. And remember, now, that sensitometry only came to Hollywood with sound.[4] Exact duplication of brightness became important in the movie field only when each release print carried a sound track along its edge.

So, in discussing tone rendition, do not forget the purpose of a picture, which is to produce a realistic and pleasing impression of the original. Now, let us set down some of the things we know about photographic reproductions, not necessarily in the order of importance, for we do not know how to do that.

Physical Facts in Photographic Reproduction

One of the most obvious differences between an original scene and a photograph is that the scene is three-dimensional and the photograph two-dimensional (excepting, of course, the effect produced by the stereoscope).

A scene is in color and most photographs are in monochrome.

A scene is boundless, and a photograph is bounded by a border which in most cases is foreign to the subject.

A photograph generally shows objects at other than their natural sizes.

The depth of field is fixed in making the picture, and the effect may be quite different from what you see in the original.

The picture always is noisy (grainy), and usually it is viewed on paper or a screen which has texture.

We have been able to describe the differences so far without mentioning tone rendition. It requires no stretch of the imagination to suppose that the effect of some of these physical differences can be partially compensated by control of the gradation in the picture.[5]

The maximum brightness in a scene usually is much larger than that in the reproduction.

The brightness range, or ratio of maximum to minimum brightness in a scene is usually different from that in the picture.

There is seldom a uniform distortion of the brightness scale in reproduction.

All of these latter items can be described in sensitometric terms. Since distortion in the sensitometric sense must exist and since some compensation for the first group of physical distortions may be possible, it should be apparent that a good deal more than physical measurement is required to produce a good picture.

Sensitometric Terminology

Before we consider how a good picture might be produced, let us recall the terminology of sensitometry so that we may at least think we know what we are talking about.

An over-all characteristic for a photographic process is shown in Fig. 1. Here the logarithms of luminances of points in the reproduction are plotted against the logarithms of luminances of corresponding points in the original. This S-shaped curve is typical of all photographic processes, showing as it does a concave upward region called the toe, a concave downward region called the shoulder, and an intermediate linear part. Various processes differ in the relative length of the three parts into which the curve may be divided. This curve is typical also in that the range of luminances in the reproduction is different from the range of luminances in the original.

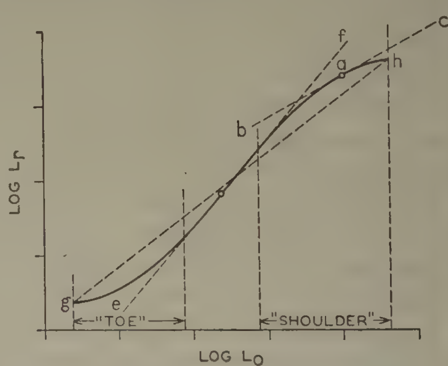


Fig. 1—Diagram showing sensitometry.
L_o = luminance of a point in the object
L_r = luminance of a point in the reproduction
Slope of $b-c$ = gradiant at a
Slope of $e-f$ = maximum gradient = γ
Slope of $g-h$ = average gradient.

The slope of this curve at any point, for example, point a, is the gradient at that point. It is customary to designate the maximum value of the gradient as gamma, in accordance with the nomenclature introduced by Hurter and Driffield.[6] The slope of the line connecting the two ends of the curve in Fig. 1 (line g-h) is called the average gradient of the reproduction. This is a measure of the change in luminance range between the original and the reproduction, or more correctly of the logarithms of these ranges.

* Decimal classification: 770.2836×R583. Original manuscript received by the Institute, December 9, 1949; revised manuscript received, July 11, 1950.
† Bell Telephone Laboratories, Inc., Murray Hill, N. J.
[1] C. E. Kenneth Mees, "The Theory of the Photographic Process," The Macmillan Co., New York, N. Y.; 1944.
[2] C. B. Neblette, "Photography—Its Principles and Practice," D. Van Nostrand and Co., New York, N. Y., Fourth Ed.; 1942.
[3] Carl W. Miller, "Principles of Photographic Reproduction," The Macmillan Co., New York, N. Y.; 1942.
[4] "Motion Picture Laboratory Practice," Eastman Kodak Co., Rochester, N. Y.; 1936.
[5] Ralph M. Evans, "An Introduction to Color," John Wiley & Sons, Inc., New York, N. Y.; 1948.
[6] F. Hurter and V. C. Driffield, "Photochemical investigations and a new method of determination of the sensitiveness of photographic plates," *Jour. Soc. Chem. Ind.*, vol. 9, p. 455; 1890.

It may not be amiss to repeat one point. The term gamma applies to the maximum gradient in the characteristic. The value of gamma is larger than the average gradient. Only if the whole characteristic were a straight line would the value of the average gradient become equal to gamma. Therefore gamma is none too significant a description of a reproduction.

Returning to the names given the three regions of the characteristic curve, Hurter and Driffield[6] called the toe and shoulder regions, respectively, the regions of under and of over exposure. They called the intermediate linear part of the curve the region of correct exposure. This choice of words unfortunately suggests that linearity in this logarithmic plot is desirable, i.e., that good pictures are a result of a simple power law relation between object and image luminances. It is only in recent years that experiments made in the Kodak laboratories[1] have demonstrated that such linearity is not necessary and possibly is undesirable in the making of good pictures. Neblette[2] points out too that it would be well to drop the designation "correct exposure," because it carries such an unfortunate connotation.

The over-all characteristic shown in Fig. 1 is not to be confused with the more common characteristic of a negative or a positive material. Both of these characteristics are combined into this curve which includes as well the effect of stray light (or flare) in the camera and of room lights on the screen in the case of a projected picture. In case the print is made by projection from the negative, the effect of stray light or flare in this process is included likewise in the over-all characteristic.

The Photographic Process

If we assume that linearity is required for good reproduction, it is hard to see how good photographs occur, since this process seldom is linear. Even if we know how a photographer proceeds in making a picture, we are still at a loss. No small part of the difficulty in understanding arises because a good photographer does not talk the kind of language that television engineers understand. His jargon and ours are different. However, let us examine each step in the over-all problem of reproducing a scene, and see if we can guess the significance of the photographer's manipulation.[7]

The various technical elements which determine the quality of a picture, aside from matters of composition are:

(1) The brightnesses in the original scene.
(2) Quality of the image produced by the camera lens, including resolution and stray light (or flare).
(3) The negative material used, including its sensitometric characteristics, and its spectral sensitivity.
(4) The exposure given the negative, including the effects of any filter that might be used.
(5) The development of the negative.
(6) After-treatment of the negative.
(7) The material chosen for the positive.
(8) Exposure and development of the positive.
(9) After-treatment of the positive.
(10) Conditions under which the print is viewed.

We have some technical information about every one of these factors and likewise some idea of how a good photographer might work. If we put these bits of knowledge together, we may learn something about tone rendition in photography.

Scene Brightnesses

Mees[1] quotes quite liberally from a paper by Jones and Condit reporting work carried out on the measurement of scene luminances in the Kodak Research Laboratories. Observations were made on 150 outdoor scenes and photographs made of each. In these scenes, the maximum observed luminance was 11,500 foot-lamberts; and the minimum 0.82. It was found that the luminance range of each of these scenes varied from 750:1 to 27:1, where the largest range occurred in sunlit scenes with objects of interest in open shade and illuminated from the sky. Quoting from Mees:

"The logarithm of 160, the average brightness scale found in this work, is 2.2. Since the available density scale of a glossy developing-out paper is approximately 1.8, the entire brightness range of an average scene cannot be rendered without some compression. Most exterior scenes have brightness scales which greatly exceed the average value, and in many cases a very considerable compression of the brightness scale must be accepted in photographic reproduction."[8]

The scenes chosen in the Kodak study undoubtedly include some which a good photographer would not attempt to photograph. He despises deep shadows and will wait for fleecy clouds to form to throw light into them. If nature fails him, reflectors or even artificial lights are used to illuminate the shadow areas. Just how much this reduces the luminance range of the subjects of good pictures is unknown but it is likely that the average value 160 to 1 of the Kodak study is close to the good photographer's maximum.[9] In this connection Mees states that the range of luminance on portrait and movie sets seldom exceeds 150 to 1.

The Camera Lens

The lens in the camera has two defects in producing the image in the negative material. The imagery may be poor (in a variety of ways) and non-imaged light may fall on the negative. The sharpness of the image has a complicated and little understood effect on the desirable over-all gradient of the photographic process. It is known that some compensation for lack of sharpness can be gained by increasing the over-all gradients.[5] This compensation is only partially explained on technical grounds.

In an otherwise perfect lens, there will be stray light on the negative due to multiple reflection from the glass-air surfaces of the lens elements. Such flare is increased by dust, moisture films, or thumb prints in the lens surfaces. Mees[1] devotes considerable space to this question, and we may summarize it in this rough way. The flare light increases by a factor of about 2 for each pair of glass-air surfaces added. The flare for a lens with four glass-air surfaces was such that the average luminance range for the 150 scenes measured was reduced from 160:1 to 68:1. The major effect of the flare light is to illuminate the shadows in the image. The gradient of the scene luminance-negative exposure characteristic therefore is less than unity in the shadows, and a toe is introduced in the over-all characteristic from this cause. Fortunately, the present trend toward anti-reflection coatings on lenses reduces the flare, provided, of course, that the lens surfaces are kept clean.

It is possible that the preference of some photographers for double-anastigmats or even such meniscuses as the single Protar, over lenses with more elements, is due to this cause. Certainly such preference exists, especially among the older photographers, and is stated in terms of the greater crispness of the photograph. In my own experience, the shadow gradation of a negative taken with a single Protar element is noticeably superior to that when the taking lens has six or eight glass-air surfaces.

The Negative Material

The photographer has a choice of a wide variety of negative materials, varying in sensitivity and spectral sensitivity, maximum density range, maximum gradient, length of the straight part of the D-log E characteristic, graininess, etc. Since there are so many different negative materials available, the selection of a particular one for a picture must depend on more than objective accuracy in tone rendering. Mees[10] shows that the toe of the characteristic of available materials may be long and sweeping, covering log-E ranges greater than one, or may be short and sharp covering much smaller ranges in log E. However, he points out that the linear parts of the characteristics of available materials are adequate to record the range of luminances of all of the 150 scenes in the Kodak study in which the camera lens (without coating) had nominal flare.

It seems that the use of a negative material with a long sweeping toe in its characteristic is not a bar to producing a good picture.[11] Since we cannot tell too much about how the typical good picture was made, and since emulsions having such characteristics have been used to make good pictures, we should not weigh linearity in this stage of the process too heavily.

Exposure of the Negative

Photographers use one of two rules in exposing the negative. The one might be stated as follows: Expose for the shadows and let the highlights fall where they may. The other is similar, except that the words shadows and highlights are interchanged. The photographer using the first rule will place the exposure corresponding to the

[7] Individual references to photographer's techniques will not be cited. The information used here has been obtained over a long period from a wide field of references and by discussion with successful photographers.

[8] See page 775 of footnote reference 1.

[9] In this connection, see footnote reference 3, page 130.

[10] See page 787 of footnote reference 1.

[11] For example, see page 454 of footnote reference 2.

deepest shadow at some point on the toe of the curve which his experience indicates is desirable. Unless the luminance range of the subject is excessive, the highlights will fall on the linear part of the characteristic. If the second rule is followed, the maximum highlight will fall near the lower end of the shoulder of the curve and the shadows will work down toward the toe. In either case, Mees' data discussed in the preceding paragraph indicate that the greater part of the exposure range of the negative falls on the linear part of its characteristic, unless the photographer makes conscious use of the more curved part of a long sweeping toe.

The exposure of the negative is modified by the use of color filters. Besides changing the over-all color sensitivity of the photographic process, color filters modify the exposure range of the negative. Part of this effect comes about because we talk of exposure in visual terms (meter-candle-seconds, for example) and the spectral-sensitivity of the negative material is quite different from that of the eye. However, the photographer takes into account the effect of the filter when he makes his exposure.

Development of the Negative

The published characteristics of any emulsion apply only to certain restricted conditions. The shape of the characteristic depends on the spectral distribution of the radiance producing the exposure and the duration of the exposure. The shape depends also, and to a marked degree, on the development.[12]

There is possible a wide range of development methods, extending from physical to the more customary chemical development. In the one case, the final image is produced by silver deposited from the developing solution; in the other, the silver halide grains in the emulsion are converted to silver. Chemical development may be accompanied by stain image formation, as is the case with pyrogallol as a developing agent. Each development procedure results in a different characteristic of the negative material. Even more important changes in the shape of the characteristic result from variations in the concentration of antifogging agents (alkali bromides, etc.) in the developer. In particular, such restrainers tend to reduce the apparent speed (sensitivity) of the negative material and to shorten the toe of the characteristic.

The photographer may make use of all of these variations. He does, however, adjust his exposures to his processing methods. Therefore, about the only generality we can draw about exposure and development is that some effort is made to place most of the middle and upper tones in the original on the linear part of the negative characteristic.

After-Treatment of the Negative

No photographer is infallible. Even in the movie industry, intensification and reduction of a negative to correct for incorrect exposure and development sometimes are necessary. However, quoting Mees:[1]

> "None of them, however, gives a final result equal to that obtained by correct exposure and development; their use is an expedient and not a practice."

For our purpose we may forget about the sensitometry of intensification and reduction.[13]

There is another type of after-treatment that is practiced in still photography, and that is the local alteration of the scale of tones on the negative through the use of stains, pencil marks, etc. Here again we may neglect safely consideration of the sensitometry of such processes.

Résumé of the Sensitometry of the Negative

Before we consider the production of a final print, let us bring together what we know about the negative. If it represents the work of a good photographer, the negative is a record of a scene in which the range of luminances was somewhat less than 150:1. Flare from the surfaces of the camera lens increased the relative exposure of the shadows, so that the range of illumination of the negative was less than half as great as the original scene, or less than 70:1. The deepest shadows were recorded on the toe of the negative characteristic, while most of the middle tones and the highlights were recorded on the linear portion of the curve. If the negative is a portrait or a movie, the maximum gradient (gamma) most likely is between 0.6 and 0.8.[2,4] In the case of a commercial photograph it may be close to unity.[2] In any case, the average gradient of the negative is most apt to be appreciably less than unity. (You will recall that gradients or gammas refer to the exponent of the input which produces the output, since these terms refer to a log-log plot.)

Positive Materials

The photographer has a very wide choice of positive materials, ranging from transparency stock (lantern slides and movie film) to all sorts of light sensitive materials on paper. He is not limited to emulsions of silver halides in gelatine, and in fact he may not use such emulsions.[13] The sensitometric characteristics of these various printing media vary. The density range required in the negative to print in each of these mediums may be different too. The good photographer, however, has chosen his procedure in making the negative with his printing process in mind. If he had changed his mind after making the negative, there are ways of altering the negative or of making another negative from it, so that the printing medium is used properly.

Let us consider first a movie positive (assuming that all of the intermediate duplication processes ordinarily used do not affect the over-all result). The characteristic curve for positive film has the general shape shown in Fig. 1, with a relatively long straight portion. Quoting from the literature:[4]

> "It is well known, however, that the shape of the characteristic curve for positive film can be modified profoundly by the composition of the developer, and many laboratories use developers which produce curves quite different from those shown in Figure 25. In some cases, for example, the transition between 'toe' and the 'straight-line' portions of the curve takes place at relatively low densities, while in other cases this transition may occur at much higher densities. In the first case the 'toe' is comparatively short, while in the second case it is much longer and is frequently referred to as 'basket-shaped.' . . .
>
> "The point of transition between the straight-line portion of the curve and the 'shoulder' is also influenced by processing conditions. However, this is generally of minor importance, as the density at which the transition occurs is, in the vast majority of cases, beyond that which is practically useful in a positive for projection."

As we have seen, the highlights of the original scene generally fall on the linear part of the negative characteristic, and the shadows are compressed to some extent. When such a negative is printed on the positive film, the denser part of the negative corresponding to the highlights will be printed on or near the toe of the positive characteristic, and the shadows in the negative will fall on the linear part of the positive characteristic. Obviously there will be some, and there may be considerable, compression of both the shadows and the highlights in the over-all process.

In case the print is made on a silver-halide-gelatine paper, the result is much the same. However, in this case the shoulder of the characteristic of the positive material cannot be avoided, so that additional compression of the shadows over that present in the negative is to be expected. It must be kept in mind that the density range in such a print cannot be greater than 1.1 to 1.8, depending on the paper surface.[2]

The pictorialist of today still may utilize media for his best prints which were used by the better photographers of a generation past. Instead of silver halides in gelatine, the light-sensitive material might be a bichromated colloid, or ferric oxalate in the presence of potassium chloroplatinate. Anderson[13] as a representative of those interested in pictorial photography, recommends platinum or platinum overprinted with bichromated gum arabic as the best process, with carbon as a second choice.

Characteristics of these printing media are presented by Miller.[3] The platinotype paper has an extremely long linear portion in its characteristic, with substantially no toe. The maximum density is not very high (about 1.2), so that Anderson[13] recommends overprinting the shadows with gum. This latter process has a density range of only 0.6 or 0.7, but it requires a correspondingly small exposure range. In combination, these two processes result in a printing characteristic which is linear over an extremely large range of densities. (It should be noted that Anderson considers the highlight gradation of platinum the best of all printing processes, indicating again that the toe covers a small part of the total exposure range.)

The characteristics of carbon are shown by Miller.[3] This process is another which produces characteristics with very short toes and long linear portions. As Anderson[13]

[12] See Part III of footnote reference 1.

[13] Paul L. Anderson, "The Technique of Pictorial Photography," J. B. Lippincott Co., New York, N. Y.; 1939.

points out, the minimum density in a carbon print often is undesirably high, and for this reason he prefers platinum.

These latter printing processes differ from silver for another reason. The density range of 1.8 is obtained with a silver print only when the surface is glossy. However, equally high density ranges can be obtained from some of these processes on matte surfaces. The reason, of course, is that the light absorbing particles in one case are imbedded in a constant thickness film of gelatine which is not present in the other cases.

The net result of this discussion is to conclude that the characteristics of paper prints or transparencies need not be much different.

Exposure and Development of the Positive

As indicated in the preceding paragraphs, the characteristic produced in a printing process depends on the processing conditions (the composition of the developer, etc.). It does not seem profitable even to try to summarize all the variables that can be introduced at this point.

It might pay to point out that the method of exposing the positive may introduce nonlinearity. If the print is made by contact, the relation between the diffuse density of the negative and the log-exposure of the positive is linear, and a one to one correspondence exists. If projection printing is used, the difference between diffuse density and printing density of the negative, under the conditions existing in the printer, disturbs this direct correspondence (the Callier effect).[1] In addition, flare light in the printer illuminates the highlights in the print more than should be. In consequence, projection printing reduces the gradient in the highlights.

After-Treatment of the Positive

The positive print may be used following development and fixation, or it may be worked over. Of the after-treatments possible, the most useful is reduction using a subtractive reducer. As Mees[1] shows, a subtractive reducer decreases all densities of a silver image equally. This, therefore, is a means for suppressing the toe of the characteristic of the printing emulsion.

This process does not seem to be used to any extent in the production of movies.[4] However, it is used commonly to "clear the highlights" of lantern slides. It is used, with the same justification, by some older photographers with silver prints on paper.

Some of the toning processes have similar reduction characteristics.[2] In these processes, changes of the color of the silver image is accompanied by a suppression of the toe of the characteristic. Obviously, it is necessary to lengthen the printing exposure when such processes are used.

Viewing Conditions

One of the favorable qualities of a paper print is that the range of luminances in it is not modified by the viewing conditions, except for specular reflection which can be avoided easily. In the case of a projected transparency, conditions are not so simple. The projection density of the print differs from the diffuse density by amounts which depend on the projector;[1] the projector introduces flare light; and the room lighting, if present, dilutes the shadows. The net effect is a decrease of the shadow gradient over that in the highlights, and usually an increase in the highlight gradient.

Over-all Tone Rendering

We have seen how each step in the production of a photograph may alter the shape of the overall relation between scene luminance and reproduction luminance. We can generalize to the extent that the over-all characteristic has the general shape shown in Fig. 1 with the proviso that the toe, the straight portion, and the shoulder of the curve each may have any relative importance. In practice there seems to be some effort to produce an over-all average gradient of unity, with as small a departure therefrom as is possible.

There is little information on the effect on the observer of the change in absolute level of luminance between the original and the reproduction. Mees[1] has some discussion of this effect, and concludes that it produces a further reduction of the gradient of the over-all process in both the shadows and the highlights.[14]

[14] See Fig. 273 of footnote reference 1.

As a part of a study of negative speeds by judgment of print quality, Jones investigated the effect of changes of the over-all characteristic, but for one scene only.[15] These results are not conclusive, since it was not possible to produce average over-all gradients much greater than unity. With this limitation, it was found that the preferred average gradient was unity, without much effect from variations of maximum gradient in the range 1.29 to 1.57.

Conclusions

It should be obvious that it is difficult to describe a good photograph in technical terms.[16] There is some evidence that the average gradient of a good photographic reproduction is not far from unity. Since the typical characteristic is S-shaped, the maximum gradient is somewhat greater than the average. We may conclude that an effort is made in good photography to produce and use a long linear over-all characteristic.

It appears further that by suitable choices in the several steps of the photographic process, the maximum gradient may fall in any region of the characteristic curve. It is pointed out in the laboratory handbook[4] that the final choice of characteristic is based on experienced criticism of the print, and that the position of the maximum gradient should depend on the part of the tone scale in which the greatest interest is centered.

The conclusions, applied to television, might be summarized as follows:

(1) The television tube should be capable of making a picture with a luminance range of the order of 100 to 1.

(2) The average gradient of the system, from light to light, averaged over an output luminance range of 100 to 1 should not be far from unity, possibly being slightly greater than 1.

(3) Means should be available to the program director to shape the over-all system characteristic so that the picture pleases him.

[15] See page 818, et seq., of footnote reference 1.

[16] Bearing on this, Anderson says, "It will also be obvious that although gamma is extremely useful to the research worker or to the laboratory technician, it has no value whatever to the pictorialist . . . ."

CORRECTION

R. C. Hergenrother and B. C. Gardner, authors of the paper, "The Recording Storage Tube," which appeared on pages 740–747 of the July, 1950, issue of the Proceedings of the I.R.E., have brought the following error to the attention of the editors:

In Appendix B on page 746, in equation (3), the term preceding the integral on the right-hand side of the equation should read

$$\frac{i_w}{S/v} \quad \text{instead of} \quad \frac{i_w}{S/2}.$$

Tone Rendition in Television*

B. M. OLIVER†, MEMBER, IRE

Summary—**This paper is a review of some of the brightness transfer characteristics which may be obtained in television using present-day apparatus and techniques. Several families of curves are presented which show the effects of varying one or more of the relevant factors, the remainder being held constant at reasonable values.**

I. Introduction

Television today is passing out of the novelty stage and more attention will be given as time goes on to the pleasing reproduction of intermediate brightnesses. Not until this is done will the full capabilities of television as an artistic medium be realized. Just as the end result in photography is secured by a proper balance of negative and positive film characteristics, so also in television the transmitter and receiver characteristics must be properly matched to secure good results. This in turn implies a standardization of both the transmitter and receiver characteristics, so that any receiver can be used to best advantage with any transmitter. But there is more to the story than merely matching the two characteristics. If we assume for the moment that linear reproduction of brightness at the receiver is desired (reproduced brightness everywhere proportional to original scene brightness) then this result could be achieved with a linear transmitter and a linear receiver, or a square root transmitter and a square law receiver, or with a logarithmic transmitter and an exponential receiver; in fact with any two single valued functions such that one is the inverse of the other. Which pair of matched characteristics is best, depends upon other considerations, such as the relative amount of disturbance produced by added noise or signal level changes which may occur between the transmitter and receiver.

This paper is a review of some of the various characteristics which are obtainable with present day techniques. Some of the virtues of certain of these characteristics as compared with others are pointed out. An attempt has been made to normalize the scales and quantities involved so that results are generally applicable and not specifically limited to the present standards as to sync. pulse height, set-up etc.

II. Definition of Terms

The following terms are used throughout the paper, and their definitions are collected here for ready reference.

B = elemental scene brightness
B_{max} = maximum scene brightness
B_{min} = minimum scene brightness
b = reproduced brightness corresponding to B
b_{max} = value of b when $B=B_{max}$
b_{min} = value of b when $B=B_{min}$
$R=B_{max}/B_{min}$ = brightness ratio of original scene
$r=b_{max}/b_{min}$ = brightness ratio of reproduced image
$X=B/B_{min}$
$x=b/b_{min}$
A = normalized signal amplitude (i.e., $A=0$ when $B=B_{min}$, $A=1$ when $B=B_{max}$
S = transmitter brightness sensitivity $=dA/(dB/B)$
s = receiver brightness sensitivity $=(db/b)/dA$
g = gradient $=(db/b)/(dB/B)$
$1/m$ = exponent of transmitter characteristic
n = exponent of receiver characteristic.

III. Transmitter Characteristics

We will call the functional relation between the elemental scene brightness and the corresponding signal amplitude at some convenient measuring point (such as the video line from the studio or the current in the radio transmitter antenna) the transmitter brightness characteristic. This does not imply that the radio transmitter itself has a nonlinear relation between video signal amplitude in, and rf signal out. On the contrary, the radio transmitter would probably in all cases be reasonably linear, and the brightness characteristic would be determined principally by the characteristics of earlier parts of the system, notably the camera and any associated corrective amplifiers.

All ordinary cathode-ray tubes exhibit a power law characteristic of screen brightness versus control grid voltage. Since such tubes are almost universally used in receivers which are otherwise fairly linear, the most natural class of transmitter characteristics to consider would appear to be those in which the output signal amplitude is proportional to some *root*, say the *m*th root of brightness. Then

$$A = K_1 B^{1/m} + K_2.$$

Adjusting the constants K_1 and K_2 so that $A=0$ when $B=B_{min}$ and $A=1$ when $B=B_{max}$, we obtain:

$$A = \frac{X^{1/m} - 1}{R^{1/m} - 1}. \tag{1}$$

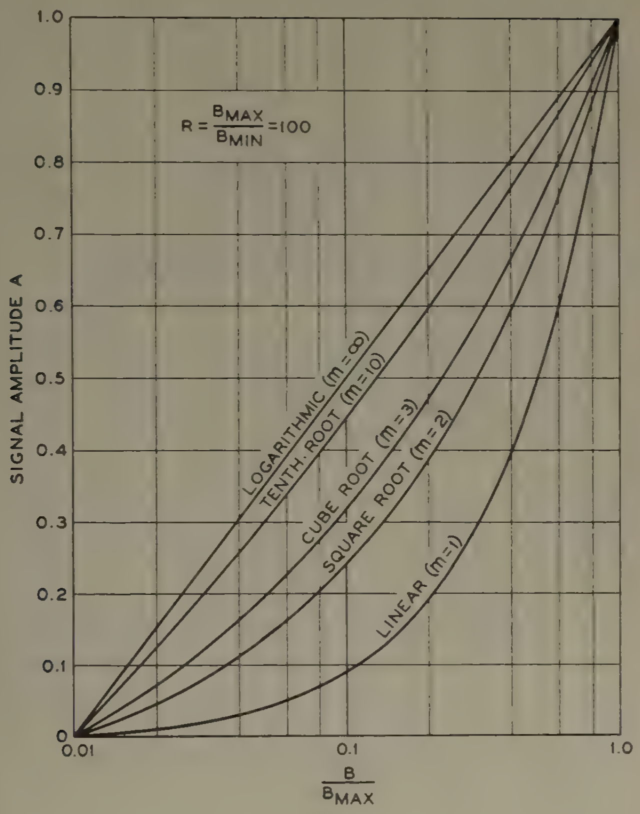
If $m=1$, the transmitter is linear, if $m=2$ it has a square root characteristic, and so on. For $m=\infty$, (1) becomes indeterminate, but the limiting form can be found by L'Hospital's rule. Thus as $m\to\infty$:

$$A = \lim_{m\to\infty} \frac{\dfrac{d}{dm}(X^{1/m} - 1)}{\dfrac{d}{dm}(R^{1/m} - 1)},$$

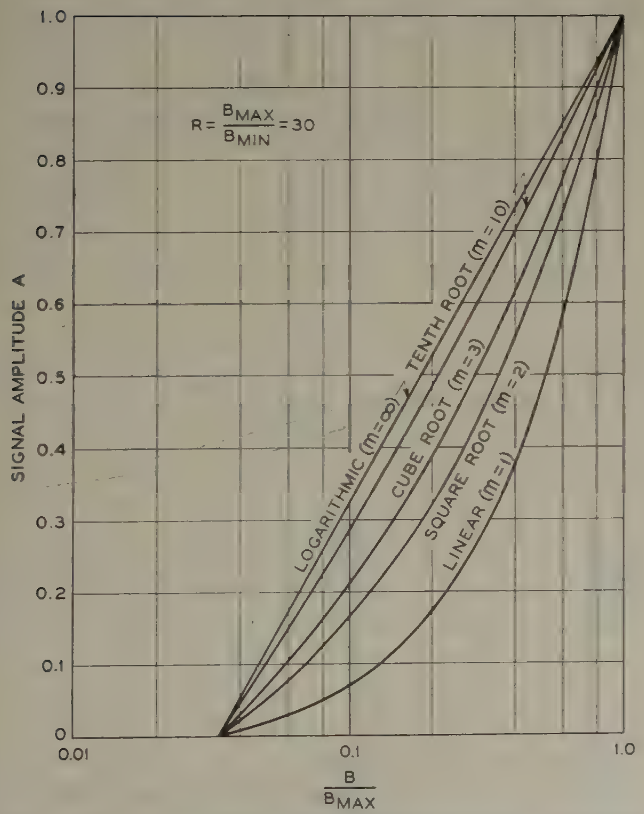
* Decimal classification: R583. Original manuscript received by the Institute, December 9, 1949; revised manuscript received, June 2, 1950.
† Bell Telephone Laboratories, Inc., Murray Hill, N. J.

and we obtain

$$A = \frac{\ln X}{\ln R}. \qquad (2)$$



(a)



(b)

Fig. 1—Transmitter brightness characteristics. (a) $R = B_{\max}/B_{\min} = 100$; (b) $R = B_{\max}/B_{\min} = 30$.

Thus a logarithmic transmitter characteristic can be regarded as the limit of an mth root characteristic as $m \to \infty$:

The characteristics (1) and (2) for several values of m are shown on Fig. 1(a) for $R = 100$, and on Fig. 1(b) for $R = 30$. Since the scale chosen for A is linear, and that for $B/B_{\max}$ is logarithmic, the logarithmic characteristic ($m = \infty$) plots as a straight line, while the linear characteristic ($m = 1$) plots as an exponential curve. The log scale for $B/B_{\max}$ was selected on the basis of the Weber-Fechner law of sensation as applied to the eye. Over the range of brightness commonly encountered in television, equal *percentage* changes in B are almost equally perceptible, i.e., over this range of B, the value of $\Delta B/B$ required to produce a perceptible change is almost constant. Now

$$d\left(\ln \frac{B}{B_{\max}}\right) = \frac{dB}{B},$$

so that equal distance increments along the abscissa correspond to equally perceptible changes in B. A linear scale for $B/B_{\max}$ would seriously compress the low brightness end of the curves. The subjective midpoint of the brightness range is the *geometric* mean of $B_{\max}$ and $B_{\min}$ and the scale used should show this point in the middle. A log scale does this. On the other hand noise and other added disturbances produce equal changes in A, regardless of its value. Thus the scale for A is chosen to be linear.

It will be noticed that the curves for $m = 2$ and $m = 3$ lie about midway between $m = 1$ and $m = \infty$. Thus, in a sense, a logarithmic transmitter is as bad a match for a 2.5-power receiver ($n = 2.5$) as a linear transmitter. This will appear more clearly in some of the later curves.

Another quantity of interest, in connection with the transmitter, is the slope of the characteristics shown in Figs. 1(a) and (b). This we will call the transmitter brightness sensitivity S, although transmitter log-brightness sensitivity would be a more accurate term. We define $S = dA/(dB/B)$. Thus from (1)

$$S = \frac{1}{m} \frac{X^{1/m}}{R^{1/m} - 1}, \quad (m < \infty), \qquad (3)$$

and from (2)

$$S = \frac{1}{\ln R} \qquad (m = \infty). \qquad (4)$$

S is a measure of how much response the transmitter gives to equally perceptible changes in B, over the range of B. Fig. 2(a) shows some curves of S for $R = 100$, while Fig. 2(b) shows the case for $R = 30$. Referring to Fig. 2(a), it will be seen that for equally perceptible changes in B, the linear transmitter has only 1/100 the response at $B/B_{\max} = 0.01$ that it has at $B/B_{\min} = 1$. The matching receiver, in this case linear, must construct an equally perceptible change in b out 1/100 the signal change at $B = B_{\min}$ that it has to work with a $B = B_{\max}$. The logarithmic transmitter, on the other hand, responds

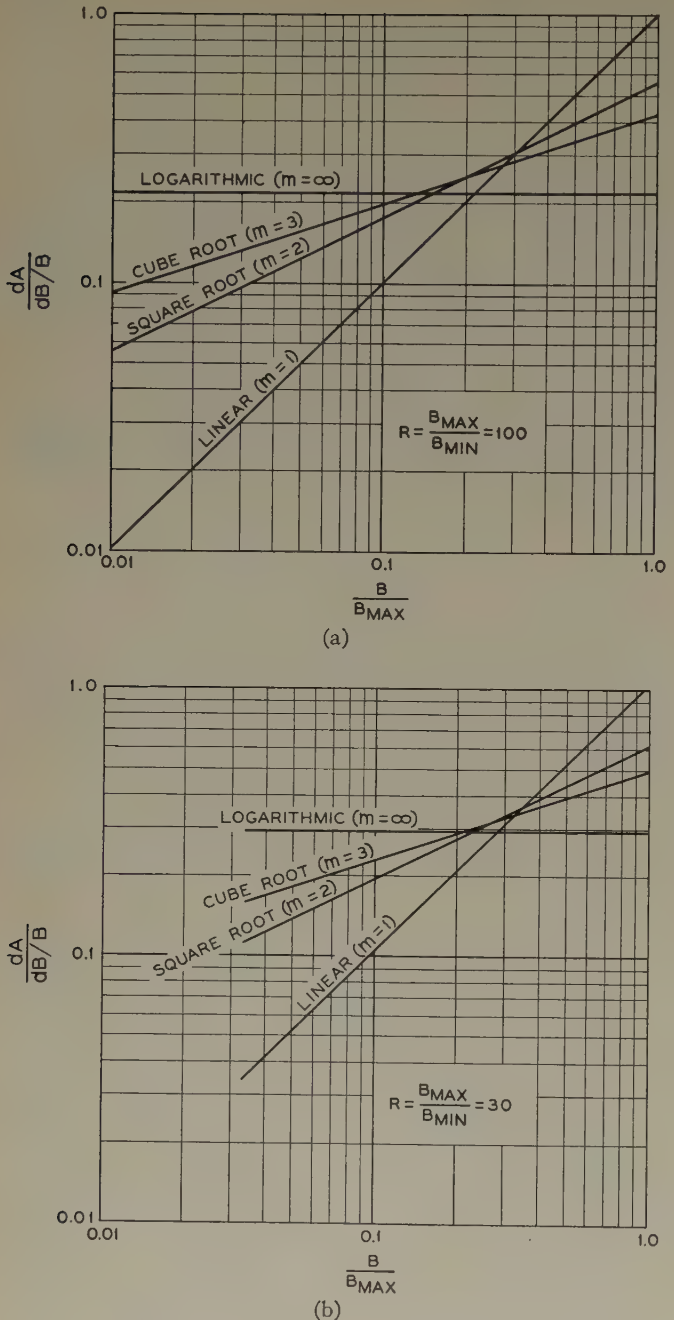


Fig. 2—Transmitter brightness sensitivity. (a) $R=B_{\max}/B_{\min}=100$; (b) $R=B_{\max}/B_{\min}=30$.

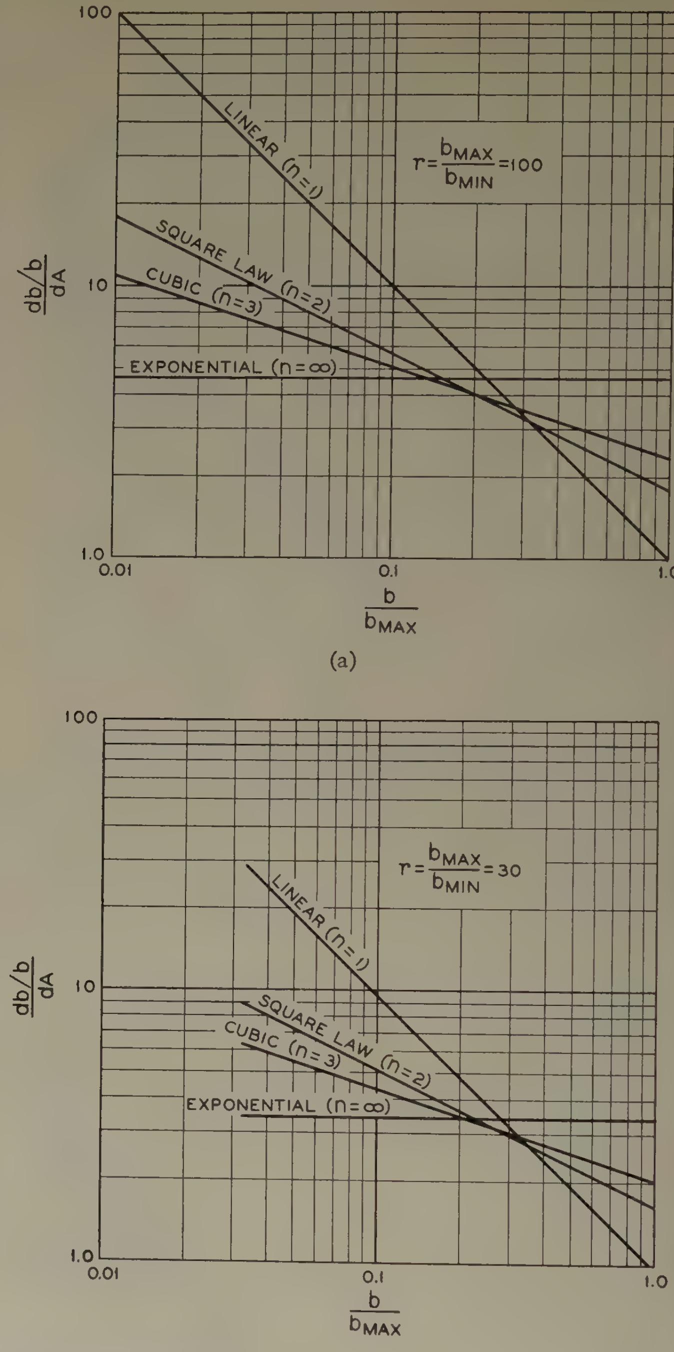


Fig. 3—Receiver brightness sensitivity. (a) $r=b_{\max}/b_{\min}=100$; (b) $r=b_{\max}/b_{\min}=30$.

equally over the range to equally perceptible changes in B.

IV. Receiver Characteristics

We will assume in accordance with the actual facts, that the receiving tube characteristic closely approximates a power law, with some exponent n, and take as the form for the receiver characteristic:

$$b = k_1 + k_2 A^n.$$

Adjusting the constants k_1 and k_2 so that when $A=0$, $b=b_{\min}$, and when $A=1$, $b=b_{\max}$, we obtain

$$x = \frac{b}{b_{\min}} = [1 + (r^{1/n} - 1)A]^n, \tag{5}$$

$$\frac{b}{b_{\max}} = \frac{x}{r},$$

for $n < \infty$. For $n = \infty$, we obtain by taking the limit as before:

$$x = e^{A \ln r} = 10^{A \log r}, \tag{6}$$

$$\frac{b}{b_{\max}} = \frac{x}{r} = e^{(A-1) \ln r}.$$

Thus the exponential receiver characteristic may be regarded as the limit approached by an nth power characteristic as $n \to \infty$.

A quantity which is very useful in describing receiver performance is the receiver brightness sensitivity (or log brightness sensitivity) s, defined as $(db/b)/dA$. From (5) we have:

$$\frac{db}{b} = \frac{dx}{x} = n \frac{r^{1/n} - 1}{[1 + (r^{1/n} - 1)A]} dA,$$

$$s = n \frac{r^{1/n} - 1}{[1 + (r^{1/n} - 1)A]} = n \frac{r^{1/n} - 1}{x^{1/n}}, \quad (7)$$

and from (7) for $n = \infty$:

$$s = \ln r. \quad (8)$$

The receiver brightness sensitivity is a measure of the response of the receiver in *relative* brightness change to a given change in signal amplitude, over the operating range. The larger s is, the smaller will be the increment in signal amplitude which just produces a perceptible brightness change. By the same token, the smaller s is, the greater will be the noise amplitude which can be present in the signal without being visible on the screen.

Fig. 3(a) shows some receiver brightness sensitivity curves for $R = 100$. Fig. 3(b) shows the case for $R = 30$. Referring to Fig. 3(a), it will be seen that the linear receiver is 100 times as sensitive to noise at $b/b_{\max} = 0.01$ as it is at $b = b_{\max}$. The exponential tube, on the other hand, is equally sensitive to noise at all brightnesses.

Let us imagine four receivers, one linear, one square law, one cubic, and one exponential, all supplied with the same initially noise-free signal, and all adjusted to the same r. If noise is now added to the signal and gradually increased in level, it will become visible first in the shadows in the linear receiver, then in the shadows in the square-law receiver, then in the shadows in the cubic receiver, and finally all over the picture in the exponential receiver. By this time, the noise would be quite bad in the linear receiver. The ratio of the value of s at $b = b_{\min}$ for an nth power receiver to the value of s for an exponential receiver, is a measure of how much greater the signal-to-noise ratio must be for the nth power receiver to keep the noise below threshold. This ratio (expressed in db) has been plotted against n in Fig. 4 for the cases $r = 100$ and $r = 30$. We see that the threshold noise penalty for a 2.5-power receiver is about 8 db; for a 5th power receiver, about 4 db; and for a 10th power receiver about 2 db. In appraising these figures it must be remembered that they apply only to noise added at points in the system between the nonlinear transducers which determine the transmitter and receiver brightness characteristics (e.g., input circuit noise), and that further they apply only when this added noise is at or near threshold. For larger amounts of noise, the difference must be estimated in other ways. For example, if the noise level is high enough to show up on an exponential receiver, it will be visible all over the picture, while on a cubic receiver under these conditions the noise would be worse in the shadows, but less visible in the highlights. It is hard to say without tests which picture would be more pleasing.

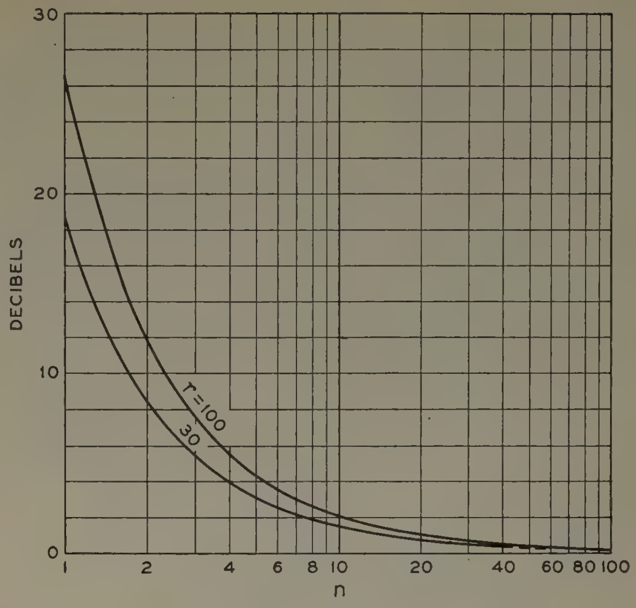


Fig. 4—Threshold noise penalty.

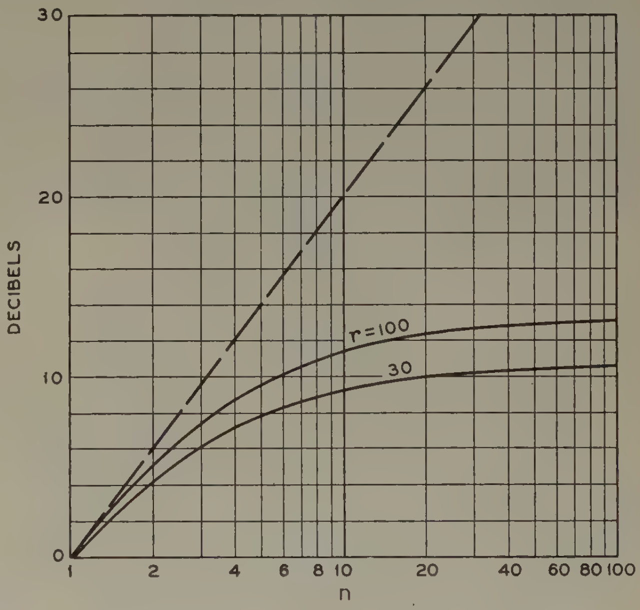


Fig. 5—Relative sensitivity to level changes.

The received signal may be disturbed in other ways besides the addition of noise or interference. When the direct path transmission is poor, as when the receiver is located behind buildings or a hill, reflection off aircraft by combining in and out of phase with the direct ray can produce large level fluctuations. Unless the receiver is provided with a fast automatic volume control, these signal level changes will appear in the picture. If the signal is clamped in the receiver at $A = 0$, then the brightness fluctuations will be most pronounced in the highlights. The relative sensitivity of various receiving brightness characteristics to level changes when the signal is clamped at $A = 0$ is thus given by the relative values of s at $b = b_{\max}$. In Fig. 5, 20 log s at $b = b_{\max}$ is plotted against n for $r = 100$, and $r = 30$ (solid curves). The dashed line in Fig. 5 shows the relative sensitivity to

level changes, for all r, when the signal is clamped at a level corresponding to $b=0$ (cutoff). The effect is larger in this case but corresponds to a pure brightness change, with no effect on picture contrast, as will be seen later.

Summarizing the above briefly, then, we may say that the lower the exponent of the receiver brightness characteristic, the greater will be the sensitivity to changes which affect the signal amplitude in the shadows, and the smaller will be the sensitivity to changes which affect the signal amplitude in the highlights.

It should be mentioned that the receiver exponent need not be fixed by the picture tube characteristic alone. The video amplifier can easily be made nonlinear so as to raise or lower the exponent considerably, but this nonlinearity must be introduced *after* the dc reinsertion.

V. Over-all Brightness Characteristics

The expressions given in the previous two sections can readily be combined to give expressions for any combination of transmitter and receiver. Aside from camera color response and halation effects which are not representable here, the over-all brightness characteristic completely defines the system so far as tone rendition is concerned, but in addition it is useful to know another quantity: the system gradient, g. This is the slope of the over-all brightness characteristic when plotted on log paper, and is defined as:

$$g = \frac{d(\ln b)}{d(\ln B)} = \frac{db/b}{dB/B}.$$

A 1 per cent change in scene brightness produces g per cent change in reproduced brightness. When $g>1$, the contrast is enhanced; when $g<1$, the contrast is reduced. In the curves to follow, g is plotted on a log scale. The rationale for this is that the suppression of contrast represented by $g=\frac{1}{2}$ or $\frac{1}{4}$, say, is as serious as the enhancement represented by $g=2$ or 4, and that therefore any value of g should plot as far from the line $g=1$ as the reciprocal of this value.

Because the mth root and nth power expressions become indeterminate when m or n is infinite, and must be replaced by the limiting logarithmic or exponential forms, four sets of expressions are necessary for the over-all characteristics. These are given below.

mth Root Transmitter, nth Power Receiver

From (1) and (5)

$$x = \left[1 + \frac{r^{1/n}-1}{R^{1/m}-1}\left(X^{1/m}-1\right)\right]^n. \tag{9}$$

From (3) and (7)

$$g = \frac{n}{m}\left[1 + \frac{R^{1/m}-1/n}{(r^{1/n}-1)X^{1/m}}\right]^{-1}. \tag{10}$$

mth Root Transmitter, Exponential Receiver

From (1) and (6)

$$x = \exp\left[\frac{X^{1/m}-1}{R^{1/m}-1}\ln r\right]. \tag{11}$$

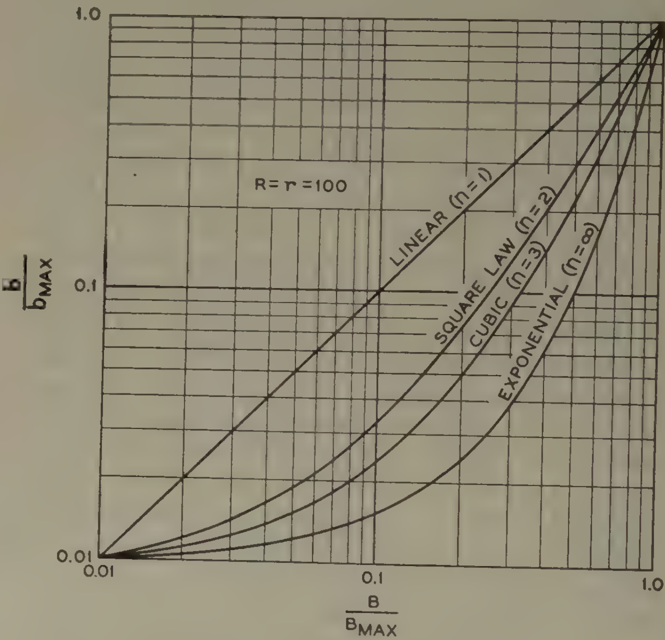
From (3) and (8)

$$g = \frac{\ln r}{m}\,\frac{X^{1/m}}{R^{1/m}-1}. \tag{12}$$

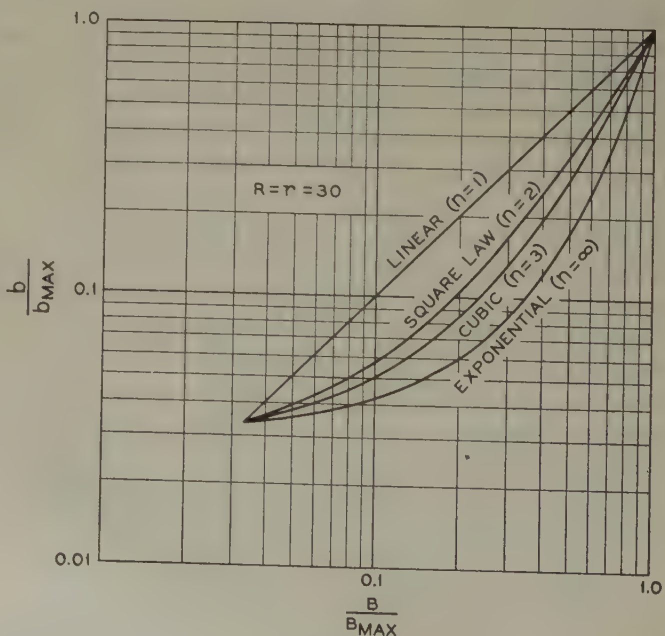
Logarithmic Transmitter, nth Power Receiver

From (2) and (5)

$$x = \left[1 + (r^{1/n}-1)\frac{\log X}{\log R}\right]^n. \tag{13}$$



(a)



(b)

Fig. 6—Over-all brightness characteristics with a linear transmitter. (a) $R=r=100$; (b) $R=r=30$.

From (4) and (7)

$$g = n\left[\ln X + \frac{\ln R}{r^{1/n} - 1}\right]^{-1}. \tag{14}$$

Logarithmic Transmitter, Exponential Receiver

From (2) and (6)

$$x = \exp\left[\frac{\ln r}{\ln R} \ln X\right]. \tag{15}$$

From (4) and (8)

$$g = \frac{\ln r}{\ln R}. \tag{16}$$

Using these expressions, the brightness characteristics and the corresponding gradient curves have been calculated for three transmitters: linear, 2.5-root, and logarithmic, together with four receivers: linear, square law, cubic, and exponential, in all combinations. The receivers are assumed to be adjusted so that the reproduced brightness range, r is equal to the original scene brightness range R. Two cases are considered; the figures designated by the subscript a are for the case $R=r=100$, while those designated b are for the case $R=r=30$.

Figs. 6(a) and 6(b) show the over-all brightness characteristics which result from the use of a linear transmitter and various receivers, while Figs. 7(a) and 7(b)

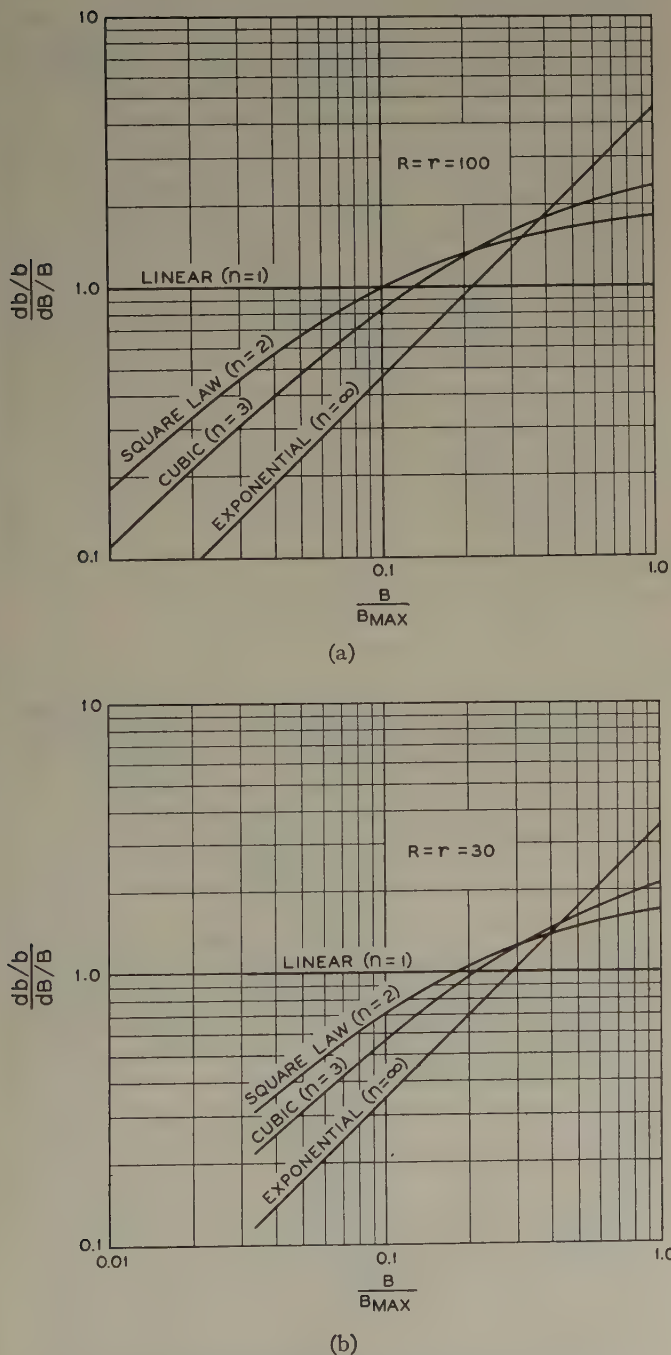


Fig. 7—Over-all gradient with a linear transmitter. (a) $R=r=100$; (b) $R=r=30$.

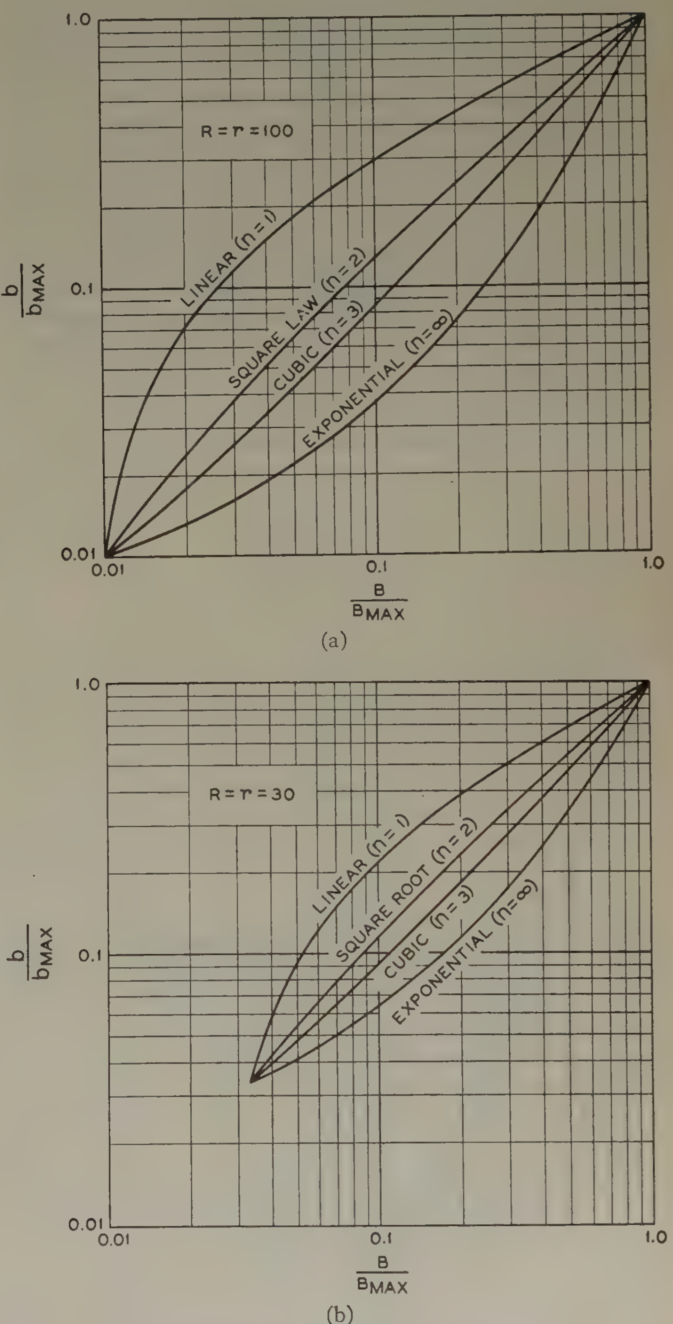


Fig. 8—Over-all brightness characteristics with a 2.5-root transmitter. (a) $R=r=100$; (b) $R=r=30$.

show the corresponding gradients. In this case, the linear receiver naturally gives linear brightness reproduction, while the (higher) power law receivers all show contrast suppression in the shadows and contrast enhancement in the highlights. When operated so that highlight saturation is avoided, many modern camera tubes are linear devices. The exponent n for typical cathode-ray tubes ranges from about two to three with the metal backed tubes lying near the upper end of this range. Thus the curves on Figs. 6 and 7 for $n=2$ and $n=3$ may be considered to bracket the range of brightness characteristics obtained with present-day camera and viewing tubes, when no gradient correction is employed. It will be seen from the curves that the contrast in the highlights is enhanced. This uses up most of the reproducing brightness range to portray a relatively narrow highlight range in the original, with the result that the shadow detail is all too dark, and the shadow contrast is suppressed.

Figs. 8(a) and 8(b) show the over-all brightness characteristics produced by a 2.5-root transmitter in combination with various receivers, while Figs. 9(a) and 9(b) show the corresponding gradients. With a 2.5-root transmitter, a 2.5-power receiver, properly adjusted, would give linear reproduction. It is evident from the curves that any receiver exponent lying between 2 and 3 is

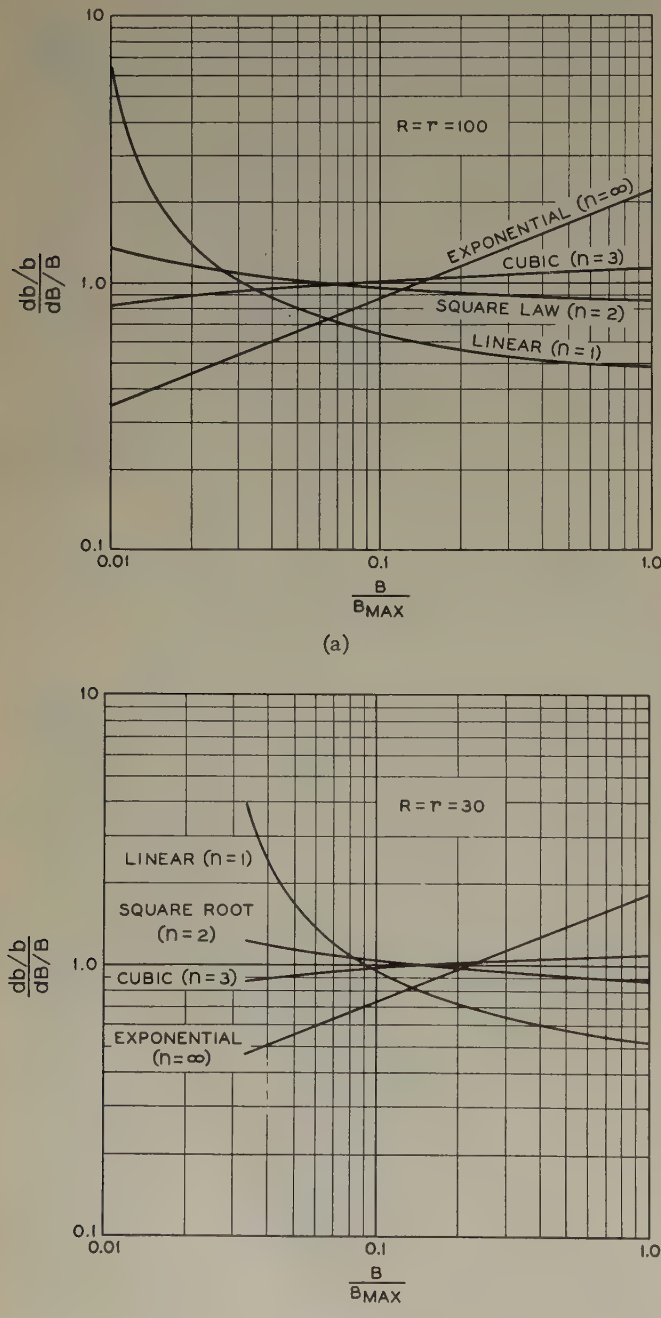


Fig. 9—Over-all gradients with a 2.5-root transmitter. (a) $R=r=100$; (b) $R=r=30$.

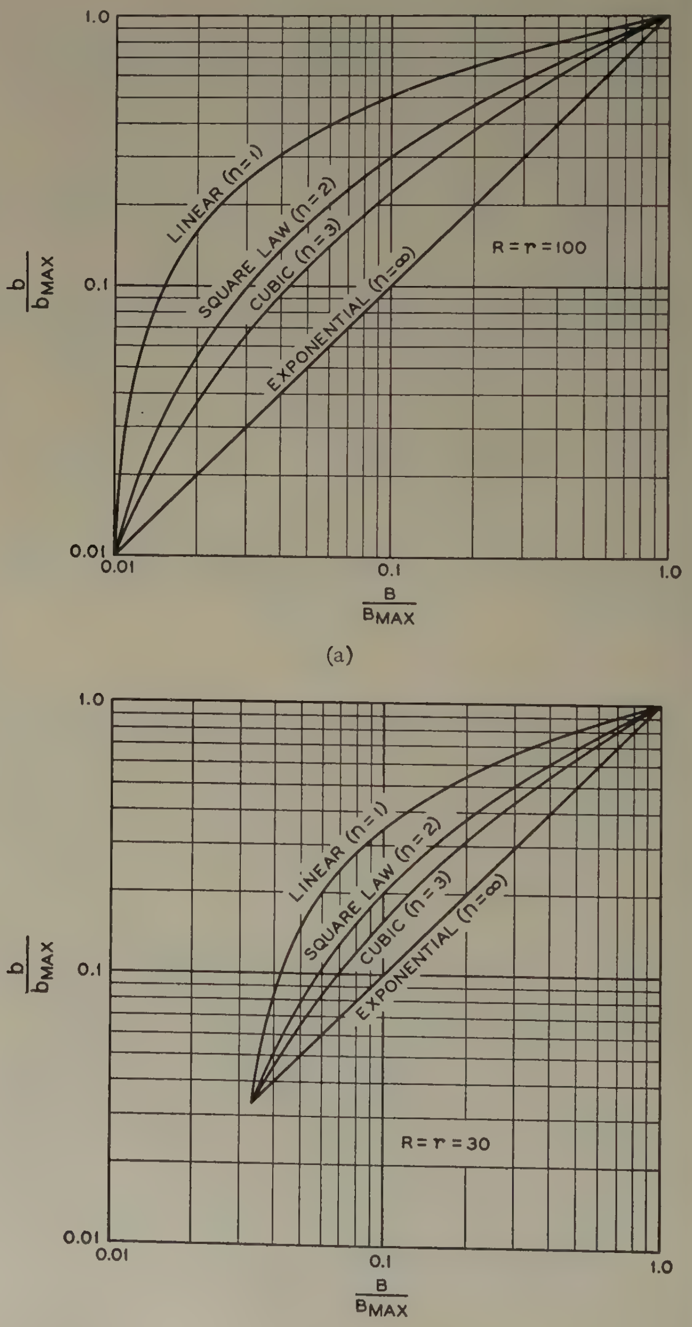


Fig. 10—Over-all brightness characteristics with a logarithmic transmitter. (a) $R=r=100$; (b) $R=r=30$.

not too bad a match. With $n=2$, the brightness in the middle of the range is only about 25 per cent too high, and with $n=3$ it is about 15 per cent too low. Also from the gradient curves it is apparent that the contrast in the reproduced image is nowhere seriously enhanced or suppressed for $2<n<3$.

Finally, Figs. 10(a) and 10(b) show the brightness characteristics obtained when the same four receivers are used with a logarithmic transmitter. Figs. 11(a) and 11(b) are the corresponding gradients. Here the situation is the reverse of that obtained with a linear transmitter. The exponential receiving tube gives linear brightness reproduction while all the (lower) power law receivers show contrast enhancement in the shadows, and contrast suppression in the highlights. For the cases $n=2$ and $n=3$ the intermediate brightnesses are reproduced much too brightly. Most of the available reproducing range is absorbed in exaggerating the contrast in the shadows.

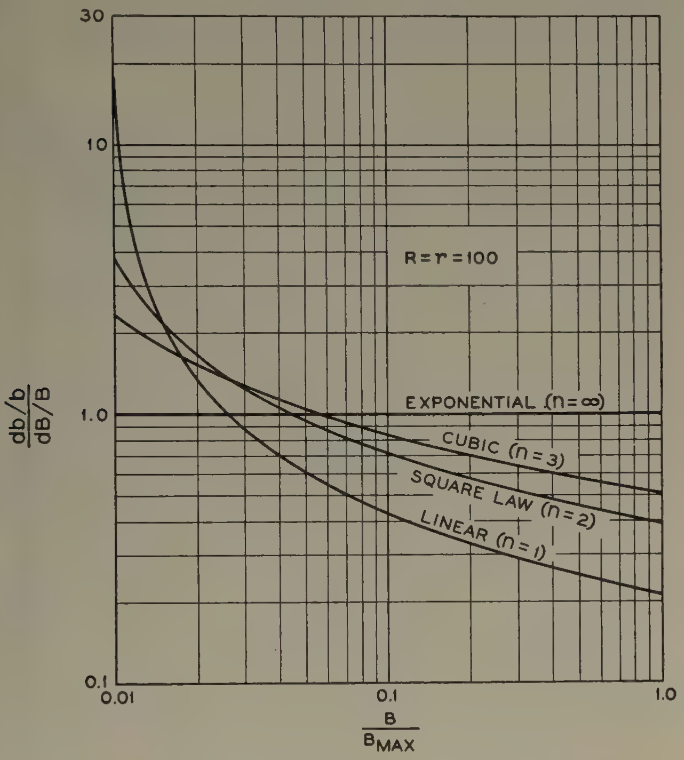
These figures show that if the brightness range of the reproduced picture is set approximately equal to the brightness range of the original subject matter, then neither a linear transmitter nor a logarithmic transmitter comes close to producing an over-all linear characteristic when used with receiver characteristics having exponents on the order of 2 to 3. Furthermore, it is evident that a 2.5-root transmitter characteristic provides a reasonably good match, with receiver characteristics having exponents lying anywhere in the range from 2 up to 3 or 4.

Up to this point, all the over-all brightness characteristics and gradients we have considered have assumed the reproduced brightness range r to be equal to the original scene brightness range R. The over-all gradient is then constant and equal to unity when the transmitter and receiver exponents are matched, i.e., when $m=n$. From (10) it will be seen that for an mth root transmitter and an nth power receiver, the gradient will be constant, and equal to n/m, if

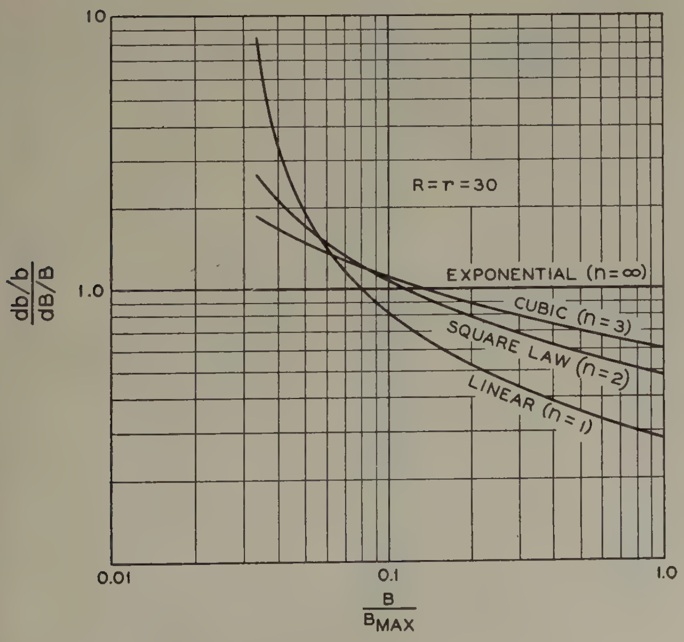
$$r^{1/n} = R^{1/m}$$

$$r = R^{n/m}.$$

This is also the adjustment which would make physical black in the image coincide with physical black in the



(a)



(b)

Fig. 11—Over-all gradients with a logarithmic transmitter. (a) $R=r=100$; (b) $R=r=30$.

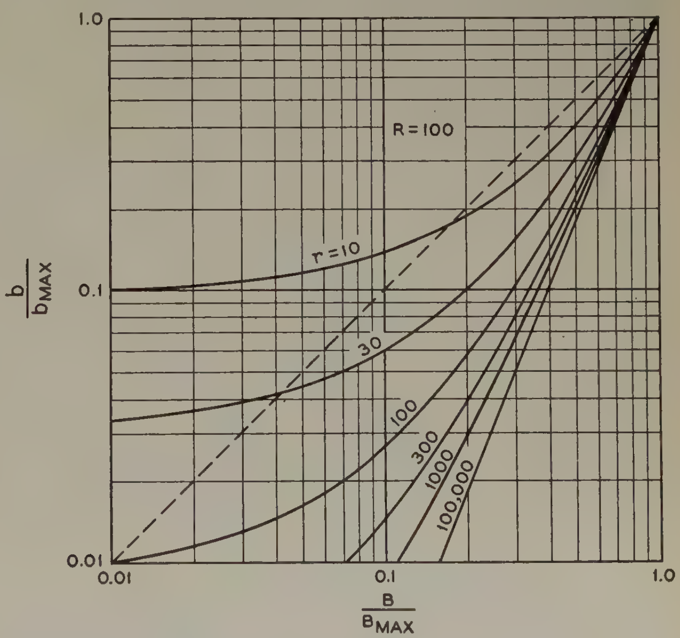


Fig. 12—Over-all brightness characteristics of a linear transmitter, 2.5-power receiver.

original, provided the assumed brightness characteristics held over that range. If g is constant in a given system we may integrate directly and obtain the same result:

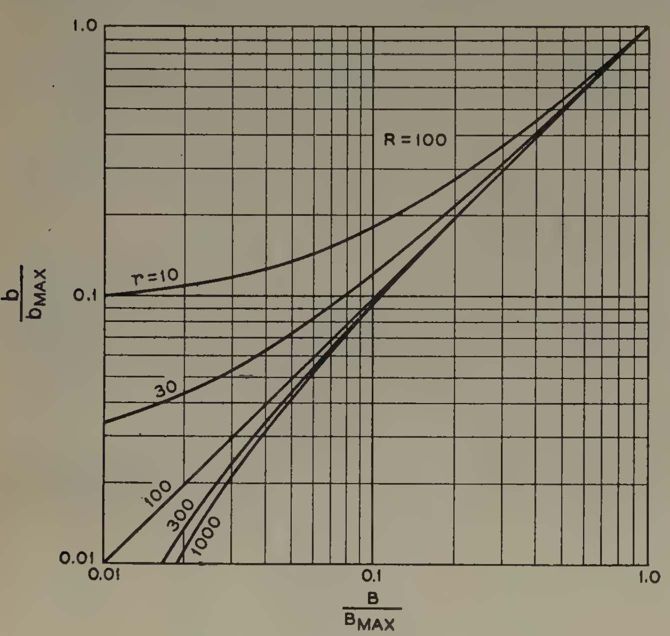
$$\int_{b_{\min}}^{b_{\max}} \frac{db}{b} = g \int_{B_{\min}}^{B_{\max}} \frac{dB}{B}$$

$$\log\left(\frac{b_{\max}}{b_{\min}}\right) = g \log\left(\frac{B_{\max}}{B_{\min}}\right)$$

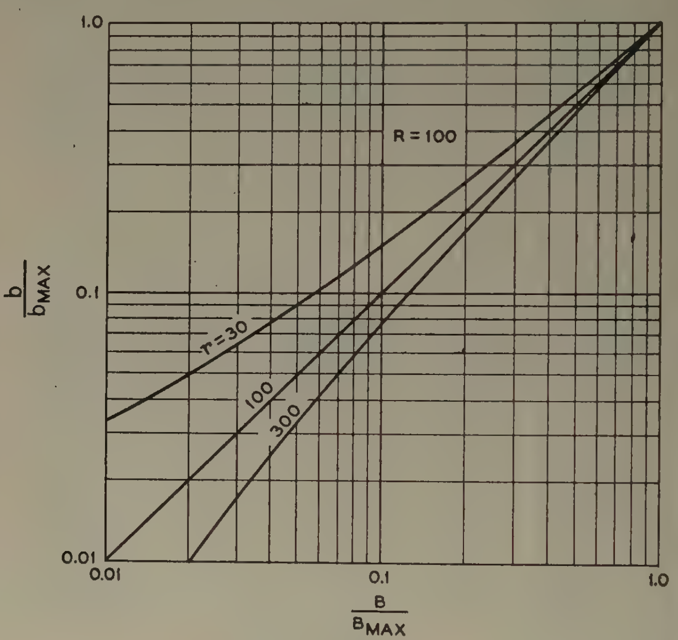
$$r = R^g.$$

The *average* gradient, $\bar{g}$, of any system, may be defined as the gradient of a constant gradient system having the same R and r as the system in question. Thus $\bar{g} \equiv \log r/\log R$.

In photography the term "gamma" is commonly used to denote the *maximum* gradient of the over-all photographic process, i.e., $\gamma = g_{\max}$. The actual gradient will be equal to γ at only one point in the range (or at most over a relatively small range of brightness). At higher or lower brightnesses, the gradient will be less than γ. The average gradient, $\bar{g}$, will therefore also be less than γ, and may in some cases, be less than unity. The fact that good pictures are often made with γ on the order



(a)



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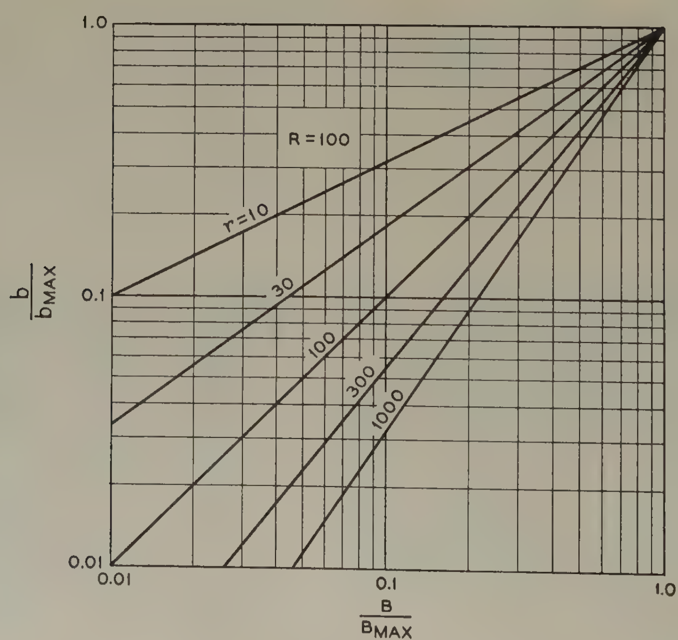

(c)

Fig. 13—Over-all brightness characteristics of (a) linear transmitter and linear receiver, $R=100$; (b) 2.5-root transmitter and 2.5-power receiver, $R=100$; (c) logarithmic transmitter, and an exponential receiver, $R=100$.

of 1.5 is thus certainly no justification for assuming that a constant gradient system with $g=1.5$ would be desirable in television.

To reproduce an original brightness range of 100 to 1 with a constant gradient of 1.5 requires a brightness range of 1,000 to 1 in the reproducing means. Conversely, if the reproducing means has a usable brightness range of 100 to 1, then the range of original scene brightnesses which can be reproduced with a constant gradient of 1.5 is only 21.5 to 1. In a constant gradient system, therefore, g cannot be much greater than unity without either demanding an excessive brightness range in the reproducer or else seriously limiting the usable brightness range of the subject matter.

In television, the reproduced brightness range is under control of the receiver operator. He can vary the so-called "brightness" and "contrast" controls to suit himself, and thus has at his disposal a whole family of brightness characteristics. Fig. 12 shows some of the possible characteristics obtainable when the transmitter is linear and the receiver obeys a 2.5-power law. The curves are all concave upward for $r<100{,}000$. For $r=100{,}000=100^{2.5}$, the gradient is constant and equal to 2.5.

Figs. 13(a), (b), and (c) show the types of characteristics obtained for various r and with $m=n=1$, 2.5, and ∞. Comparing these figures, it will be seen that when $r\neq R$, the system gradient becomes more nearly constant as $m=n$ is increased. With the matched linear system ($m=n=1$) of Fig. 13(a), the increase or decrease in r is produced principally by gradient changes in the shadows with relatively little effect upon the highlights. Small changes in r distort the shadow reproduction greatly. For higher $m=n$ the deviation from linear reproduction (i.e., $g=1$) is spread more uniformly over the brightness range, and relatively large changes in r can be accommodated without serious contrast suppression or enhancement in any particular part of the range. Finally, a logarithmic transmitter, exponential receiver combination ($m=n=\infty$) is inherently a constant gradient system as may be seen from Fig. 13(c). Regardless of r, the gradient will be constant over the range and will be given by $g=\log r/\log R$.

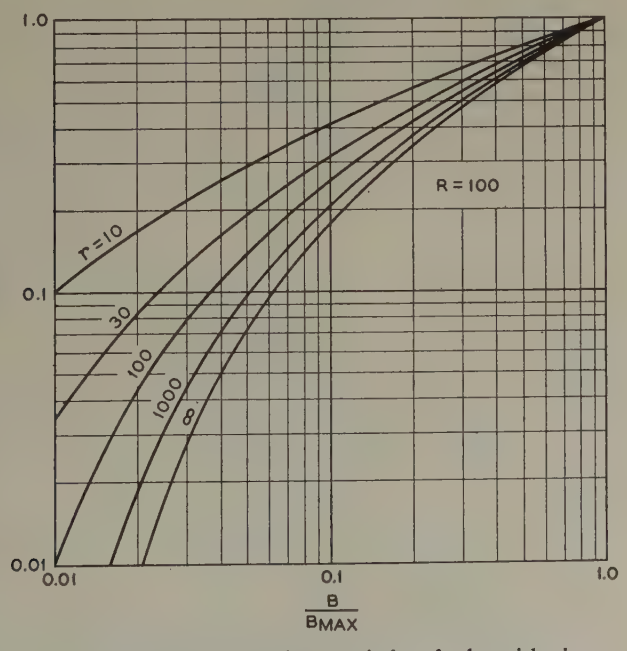

Fig. 14—Over-all brightness characteristics of a logarithmic transmitter, 2.5-power receiver; $R=100$.

Fig. 14 shows part of the family of characteristics obtainable with a logarithmic transmitter and a 2.5-power receiver. All these characteristics are markedly convex upward. Contrast, and camera noise in the shadows is enhanced for all values of r greater than 14. No setting of the controls results in a linear characteristic (except the trivial case $r=1$). The characteristics are, if anything, less usable than those for the linear transmitter.

VI. Change of Operating Conditions

In this section we shall assume that transmitter and receiver are matched, i.e., that $m=n$, and investigate the effect on the brightness characteristic of a change in operating conditions, for various values of n.

Suppose the bias control in the receiver is maladjusted, or the bias potential drifts, or the setup in the signal changes. What effect does this have? If the bias control and gain control in the receiver are set initially so that $r=R$, then the brightness characteristic will be a straight line of unity slope for all $m=n$, as shown by the line marked A in Fig. 15. If now, for any reason,

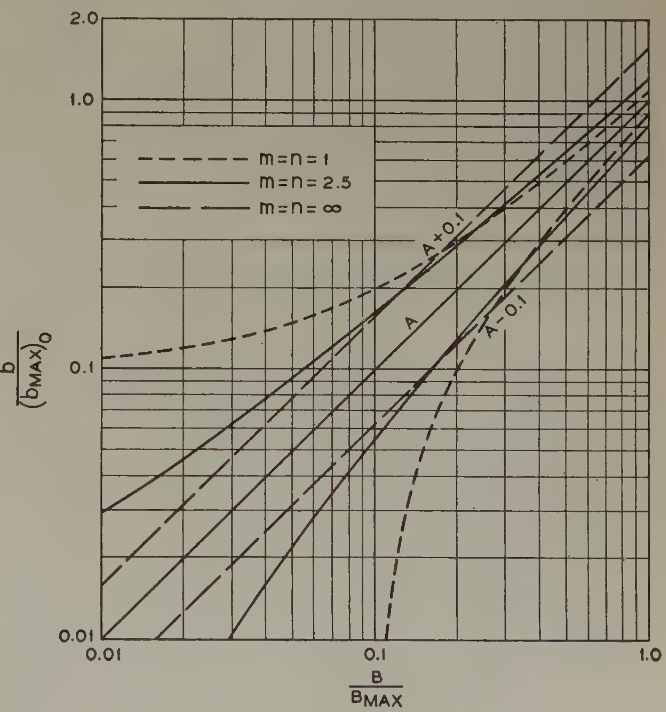

Fig. 15—The effect of 10 per cent bias shift.

the applied signal shifts by 10 per cent in the white direction, so that all signal amplitudes are changed from A to $A+0.1$, the resulting characteristics for different

$m=n$ are as shown by the curves marked $A+0.1$. Similarly if the applied signal shifts 10 per cent in the black direction, we obtain the set of curves marked $A-0.1$. Fig. 16 shows the gradient curves under these same conditions.

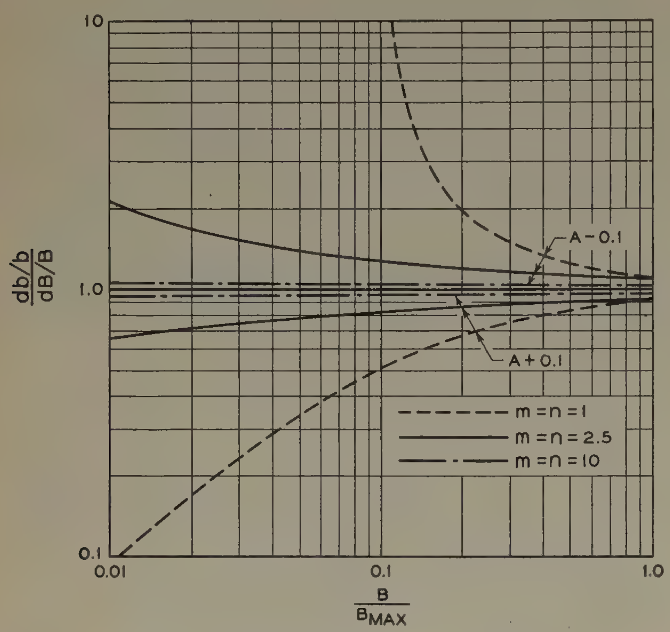

Fig. 16—The effect of 10 per cent bias shift.

Obviously the linear system ($m=n=1$) is very sensitive to these bias changes. With such a system, the setup at the transmitter and the bias control at the receiver would be very critical. Small changes would wreck the picture quality. The way in which the brightness characteristic for the linear system swings around at the low brightness end with small bias shifts is in keeping with the high noise sensitivity of a linear receiver for low brightness. Both bias changes and noise are signals *added* to the correct signal.

As the system exponent is increased, the effect of bias shift becomes less at low brightnesses and greater at high brightnesses, as one might expect. Finally for $m=n=\infty$, a bias shift produces only a change in brightness. The characteristics remain straight lines of unity slope, and all reproduced brightnesses are increased or decreased by the same factor.

Fig. 17 shows what happens to the initially linear brightness characteristic when the signal amplitude is changed by 10 per cent; that is from A to $1.1A$ or $0.9A$. In this figure it is assumed that the signal is clamped at $A=0$ (black level) in the receiver. As a result, no change in brightness is produced at $b=b_{\min}$, but the gradient is changed to 1.1 or 0.9 at this brightness for all system exponents. This change in initial gradient is maintained over the entire range for the logarithmic—exponential system ($m=n=\infty$) and the effect may be described as a "gradient change of 10 per cent" with the brightness corresponding to the clamping level held constant, With the linear system ($m=n=1$) the initial contrast change at $b=b_{\min}$ is not maintained over the range, and the gradient rapidly reverts to unity at the higher brightnesses. As a result the characteristics for $m=n=1$ diverge less than those for $m=n=\infty$, and less brightness change in the highlights is produced. The curves for $m=n=2.5$ are naturally intermediate to the others. The highlight brightness change produced by the 10 per cent signal amplitude change is about 10 per cent

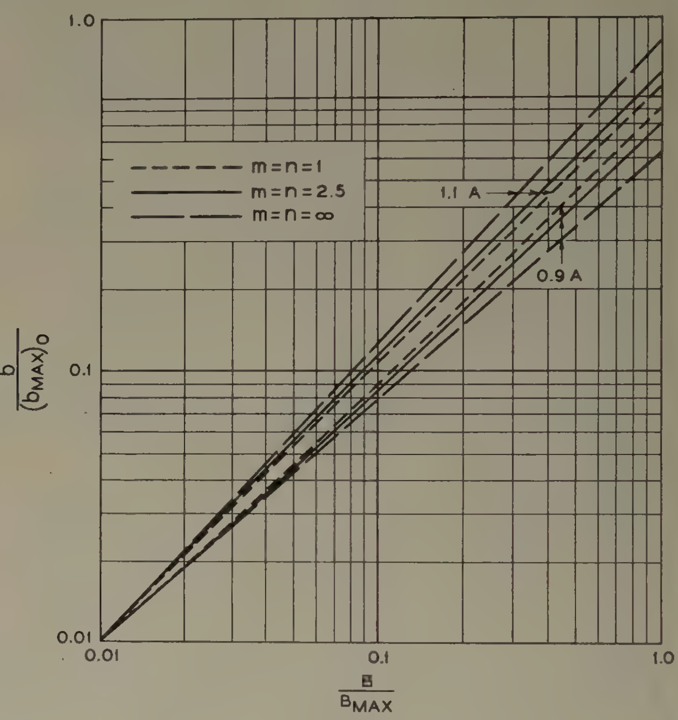

Fig. 17—The effect of 10 per cent change in signal amplitude, signal clamped at $A=0$

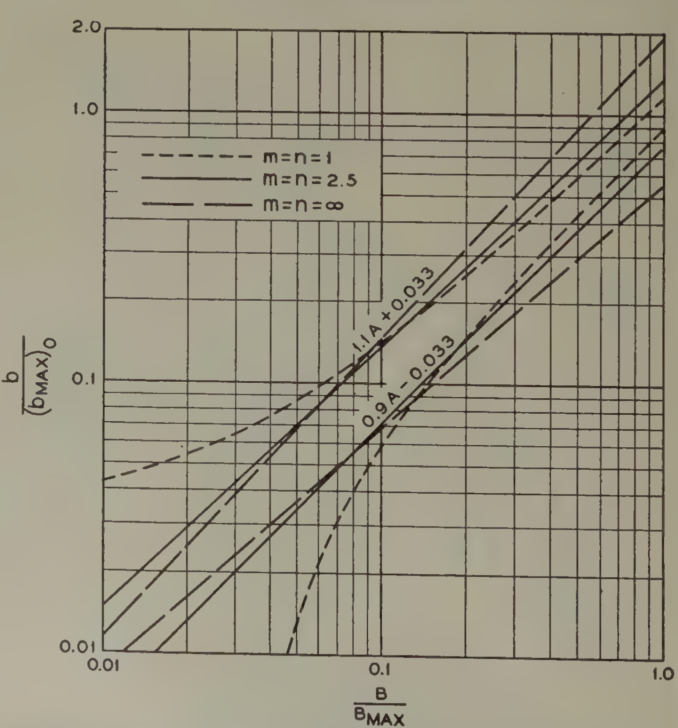

Fig. 18—The effect of 10 per cent change in signal amplitude, signal clamped at $A=-1/3$.

for $m=n=1$, 23 per cent for $m=n=2.5$, and 59 per cent for $m=n=\infty$. These figures are in accord with the earlier curves for the relative sensitivity of various receivers to level changes.

Most receivers do not clamp the signal at black level, but rather reinsert the dc by "bouncing" off the sync peaks. This corresponds to clamping the signal at $A=-\frac{1}{3}$. The effect of a 10 per cent level change under this type of operation is shown on Fig. 18. The applied signal is now changed from A to $1.1A+0.033$ or to $0.9A-0.033$ and the corresponding curves are so marked. The effect on the system for which $m=n=2.5$, it will be seen, is substantially that of a pure brightness change.

In many more recent receivers, the dc developed by the second detector is preserved by using dc coupling in the video amplifier. Since zero rf signal corresponds roughly to the maximum scene brightness, this amounts to clamping the signal at approximately $A=1$. The brightness characteristics resulting from a 10 per cent signal amplitude change under this type of operation are shown in Fig. 19. The highlight brightness now is unaffected, as we might expect. The linear system now shows the greatest variation, and the logarithmic-exponential system the least, while the 2.5-root-2.5-power system again is intermediate.

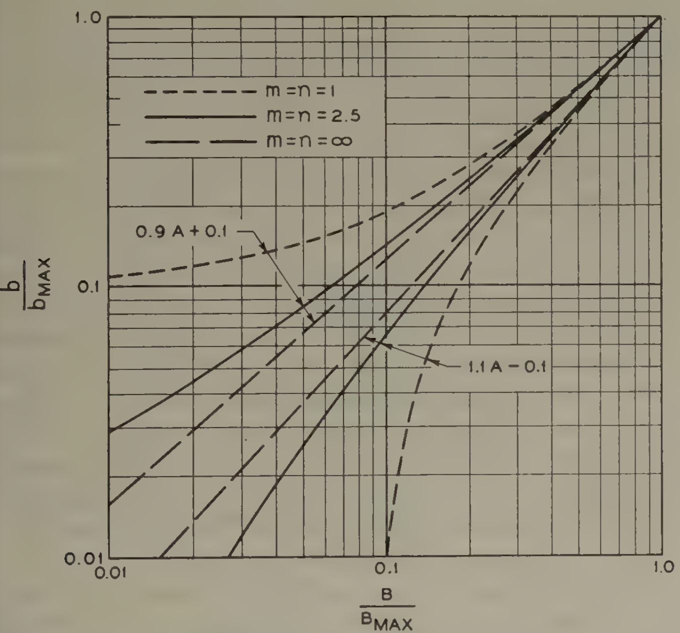

Fig. 19—The effect of 10 per cent change in signal amplitude, signal clamped at $A=1$.

Apparently, then, a linear receiver is undesirable because of its extreme sensitivity to changes which affect the signal amplitude in the shadows. Certainly n (and m) should be at least two to avoid undue trouble from this cause. On the other hand, unless the signal is clamped somewhere near "white" level ($A=1$), very high values of n (and therefore m) are undesirable from the standpoint of level changes. If the use of dc coupled video amplifiers were adopted as standard practice, however, there would be little reason not to make m and n both quite large.

TABLE I

System	Good Features	Bad Features
n Small (any m)	Insensitive to level changes if clamped at $A=0$ Good correction for stray light possible	Sensitive to level changes if clamped at $A\neq 0$ Sensitive to bias changes and added noise
n Large (any m)	Fairly insensitive to level changes if clamped at $A\approx 1$	Sensitive to level changes if clamped at $A=0$
$m\ll n$	Insensitive to bias changes and added noise	Contrast in shadows suppressed Contrast in highlights enhanced Small usable range of scene brightness Very sensitive to stray light
$m\gg n$	Large usable range of scene brightness Less sensitive to stray light than $m\leqq n$	Contrast and camera noise in shadows enhanced Contrast in highlights suppressed
$m\approx n$ (both small)		r and R must be nearly equal to avoid serious contrast distortion in shadows
$m\approx n$ (both large)	r can be varied widely with-serious distortion in any one part of brightness range	

VII. Stray Light

The effect of a uniform ambient illumination of the reproducing screen is to add a constant brightness increment to all parts of the reproduced picture. If the over-all system is adjusted to a linear characteristic ($g=1$) in the absence of stray light, then the principal effect of the stray light will be to "fill up" the shadows and reduce the contrast in those portions of the picture. The effect of stray light can be reduced by readjusting the receiver so that the contrast in the shadows is again increased. Not all receiver characteristics are equally suitable in this respect.

A linear receiver has the property that a simple bias change adds or subtracts a constant brightness increment over the entire brightness range (see Fig. 15). By merely changing the bias control on a linear receiver, therefore, one can substract off the brightness added by the stray light, so that over the brightness range between the highlight brightness and that produced by the stray light alone, linear reproduction is restored. At signal amplitude corresponding to brightnesses below that produced by the stray light alone, the receiver is, of course, completely cut off, and all detail in these portions is lost. For example, if the stray light produced a screen brightness of one-tenth the highlight brightness, then the linear receiver might be biased so that the operation were along the curve $A-0.1$, $m=n=1$, in Fig. 15.

The total screen brightness would then be constant at $b/b_{max}=0.1$ for $0<(B/B_{max})<0.1$ and proportional to B/B_{max} for $0.1<(B/B_{max})<1$. This of course represents a lot of stray light and the characteristic obtained may not be especially pleasing. The real point, however, is that if the brightness produced by the stray light alone is equal to or less than b_{min}, then with a linear tube the effect may be completely cancelled over the entire brightness range of the picture. This is not possible for $n>1$ and is less possible, so to speak, the larger n is.

However, for a given receiver exponent n, the effect of stray light on the picture will be less, the larger the transmitter exponent m is made. This is because with large m, less of the original detail is reproduced at low brightness (see, e.g., Fig. 10) where the effect of stray light is most pronounced. The shadow detail is more quickly raised above the background than with low m. Any matched ($m=n$) system, therefore, would presumably be better from the standpoint of stray light than an uncorrected system for which $m\cong 1$, $n\cong 2.5$.

VIII. Conclusion

Table I is a tabular summary of the principal characteristics of various systems.

Because the receiver brightness characteristic affects the susceptibility of the reproduced picture to noise, level changes, and other disturbances, whereas the only function of the transmitter brightness characteristic is to match the receiver characteristic so that good pictures are readily obtained without highly artificial lighting conditions, it follows that the receiver characteristic should be chosen first. In the preceding sections we found that the best receiver exponent depended to a certain extent upon the reference level at which the signal in the receiver is clamped, so that there is no one ideal receiver exponent, unless all receivers are standardized in their method of dc reinsertion. A decision as to the standard receiver brightness characteristic involves therefore, some speculation as to future practice in dc reinsertion. Also the cost (if any) of modifying the receiver brightness characteristic from that inherent in the viewing tube itself is an important factor.

Once the receiver brightness characteristic is standardized, it does not follow that the standard transmitter characteristic should be the inverse. It is not proven here that the over-all characteristic should be linear (although this is probably true in the case of motion picture film pickup). The best over-all brightness characteristic *might* be one which enhanced the contrast somewhat in the brightness range of principal interest at the expense of some compression in other parts of the range. Or it might even be a constant gradient system with g slightly less than unity so that wider original brightness ranges could be entirely reproduced in the limited range of the receiver.

The best system is probably *not* that represented by a linear transmitter and a 2.5-power receiver, *nor* that of a logarithmic transmitter with such a receiver. Something between these extremes is probably indicated. Further study of the experience with various photographic characteristics, as well as psychometric tests with various television systems, must be made before an intelligent answer can be given.

Appendix

Naming the Knobs

In an exponential receiver, bias changes would produce pure brightness changes with no effect on gradient. The bias control in an exponential receiver would therefore properly be called a "brightness" control. Gain changes, on the other hand, would produce uniform changes in gradient over the range and the gain control would therefore be in truth, and not merely by definition, a "contrast" control. However, a majority of present-day receivers are approximately 2.5-power law devices, with the signal effectively clamped at sync peaks. As may be seen from Figs. 15 and 18 the bias ("brightness") control and the gain ("contrast") control each affect both the brightness and contrast. But the gain ("contrast") control certainly produces more nearly a pure brightness change than does the bias ("brightness") control, so the knobs are, in a sense, mislabeled.

This brings up a possibility which may be worth considering. If the signal is clamped at a level which corresponds to physical black, and the bias control is set so that this level also corresponds to cutoff ($b=0$) in the receiver, then gain changes will produce pure brightness changes; regardless of the receiver exponent n, The gain control is now the brightness control. Of course, this system cannot be used for $n=\infty$, for then the level corresponding to physical black is at $A=-\infty$, in our notation. However, for n in the vicinity of 3 to 4 the system is quite practical. For example, if the physical-black level ($B=b=0$) is made to coincide with sync peaks, and if $n=3$, then at the present standard blanking level, b/b_{max} would be equal to $(1/4)^n=1/64$. A brightness range of 64 to 1 could thus be achieved without the picture signal dropping below present blanking level. Or if $n=3.3$, the range would be 100 to 1.

Since we would then clamp on sync peaks there would be no reason any longer for the "back porch"—at least not for a very long one. We might as well let the sync pulse occupy the entire horizontal retrace time. This would provide complete blanking of the return traces except during the vertical return.

The bias control, with this type of operation, would affect the reproduced brightness range and the gradient in the shadows. A good name for this knob might be "blacks," or "background," or "shadows." With $n>3$, as we have seen, the adjustment of the bias control is not very critical, and the control might even be removed from the front of the receiver.

A Rooter for Video Signals*

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Summary—This paper describes a device which takes the *n*th root of the instantaneous amplitude of a video signal. Its function is to linearize the over-all transfer characteristic, and thus to improve the picture quality in a television system using linear camera tubes and conventional cathode-ray viewing tubes.

Introduction

MANY TELEVISION camera tubes (e.g., the orthicon) are fairly linear devices, that is the output signal current is roughly proportional to the brightness of the area being scanned. On the other hand, in most cathode-ray tubes the relation between screen brightness and grid voltage is quite nonlinear. Over the useful range of screen brightness the characteristic can be quite closely approximated by a power law:

$$b = K(e_g - E_0)^n \tag{1}$$

where

b = screen brightness
K = a constant
e_g = grid voltage
E_0 = a reference voltage (approximately cutoff).

The accuracy of this power-law approximation may be judged from Fig. 1 which shows the measured[1] characteristics for two tubes, a 10BP4 and a 10FP4. On the log scales used, a true power law would plot as a straight line of slope n. The actual characteristics are quite close to this with $n=2.16$ for the 10BP4, and $n=2.7$ for the 10FP4. The approximation is quite good over the entire useful brightness range.

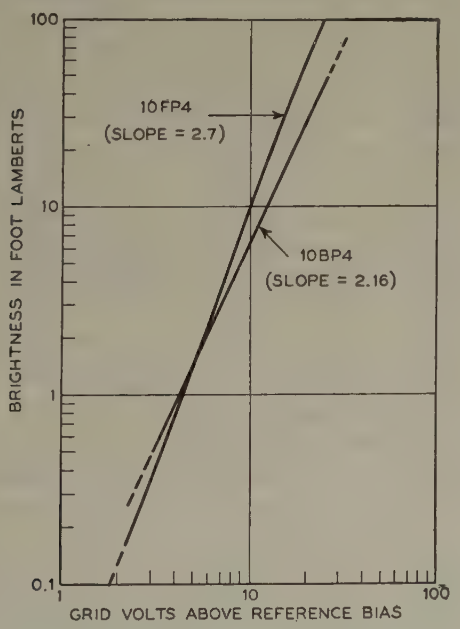

Fig. 1—Viewing-tube brightness characteristics.

When such a picture tube is connected by a linear transmission system to a linear camera tube, a whole family of over-all brightness characteristics may be obtained, depending on the settings of the bias and gain controls, but none of these characteristics is linear. If the reproduced brightness range is set approximately equal to the original scene brightness range the resulting characteristic will show contrast enhancement in the highlights and severe contrast suppression in the shadows.[2] All intermediate brightnesses are reproduced much too darkly; areas which should be light grey are substantially black. If one attempts to raise the brightness of these areas by means of the bias control, the principal effect is to destroy all true blacks in the picture The picture then appears fogged (light-struck) and the return lines show long before the areas in question have been raised to their proper relative brightness.

To straighten things out we need to take the nth root of the video signal (measured with respect to black level) before this signal is applied to the cathode-ray tube. Only then will blondes be blondes and the men clean-shaven. The following sections describe a nonlinear device designed to perform this function. For rather obvious reasons it has come to be known as the "rooter."

This predistortion, this "rooting" of the video signal, should be done near the camera and not at thousands of receivers. Not only is this economical, but it is also preferable for technical reasons. The rooter necessarily increases the system gain in the shadows, and if used at the receiver would increase the visibility of receiver noise in these portions of the picture. The present rooter circuit was designed to be inserted in a 75-ohm coaxial line. Many of the present tubes could be eliminated if the circuit were to be designed as an integral part of a camera preamplifier.

Principle of Operation

Fig. 2 shows a block diagram of the rooter. Video signals from the linear camera and preamplifiers are applied to the input of a feedback amplifier. At the output of this amplifier, the dc and low-frequency components of the signal are reinserted by means of a clamp circuit actuated by the horizontal drive pulses from the sync generator. The amplified and restored video

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[1] Measured by the method described by M. W. Baldwin, "Measurement method for picture tubes," *Electronics*, vol. 22, pp. 104–105; November, 1949.

[2] B. M. Oliver, "Tone rendition in television," Proc. I.R.E., pp. 1288–1301; this issue.

signal is applied to a linear current generator which drives a nonlinear impedance with a current directly proportional to the brightness of the area being scanned. The output stage is, in turn, driven by the voltage across this nonlinear impedance—the "rooted" signal—and delivers its output to the line. We merely need, then, a suitable nonlinear impedance.

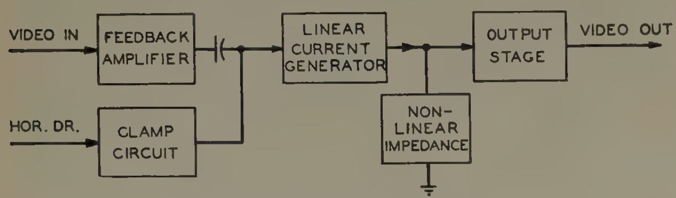

Fig. 2—Block diagram of rooter.

The plate current-versus-grid voltage characteristic of a triode, like the brightness characteristic of a cathode-ray tube, can be closely approximated by a power-law relation. In some tubes the exponent of this power law is quite high. Fig. 3 shows the plate current-versus-cathode voltage characteristic of a 2C51 with 150 volts on the plate and -7 volts fixed bias on the grid. Under

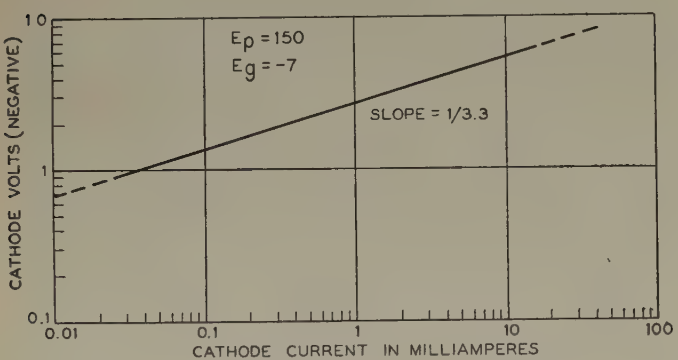

Fig. 3—Cathode current-voltage curve of 2C51.

these conditions, the plate current is quite closely proportional to the 3.3 power of the cathode voltage, i.e.,

$$I_p = KE_c^{3.3}. \quad (2)$$

Now either I_p or E_c may be considered to be the independent variable here. If the cathode is driven from a high impedance source so that the cathode *current* is specified, then the cathode will assume the potential required by (2) and this potential will be proportional to the 3.3 *root* of the applied current. This is the principle used in the present rooter.

Circuit Details

Fig. 4 is a schematic diagram of the complete rooter circuit including the clamp circuit.

The feedback amplifier comprises the first two tubes (404-A's) in the upper row. A 75-ohm potentiometer terminates the input cable and serves to adjust the input level to the amplifier to the proper value of 0.5 volt maximum peak to peak. The applied signal must be of the standard polarity, i.e., black negative, and should not contain any excursions more negative than black level (no sync pulses). The feedback amplifier has about 20-db loop gain from low frequencies up to 4 Mc. Above 4 Mc, the loop gain falls first at 12, and then a 6 db per octave, and gain crossover is at about 14 Mc. The external gain is 26 db and is flat within about 0.2 db to 10 Mc.

The output signal from the feedback amplifier (10 volts maximum peak to peak) is applied through a 1,000-$\mu\mu$f coupling condenser to the grid of the third 404-A. The potential drop across the 1,000-$\mu\mu$f condenser is adjusted as required at the start of every line, by current from the clamp circuit. The clamp circuit makes ground potential on the grid of the third 404-A correspond to black at all times. The potential of this grid thus varies over the range 0 (black) to +10 volts (maximum white) depending on the signal. This tube is the linear current generator. It is linear because of the 18 to 20 db of local feedback provided by the unby-passed cathode resistor. It is a current generator because the plate resistance is very high compared with the load impedance. Since this stage must have flat transmission clear down to dc, the screen voltage is obtained from a gas-tube regulated supply rather than from the usual dropping resistor—bypass condenser combination.

The "black current" in the driver stage just described (i.e., the plate current corresponding to black in the picture) is about 4 ma. All but about 100 μa of this current are drawn through the 24,000-ohm fixed and 0 to 25,000-ohm variable resistors from $B+$. The remainder of the current demanded by the driver tube flows through the left half of the 2C51 and the 100-ohm resistor in series with the cathode of this tube. The nonlinear impedance shown in the block diagram thus consists of the cathode impedance of a 2C51 in series with 100 ohms, and a shunt of 24,000 ohms to 49,000 ohms around this combination. The effect of the linear resistances is to dilute the nonlinear resistance of the 2C51 cathode so that the combination obeys a power law with a smaller exponent than the 2C51 alone, as will be shown later.

The grid of the rooter tube (the left half of the 2C51) is connected to the screen supply of the driver tube. This screen supply thus effectively becomes the "ground" or reference end of the nonlinear impedance. Variations in this supply voltage will cause corresponding variations in the plate potential of the driver tube. In order to suppress these variations in the output stage (the right half of the 2C51) the cathode of this section is also returned to the screen supply. Bias for this section is obtained by the tube current as well as the screen supply current flowing through a 560-ohm cathode circuit resistor. Since plenty of signal is available to drive the output stage, this resistor is left unby-

passed. This improves the linearity and reduces the input capacity. Direct-current coupling is used into the output stage grid. This also reduces the stray capacity by eliminating a bulky condenser, and furthermore reduces the peak-to-peak plate current swing in the output stage (between extremes of pictures material) by about 4 db.

A meter is provided to read either the plate current of the rooter tube or one tenth the plate current of the output stage. Under no signal (black picture) conditions the rooter-tube plate current is set by the left hand 25,000-ohm control to about 100 μa, and the plate current in the output stage is then set by the right-hand 25,000-ohm control to 10 ma (1 ma on the meter).

The lower part of Fig. 4 shows the clamp circuit. In the film scanner application for which this circuit was designed, the reference black in the video signal is obtained as follows. The flyback time of the camera horizontal sweep was made less than 6 microseconds. For the last few microseconds of the horizontal blanking time, the camera is therefore able to scan an unilluminated area to the left of the picture area on the photocathode. Flyback in the camera horizontal sweep is initiated by the leading edge of the horizontal drive pulses. A suitable clamp pulse in the rooter is obtained by generating from each horizontal drive pulse, another pulse delayed by slightly more than the camera retrace time. This allows the camera flyback to be completed and assures that the reference black edge is being scanned when clamping occurs.

Presumably the same method could be used with live pickup cameras, provided an opaque field mask were were used. In a spot scanner, beam blanking during flyback might provide a suitable black reference, and in this case the delay multivibrator (the first tube in the clamp circuit) could be eliminated.

Operating Characteristics

In the actual circuit the rooter tube is operated with about 100 μa of black current. There are several reasons for this. In the first place, this black current will vary several microamperes as the components warm up or age. If one tried to operate with zero black current, he would be walking right on the edge of a cliff. Any further decrease in the black current in the driver tube would find the rooter tube completely cut off and the cathode presenting infinite impedance. The load impedance in the plate of the driver would then consist of the shunt resistors to $B+$, i.e., 24,000 ohms to 49,000 ohms, and relatively enormous distorted voltage waves would be applied to the output stage during the darkest portions of the picture. Besides this effect, the impedance of the cathode of the rooter tube would be too high to preserve sufficient bandwidth in the shadows if the black current were less than about 100 μa.

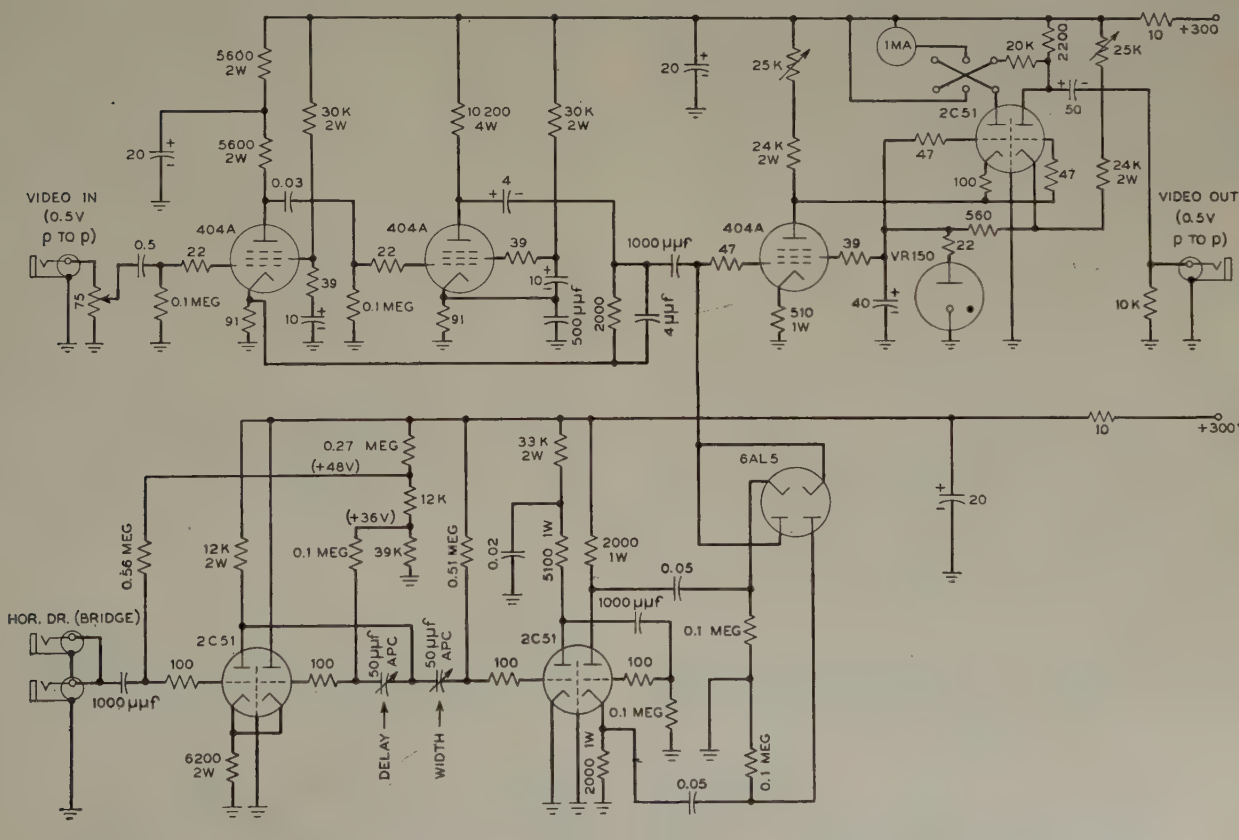

Fig. 4—Schematic of rooter. Capacitance in μf, resistance in ohms, (k=1,000 ohms) unless otherwise specified.

The presence of this black current in the rooter tube means that the characteristic is no longer as given in Fig. 3. Rather one must plot the *change* in cathode voltage versus *change* in cathode current from the operating (black current) point. Thus the curve of Fig. 2 must be shifted down 1.36 volts and to the left 0.1 ma. When this is done one obtains the solid curve of Fig. 5. The 45° dotted lines on Fig. 5 are the current-voltage curves of the 100-ohm series resistor and an assumed value of 33,000 ohms for the shunt-feed resistor. The total resultant characteristic may now be obtained by adding the ordinates of the tube characteristics and the 100-ohm line and then adding the abscissas of that curve and the 33,000-ohm line. This resultant is the operating curve of the rooter, and doesn't look much like the *n*th root law we are seeking.

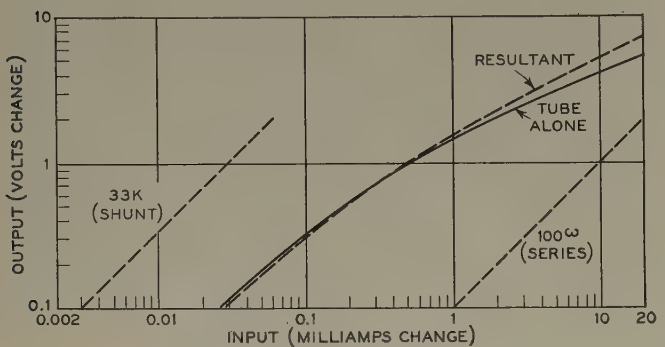

Fig. 5—Rooter interstage characteristic.

However, one must remember that an arbitrary constant voltage may be added to the output voltage. This corresponds to changing the bias on the cathode-ray tube (or the setup in the line amplifier following the rooter). When the resultant curve of Fig. 5 is shifted up by 0.4 volt, the result is the solid curve of Fig. 6.

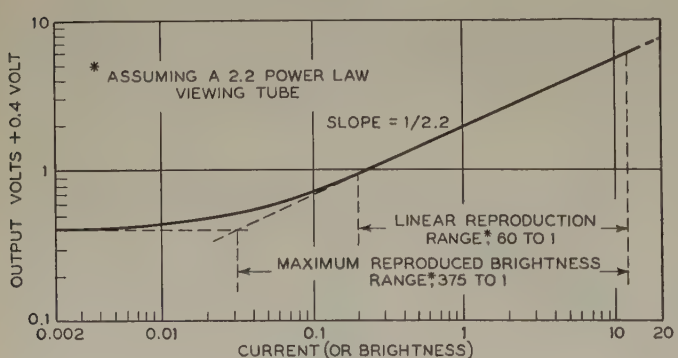

Fig. 6—Effective rooter characteristics.

While the abscissa of Fig. 6 is the same as Fig. 5, i.e., "milliamperes change from operating (black) current," this abscissa may also be interpreted as proportional to subject brightness since the camera is assumed linear. We see that the effective characteristic is a good 2.2-root law over a subject brightness range of about 60 to 1. If a 2.2-power-law cathode-ray tube is used, the reproduction will be linear over this range while the reproduced brightness corresponding to physical black in the subject will be about 1/375 of the reproduced highlight brightness. All this is a distinct improvement over the situation with no rooter.

As the cathode impedance of the rooter tube increases, the bandwidth of the rooter decreases. The decrease is not serious over the useful brightness range of the reproducing tube. Assuming a 2.2-power-law tube and a highlight brightness of 100 foot-lamberts the following figures are approximately correct:

Reproduced brightness	*Response is down 3 db at*
>7 foot-lamberts	>10 Mc
4 foot-lamberts	8 Mc
1.5 foot-lamberts	5 Mc
1 foot lamberts	4 Mc.

The bandwidth is thus greater than 10 Mc over a 14-to-1 brightness range and greater than 4 Mc over a 100-to-1 brightness range.

Performance

The rooter described above is in use in a laboratory film scanner employing a modified Farnsworth dissector as a camera tube. The improvement in picture quality as a result of the rooting of the signal is rather spectacular, particularly in low-key pictures.

One fear we had at the outset was that camera noise in the shadows would be intolerable when the rooter was used. The small signal gain of the system is about 18 db greater in the shadows when the rooter is used than without it. Noise in the shadows is therefore enhanced this much. However, with the signal-to-noise ratios present in the film scanner, the effect is not nearly so serious as had been anticipated. It is true that the shadow noise is increased, but so is the shadow detail, and the over-all effect is much more pleasing. (The rooter, after all, does not *produce* any noise in the shadows, it merely prevents the reproducing tube from suppressing this noise, along with the shadow detail, as it otherwise would.) With poorer initial signal-to-noise ratios, the results would undoubtedly be different. Not only would the higher noise level in the shadows be more disturbing in itself, but the rectification of this noise by the viewing tube would dilute the blacks with unwanted light, and thus obscure the effects of the rooting.

In many respects the signal from the dissector tube, or from a flying spot scanner, is ideal for rooting. In the first place, these devices are strictly linear, and there are no appreciable shading signals, or spurious modulations of the signal from one part of the picture by detail in other parts (such as halations, and the like). Secondly, a true black reference may be obtained; there is no question as to where to begin rooting. Thirdly, the noise which is already low in the high-

lights (or can be made so with enough light) is even less in the shadows. Let us consider this last point more fully.

The action of an ideal rooter may be expressed as

$$y = x^{1/n},$$

where

x = normalized input signal amplitude
y = normalized output signal amplitude
(x and $y=0$ for black, 1 for maximum white)
n = exponent of the rooting law.

The small signal transmission of the rooter is then given by

$$\frac{dy}{dx} = \frac{1}{n} x^{(1/n)-1}.$$

A small input disturbance Δx, will produce an output

$$\Delta y = \frac{1}{n} x^{(1/n)-1} \Delta x.$$

Now in a properly designed dissector or flying spot scanner, all the noise will be shot noise and the noise power will be proportional to brightness. Therefore, for such a system:

$$\Delta x = N x^{1/2}$$

where N = root-mean-square noise amplitude in the highlights. Thus the root-mean-square noise amplitude out of the rooter will be

$$\Delta y = \frac{1}{n} x^{(1/n)-(1/2)} N.$$

For $n=2$, i.e., a "square rooter," this reduces to

$$\Delta y = \frac{N}{2}.$$

The noise in the output signal is now independent of the brightness x, and everywhere 6 db *less* than in the highlights of the input signal. The resulting signal is indistinguishable from that which would be obtained by adding an independent noise of root-mean-square amplitude $N/2$ to a noise-free signal.

For a 2.5-root device, $n=2.5$, and

$$\Delta y = \frac{N}{2.5} x^{-1/10}.$$

Here the output noise is about 8 db less for $x=1$, and 4 db less for $x=0.01$.

Thus, if the camera noise is all shot noise, then over a 100-to-1 brightness range ($x=0.01$ to $x=1$), rooting of the type described will always produce less noise amplitude in the rooted signal than in the highlights of the unrooted signal. This is shown in Fig. 7.

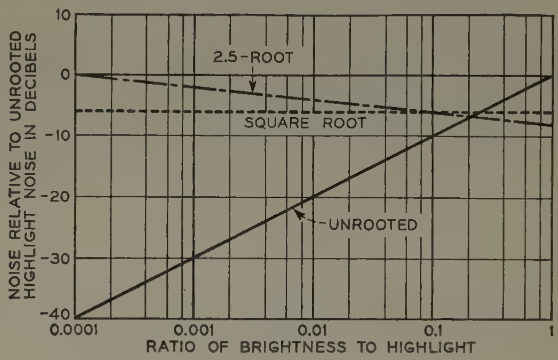

Fig. 7—Rooting of shot noise in signal.

Fig. 8 shows the situation when the camera noise is independent of brightness. In this case, the output noise exceeds the input highlight noise for all brightnesses less than one fourth to one fifth of the highlight.

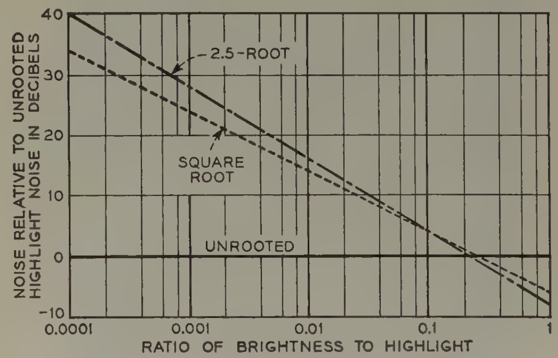

Fig. 8—Rooting of noise independent of signal.

In the film scanner in which the present rooter is used, the camera noise is all shot noise and in the highlights the ratio of peak-to-peak signal to root-mean-square noise is about 40 db. In the output of the rooter, the noise therefore is about 44 to 48 db down on the peak-to-peak signal depending on the brightness. From these figures it appears that a rooter might also be used to advantage with flying spot scanners. The successful use of a rooter with present image orthicons, where the noise is, if anything, greater in the shadows than the highlights, and where black level is harder to obtain accurately, appears questionable.

Methods of Calibrating Frequency Records*

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Summary—**A frequency record cut in lacquer, while an FM calibrator was used to measure the amplitude of motion of the cutting stylus, was processed and calibrated by the following means: a "static" method in which the amplitude of the recorded groove was measured by means of a displacement sensitive gauge; a variable speed method in which the velocity was measured with a pickup while the same reproduced frequency was maintained by rotating the disk at different turntable speeds; the reflected-light-pattern method commonly used in evaluating the recorded velocity; and by direct measurement of the recorded amplitude as pictured in photomicrographs.**

I. Introduction

RESPONSE MEASUREMENTS of disk reproduction systems, or of pickups, require the use of records of known frequencies and recorded levels. Normally the reproduced frequency will be the same as that used during recording unless a difference in speed exists between the playback and recording turntables. If a difference does exist, the exact frequency can usually be determined, easily and accurately, by comparing it with a known frequency and thus presents no problem. Evaluation of the recorded level, however, is more difficult and requires a knowledge of the amplitude of the recorded wave, or of its velocity, which is a function of amplitude and frequency.

It is the purpose of this paper to discuss several methods of calibration that were used during the evaluation of a particular frequency record. Since the calibration principles with which we are concerned are widely different, it seems desirable to describe the methods first, setting forth their advantages and limitations, and then to discuss the results of our experiments. It is thought that by doing so, the reader, having the different principles of calibration in mind, will find the comparison less confusing. For the same reason, it is thought desirable to explain some of the terms commonly used in disk recording before proceeding with the calibration methods.

II. General

A. Constant Amplitude and Velocity Recording

It is the usual practice in recording phonograph records to cut a groove from the outside towards the center of the disk in such a manner that the unmodulated groove has the form of a spiral. When modulation is applied, the stylus moves at right angles to the spiral and varies about the mean in accordance with the frequency and amplitude of the signal being applied. If the change in the spiral radius for each revolution of the turntable is slight, high amplitudes of modulation may cause the stylus to cut into the adjacent grooves. To avoid this form of overcutting it has become common practice to cut the lower frequencies, where the high amplitudes are apt to occur, on a constant amplitude basis. That is, signals having the same voltage as applied to the recording amplifier, but of different frequencies, are recorded with equal amplitudes. This form of recording is illustrated by f_1 and f_2 of Fig. 1(a). For the higher frequencies it is the usual practice to design the cutter so that the amplitude of cut decreases in proportion to the frequency increase or so that the velocity of stylus motion remains constant. Constant velocity recording is illustrated by f_2, f_3, and f_4 of Fig. 1(a).

Actually, constant velocity and constant amplitude recording characteristics are not strictly adhered to in cutting phonograph records. It is common practice to pre-emphasize, or tip up, the higher frequencies in recording in order to obtain a better signal-to-noise ratio, and in addition, some recording companies prefer a slight pre-emphasis at the lower frequencies. The cutter is usually designed, however, for constant-amplitude and constant-velocity operation and the pre-emphasis is obtained by electrical means. The "crossover" or "transition" frequency between the two modes of operation varies with recording companies, but it usually occurs between 300 and 500 cycles. For the frequency record under discussion the transition frequency was 500 cycles; frequencies lower than this were recorded on a constant-amplitude basis, while the velocity was maintained constant for the higher frequencies.

B. Stylus Velocity

If we consider the excursion of the cutting stylus when a sine-wave signal is applied, we may express the equation of displacement as

$$y = a \sin wt \tag{1}$$

where

y = the displacement

a = the amplitude

$w = 2\pi f$

t = time.

To obtain an equation in terms of velocity, or change in displacement with respect to time, we differentiate the above equation.

Then

$$v = \frac{dy}{dt} = aw \cos wt \tag{2}$$

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where

v = stylus velocity.

Stylus velocity becomes greatest where the displacement curve (1) crosses the zero axis, which is the point where the slope is the greatest (see Fig. 1(b)). Mathematically, the maximum stylus velocity is expressed as

$$V_{\max} = 2\pi f a \tag{3}$$

and occurs whenever $\cos wt = 1$.

From these equations it is readily seen that in order for the slope, or the velocity, to remain constant, the amplitude must decrease as the frequency is increased (see f_2, f_3, and f_4 of Fig. 1(b)). Likewise, for the amplitude to remain constant, the velocity, or slope, must decrease in proportion to the frequency decrease. This is illustrated in Fig. 1(b) by f_1 and f_2. In calibrating sine-wave recordings by the optical pattern method, as will be shown, it is the maximum slope of the recorded groove that determines the last point of reflection, and hence, the width of the optical pattern. The measurements, therefore, give results in terms of maximum or "peak" velocities.

C. Groove Velocity

The speed of the medium relative to the stylus point is usually designated as "groove velocity." It depends

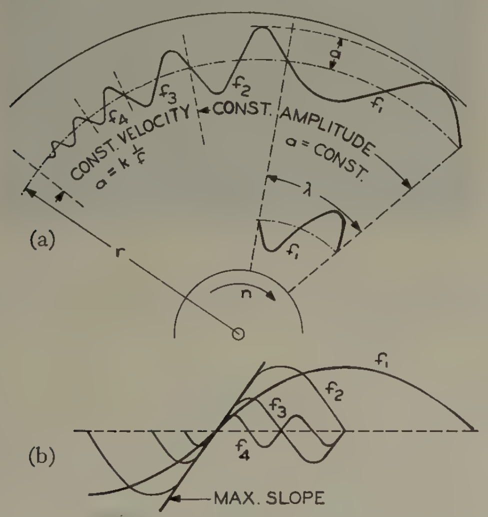

Fig. 1—Constant amplitude recording is illustrated in (a) by frequencies f_1 and f_2 and constant velocity by frequencies f_2, f_3, and f_4. In (b) note that the slope is the same for frequencies f_2, f_3, and f_4 which were plotted on a constant-velocity basis. For constant-amplitude recording the slope decreases as the frequency decreases as shown by f_1.

upon the turntable speed and the radial distance from from the center of the record to the point where the stylus contacts the groove. Mathematically it is expressed as follows:

$$V_g = 2\pi r n$$

where

V_g = groove velocity
r = radial distance from the center of the disk to the point in question
n = turntable speed.

Groove velocity reaches a maximum at the outside of the disk where the radius is the greatest and minimum at the inside. The change in groove velocity from outside to inside results in a decrease in wavelength as illustrated in Fig. 1(a) by the two traces of f_1, one near the outside and the other near the center of the disk. Both traces must be maintained in the same angular displacement if the frequency is to remain the same. The maximum slope of the inner trace of f_1 (Fig. 1(a)) is greater than the outer slope, and the increase is in proportion to the decrease in radial distance to the center of the record.

III. Methods of Calibration

A. Optical Method

A widely used method of evaluating the velocity of modulation is an optical one based upon the light reflected from the side walls of the recorded groove.[1,2] When using this method, a light pattern is visible to an observer in front of the record when parallel light rays strike the disk at a slight angle with respect to its surface. Fig. 2 is a photograph of the light pattern of a

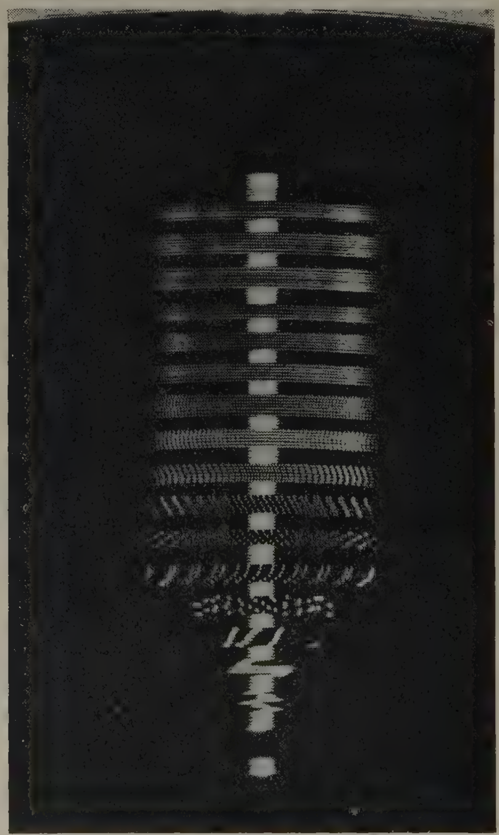

Fig. 2—Photograph of a light pattern of a frequency record. A close examination will show that the pattern is made up of numerous points of reflection.

[1] G. Buchmann and E. Meyer, "A new optical method of measurements for phonograph records," *Elek. Nach. Tech.*, vol. 7, pp. 147–152; April, 1930. Translated by J. M. Cowan, *Jour. Acous. Soc. Amer.*, vol. 12, pp. 303–306; October, 1940.

[2] B. B. Bauer, "Measurement of recording characteristics by means of light patterns," *Jour. Acous. Soc. Amer.*, vol. 18, pp. 387–395; October, 1946.

frequency record and shows the optical pattern that is visible to the observer. The light pattern is made up of numerous points of reflection from various parts of the recorded waves. For an unmodulated groove, reflections are visible along a radial line parallel to the incident light rays. When the groove is modulated, reflections occur either side of center at points where the slope, due to modulation offsets the angle due to the curvature that exists because the unmodulated groove is cut in the form of a spiral. When the mean curvature of the groove due to its spiral form exceeds the maximum slope of the recorded wave, the light is reflected back at an angle outside of the scope of the observer, and fixes the boundary where reflections are no longer visible. The width of the optical pattern is dependent upon the maximum slope of the recorded wave. For frequencies recorded at the same velocity, the maximum slopes of the waves are the same and hence the width of the light patterns equal. This correlation holds true across the entire surface of the disk, for as the recorded diameter is decreased, increasing the mean curvature, the wavelength is decreased and its slope increased, all in the same proportion. A measure of the width of the pattern permits calculation of the absolute magnitude of the recorded velocity.

It can be observed in Fig. 2 that the fifth recorded band from the bottom (400 cps) shows points of reflection that are distinct and separated a maximum amount at the center of the pattern. Toward the edges of the pattern, the reflection points come closer together and finally merge to form a bar of reflected light. Beyond the bar of reflection, points again appear, becoming smaller in size, and these eventually disappear.

Fig. 3 was constructed in order to illustrate how the bars are formed. Assume that the light is coming from above at a slight angle with the surface of the page. The

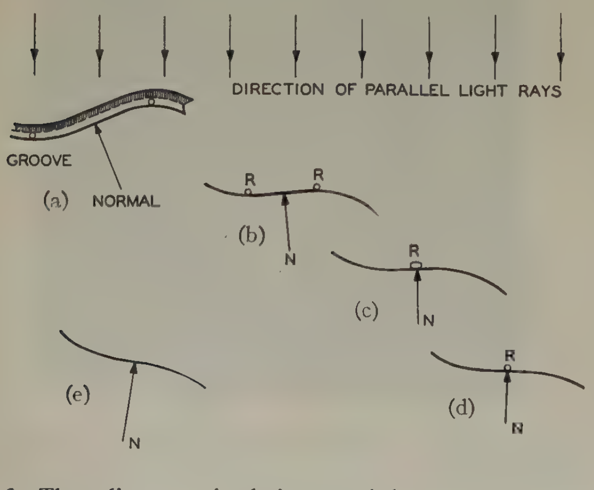

Fig. 3—These diagrams, simulating recorded grooves, show how the parallel light rays are reflected from portions of the groove sidewalls to the observer. Note that when the recorded wave is tilted, as occurs when it nears the edge of the pattern, the reflection points, indicated as circles, come closer together, finally form a bar where the two merge, then appear again as a point, and eventually are no longer visible to the observer.

reflections that occur from the side walls of the simulated recorded groove are indicated as circles. When a portion of the recorded wave is in the position illustrated by Fig. 3(a), the points of reflection occur at the peaks and valleys and are separated one-half a wavelength. As we progress towards the edges of the light pattern where the recorded wave is now rotated with respect to the incident light rays due to curvature of the spiral, the reflecting points come closer together (Fig. 3(b) and merge into a bar as illustrated by Fig. 3(c). According to theory, the last point of reflection should occur as illustrated in Fig. 3(d), where a perpendicular at the point of maximum slope of the recorded wave, is parallel to the direction of the light rays. If the wavelength is further rotated as in Fig. 3(e), the surfaces can no longer reflect light back to the observer.

To illustrate this condition a photograph, Fig. 4, was taken near the boundary of an optical pattern while an additional light was spotted on the surface of the disk

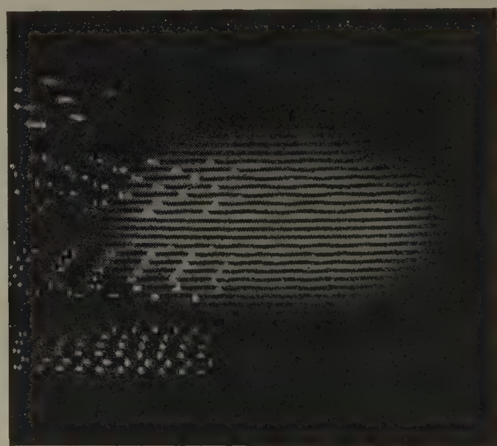

Fig. 4—A photograph near the boundary area shows the reflected points of light from the groove walls. The modulated groove is made visible by using another light to obtain a reflection from the flat surface of the disk.

to show the grooves. The merger of the two points into a bar of reflected light is clearly shown. Beyond the bar of reflection, points are again observed and these become smaller in size and finally disappear. Selecting the proper reflection points for measurements presents a problem, especially where the recorded level is to be determined accurately, for the final reflection points are not readily determined in many cases.

For recordings made at 78 rpm where the wavelengths are long, the slopes gradual and the pattern narrow, the boundary at which the reflections cease is usually sharp and well defined. But at $33\frac{1}{3}$ rpm, where the wavelengths are shorter and the slopes steeper, the pattern is wider and usually diffused and the boundary is difficult to determine. The indefinite boundary is probably due to the fact that the reflecting surfaces of the side walls have become smaller, and surface irregularities being relatively larger result in a greater diffusion of light. A sharper boundary can oftentimes be obtained

by rotating the disk so that the outermost points of reflection are continuously visible, and this expedient is resorted to in many cases. It is difficult to obtain photographs while the disk is rotating, however, since any wobble or eccentricity will result in a blurred pattern.

Better reflective surfaces may be obtained for calibrating purposes by using the metal molds or stampers from which the pressings are formed. The use of a stamper, which is a negative, with its ridges instead of grooves, is of interest, for the light pattern is now made up of reflections from the tops of the ridges, or the portions which form the bottoms of the grooves in the pressing. Since contact with the stylus is usually near the top of the groove during playback, it might be considered more appropriate to calibrate the pressing and to do so near the line of contact. This may be true; however, if the edge of the groove is included in the reflected pattern and is deformed and rough as it often is, the included surface may not be a true representation of the modulated groove and, therefore, not an ideal surface for calibration either.

The light-pattern method is difficult to apply at low frequencies since it is customary to record the lower frequencies at constant *amplitude* and not constant *velocity*, and as a result, the width of the light pattern decreases in proportion to the frequency decrease. Judging the relative flatness of recorded level by comparison then becomes difficult and, usually, measurements must be made. For a fixed position of the disk the outermost points of reflection may not be visible to the observer due to the nonoptimum position of the recorded wave with respect to the mean curvature of the groove, as can be observed in Fig. 2. This can be overcome by rotating the disk, as mentioned before, so that reflections from the outermost point are always visible.

Another factor that makes it difficult to obtain a sharply defined boundary is the appearance of interference patterns, colored or dark, that are observable throughout the reflected areas and, at some of the higher frequencies where these appear, a true pattern width is difficult to determine. A slight lateral shift in position of the observer will often change the interference patterns enough to make measurements possible.

Our measurements and observations were made using a small-filament heavy-current (10 volts, 7.5 amperes) lamp of the type used in film sound recording and reproduction. A lens, f/6.0, having a focal length of 23 inches was used to obtain parallel light rays.[3] For direct measurements a universal type of measuring microscope having the optical system on a sliding carriage with a vernier scale was used. A telescopic objective lens was employed in this case since measurements were made at a distance of about 10 feet from the disk. The disk was suspended vertically with its center at the same height above the floor as the light and the telescope. The angle between the surface of the disk and the observer was the one that gave the brightest pattern. Observations were tried close to the disk so that the individual reflection points were plainly discernible. Where the area of the field within focus is small and only limited numbers of reflections are visible, difficulty was encountered in determining the exact boundary. For this reason, increased spacing between disk and microscope so that more boundary area was included was found to be more desirable. Photographs were made for examination and subsequent calibration using a toolmaker's microscope which has a two-way sliding carriage with micrometer adjustment.

Buchmann and Meyer[1] gave the following equation for determining the recorded velocity from the width of the optical pattern:

$$v = \pi b n \tag{4}$$

where

v = recorded velocity
b = width of the optical pattern
n = turntable speed at which the disk was recorded.

They point out that in order to maintain good accuracy the distance between the observer and the disk should be many times the width of the optical pattern.

Bauer[2] has studied the problem and recommends that both reflected patterns, the one nearest the light source and the other beyond the center of the disk, be measured and combined in the following equation to give the true value of recorded velocity:

$$V = 2\pi n \frac{b_o b_i}{b_o + b_i} \tag{5}$$

where

b_i = width of the nearest reflected pattern
b_o = width of the outside reflected pattern
n = turntable speed at which the disk was recorded.

The observer should shift position slightly in order to maintain the same angle with respect to the disk when measuring b_i and b_o. If the groove shape is symmetrical and the distance between the observer and the disk is great, then the two pattern widths should be equal.

Separate pattern measurements may be used as follows:

$$v_i = n\pi b_i\left(1 + K\frac{R}{D}\right) \tag{6}$$

where

b_i = the width of the inner pattern (the one nearest the light source)

$$K = \frac{1}{1 + \cos B}$$

[3] An ordinary unfrosted 120-volt lamp when placed about 15 feet away from the disk, and without a lens, has been found satisfactory for such purposes.

B = angle between the observer and the surface of the disk

R = radius (distance from the center of the record to the groove being examined)

D = distance between the observer and the disk.

For the outside pattern the equation where the subscript o refers to the outside pattern is

$$V_o = n\pi b_o\left(1 - K\frac{R}{D}\right). \tag{7}$$

B. Calibrating While Cutting

A method of measuring the amplitude of stylus excursion while cutting has been described in articles[4,5] on the FM calibrator, and good correlation with the light-pattern method has been observed, especially for disks cut at 78 rpm. Although the calibrator may accurately measure the excursion of stylus, it does not necessarily follow that the amplitude of the resulting recorded wave is the same. For springback or cold flow of the medium and if a lacquer recording stylus is used, the width of the burnishing surface,[6] may alter the groove shape and decrease the recorded amplitude especially at the higher frequencies. Burnishing surfaces are flats formed at a particular angle along the cutting edges of lacquer styli in order to polish the side walls and so produce quieter grooves. The burnishing edge causes some lateral loading which has been measured with the FM calibrator. Wax cutting styli do not have burnishing edges and therefore do not alter the groove shape or affect the high-frequency response when cutting wax.

The frequency record[7] under discussion was made while using the FM calibrator. The approximate recording level for each frequency was determined beforehand and level adjustments made while cutting the unmodulated grooves between the frequency bands. A final adjustment was made immediately after the signal was applied. Such procedure results in an initial level that may not be correct and measurements must therefore be based upon the latter part of the recorded band. This secondary adjustment refinement is believed unnecessary as in most cases the final adjustment was found to be slight.

The master lacquer was cut laterally at $33\frac{1}{3}$ rpm, using a stylus having a bottom radius less than $\frac{1}{2}$ mil. Grooves were cut deep enough to accommodate playback styli up to 3 mils radius and still maintain contact along the side walls below the record surface. A theoretical crossover point of 500 cycles, based upon the NAB Standard lateral recording characteristic, was chosen and frequencies above this value were recorded at constant stylus velocity while those below were cut at constant stylus amplitude. The frequencies near the crossover point were recorded at levels which would result in a smooth gradual changeover from constant amplitude to constant velocity. Frequencies from 12,000 to 1,000 cycles were recorded at 1,000-cycle intervals. To assure frequency accuracy the oscillator was set for the correct Lissajous figures on an oscilloscope while using a 1,000-cycle tuning fork as the standard. Below 1,000 cycles, frequencies were recorded which have been found useful in testing, and, where possible, these were also compared with the tuning-fork standard. These precautions were taken even though a rim-driven turntable was used, as it was thought worth while to maintain the correct ratio between frequency bands. An intermodulation band of 400 and 4,000 cycles was recorded near the outside of the 12-inch disk, for stamper control. A slight spiral was introduced just ahead of the 10,000-, 5,000-, and 1,000-cycle bands for visual identification of these bands. After processing, unfilled vinyl pressings were made and these have been subjected to the calibration tests, described in this article.

C. Static Method of Calibration

If the amplitude and the frequency is known, the velocity of the recorded wave can be calculated. For the purpose of measuring the amplitude a sensitive electromechanical gauge, previously used for determining the force at the stylus tip while cutting,[8] was tried. The cutting stylus was replaced with a playback tip having a radius of 2.5 mils. The gauge was attached to a horizontal rod extending radially over and pivoted vertically about the center of the disk. Contrary to usual practice, the disk was maintained stationary while the stylus was pulled along the groove. It was moved slowly and

Fig. 5—This shows the pickup arrangement used to measure the amplitude of cut of the recorded groove. Amplitudes as low as 25 microinches were readily measured.

[4] H. E. Roys, "Experience with an FM calibrator for disk recording heads," *Jour. Soc. Mot. Pic. Eng.*, vol. 44, pp. 461–471; June, 1945.

[5] A. Badmaieff, "Push-pull frequency modulated circuit and its application to vibrating systems," *Jour. Soc. Mot. Pic. Eng.*, vol. 46, pp. 37–51; January, 1946.

[6] C. J. LeBel, "Properties of the dulled lacquer cutting stylus," *Jour. Acous. Soc. Amer.*, vol. 13, pp. 265–273; January, 1942.

[7] RCA Test Record 12-5-25 (460625–6).

[8] H. E. Roys, "Force at the stylus tip while cutting lacquer disk recording blanks," Proc. I.R.E., vol. 35, pp. 1360–1363; November, 1947.

smoothly by the simple expedient of wrapping a thread, attached to the gauge, around a shaft of small diameter. The action was so smooth and fine that the stylus tip could be stopped at the crest of even the shortest wavelength. The equipment is pictured in Fig. 5.

The method was found to be slow and tedious, but eventually sufficient measurements were made to determine the effectiveness of this method.

D. Variable Speed Method

This method is not new; it was used by Kendall and Burt as early as 1929, to measure the response of pickups. They also used the method to check the calibration of some early Victor Constant Note records. In addition, a published article[9] reveals that it was used for calibration purposes in Germany in 1929 at the same time Kendall and Burt were independently doing their work.

The principle of operation when calibrating records requires an adjustment of the speed of the turntable so that the recorded bands are reproduced at some common reference frequency. Such procedure eliminates the necessity of amplifier and pickup calibration. In practice, where a wide range of recorded frequencies is involved, several reference tones are usually necessary but these can be selected so that a good crosscheck can be obtained between them.

The variable speed turntable[10] used for these tests operates from 5 to 80 rpm and reference tones of 2,000, 700, and 200 cycles were used with an additional crosscheck at 1,000 cycles. These reference tones adequately covered the range of the $33\frac{1}{3}$ rpm frequency record which extended from 12,000 to 30 cycles. When the recorded frequency differs greatly from the reference tone, noise becomes a problem and band-pass filters were found necessary for attenuating the noise either side of the passed frequency.

E. Photomicrographs of Grooves

A method of determining absolute groove modulation amplitude by the use of photomicrographs is relatively simple and seems to give excellent results. A metalloscope capable of making photographs with a range of magnifications from 26 times to 2,000 times was used for these tests. It was found that a highly reflecting surface was most suitable for photographing with this equipment. Therefore, all photomicrographs were made from a chrome-faced "stamper" which is a negative metal part used for pressing records.

When record grooves are examined at high magnification, it becomes apparent that the groove walls and bottom are not perfectly smooth. Fine lines, caused by cutting stylus imperfections, are visible along the grooves (see Fig. 6). Although these markings might normally be considered undesirable, they prove to be of considerable value in making measurements of groove modulation. Any of these lines at or near the bottom of the groove which display clear sinusoidal wave forms may be focused on. This gives a much clearer tracing than when focused on the edge of the record groove which is usually distorted or indistinct.

To insure the accuracy of magnification, the metalloscope was calibrated with a stage micrometer. A photograph of the micrometer scale was also taken for verification.

The photomicrographs may be measured with a toolmaker's microscope very easily. A toolmaker's microscope is equipped with cross hairs and a table which can be moved with micrometer adjustments in either direction. These adjustments are calibrated very accurately with divisions of 0.0001 inch. If the photographs of the higher frequencies are magnified 250 times, one division of the microscope will correspond to 0.4 microinches. A 10,000 cps signal recorded with a normal stylus velocity of 5 cm per sec will have an amplitude of approximately 62.5 microinches peak to peak. Therefore, one division of the microscope scale will be well under 1 per cent of the lowest amplitude that is likely to be encountered.

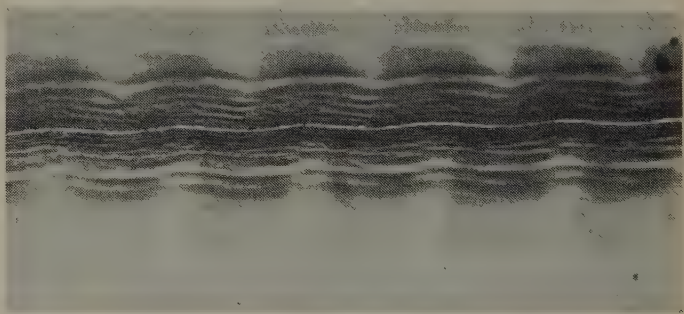

Fig. 6—A photomicrograph of a 6,000-cycle recorded wave. Because of slight nicks or imperfections in the cutting stylus the sine-wave trace is plainly visible, thus permitting a measurement of the amplitude of the recorded signal. A magnification of 250 times was used to obtain this picture.

IV. Results

A. Static and Variable Speed Tests

The static and variable speed methods gave similar results. Below 1,000 cycles good agreement was observed between the two methods, and also with the FM calibration measurements which were made while cutting (see Fig. 7).

Above 1,000 cycles a progressively greater reduction in high-frequency response was observed with both methods and this loss increased as the vertical force on the stylus was increased (see Fig. 8). Playback loss,[11] or reduction in amplitude of stylus motion due to tip size, relative to the recorded wavelength and yield of the

[9] E. Meyer and P. Just, "Frequency curves of electrical pickups and mechanical gramophones," *Elek. Nach. Tech.*, vol. 6, pp. 264–268; July, 1929.

[10] H. E. Haynes and H. E. Roys, "A variable speed turntable and its use in the calibration of disk pickups," Proc. I.R.E., vol. 38, pp. 239–243; March, 1950.

[11] O. Korner, "Playback loss of phonograph records," *Jour. Soc. Mot. Pic. Eng.*, vol. 37, pp. 569–590; December, 1941.

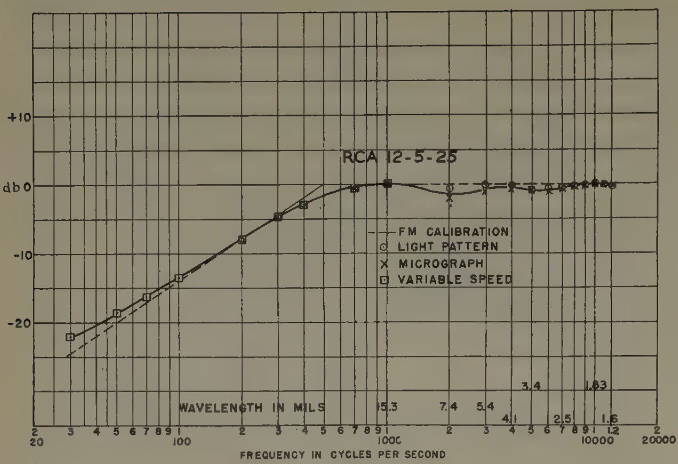

Fig. 7—Response characteristic of the record used for the investigation. The dashed curve shows the intended characteristic. The points obtained by the different methods of calibration are indicated.

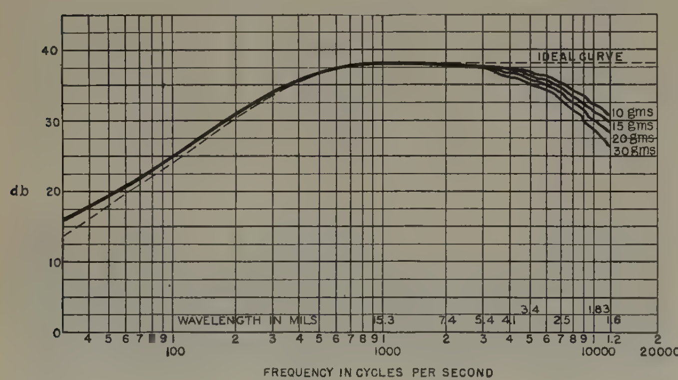

Fig. 8—Results obtained by the static method are similar to those obtained by the variable speed method illustrated here. A loss in high-frequency response exists due to the yield of the record material. The loss is dependent upon the vertical force and the radius of the stylus tip.

record material still exists even though the rotational speed of the record is reduced during playback. Decreasing the turntable speed during reproduction does not alter the recorded wavelength. Therefore, both methods are suitable for low-frequency calibration purposes only, and not higher frequencies where the wavelengths are short with respect to the stylus radius. The data illustrated in Fig. 8 were obtained with a lightweight magnetic pickup having a stylus tip of 2.9 mils radius. These data indicate that the wavelength should be at least three times the stylus radius to minimize the playback loss. The record was a vinyl pressing.

Both methods, however, offer excellent means of studying playback loss while maintaining substantially constant mechanical impedance of the pickup. In the static method where the stylus moves along the groove at an extremely low rate, the mechanical impedance is essentially that of the stiffness of the moving system. For the variable speed method the mechanical impedance of the pickup depends upon the frequency at which the tests are made. For any reference frequency it should remain constant and for one such as 2,000 cps, which may be near the fundamental resonance of the moving system (free vibration in air) where the pickup is easy to actuate, the impedance should be low.

While making measurements with the static method, a slow continuous yield of the record material was noted and considerable judgment had to be exercised during these tests. What effect the dynamic elastic properties of the material may have on record reproduction is not known yet; however, it is not believed to be a serious factor.

For ease of manipulation and speed of operation the variable speed method is preferred to the static method.

B. Photomicrograph Results

Good results were obtained with this method, especially at the higher frequencies where the wavelengths are short so that many of them appear within the focusing area. At the lower frequencies a decrease in magnification of the camera is necessary in order to obtain a sufficient number of wavelengths within the focusing field since they are now longer. Accuracy is also impaired as the curvature of the modulation approaches the average curvature of the record groove. The success of the method depends largely upon tiny imperfections of the cutting stylus which is inconsistent with good recording practice where every effort is made to obtain smooth quiet grooves free from scratches.

When the method can be applied, it offers an excellent means of measuring the amplitude of cut and thus determining the recorded velocity independently of the optical pattern method. The method is less reliable for the low frequencies where the wavelengths are long and the curvature of the spiral must be taken into account.

C. FM Calibrator Results

The FM calibrator used to measure the amplitude of stylus motion while recording gave results that checked extremely well with the other methods both at the high and low frequencies. The good agreement at the high frequencies is unusual and the method cannot be relied upon when cutting lacquer where springback may occur. When cutting wax with a sharp-edged stylus without burnishing surfaces, the method may be quite accurate. It is reliable and accurate at the lower frequencies and so may supplement the optical method where difficulty is usually encountered in evaluating the longer wavelengths. In this particular record, a discrepancy at the low-frequency end between expected and actual recorded level resulted due to a forgotten low-frequency loss in the calibrating amplifier that caused us to raise the recording level in order to obtain the proper meter reading.

D. Optical Method

The optical method still appears to be the best overall method of calibrating recorded disks. If an absolute velocity measurement is wanted, the width of both reflected patterns must be measured and averaged.[2] Some judgment is required in determining the boundary especially at the higher frequencies. A study of the photograph of Fig. 4 reveals that it is difficult to decide which spot of reflected light represents the case of Fig. 3(d) and hence is the one that should be used. Richard Bierl[12] concludes from a geometrical study that the last point of reflection does not appear at the point of maximum slope but the error even for unfavorable conditions is less than one per cent. It is quite likely that some of the minute spots are reflections beyond the optimum point and are visible only because of irregularities in the record groove. This being the case, the boundary that we are seeking must be somewhere between the bar of reflection and the last-minute spot. In our case, the attempt was made to obtain an absolute measurement of the 1,000-cycle band and a relative width measurement of the other bands. A point between the reflected bar and the last tiny point was taken, one that appeared about equally discernible at each different high-frequency band.

Fig. 9—Photograph of the optical pattern without any color filter.

Photographs of the reflected pattern of the metal stampers were taken, some of which were made with a red filter before the camera lens so that the result was essentially that of using a monochromatic light. In this particular case a very sharp image was obtained and it took several different exposures without the red filter to obtain a negative of equivalent sharpness (see Fig. 9). However, there appears to be no advantage in using a monochromatic light source. When interference patterns are present, and they were observed in all cases, it appears desirable to use a system of uniform color sensitivity so that the color effect of the interference patterns is averaged out.

For low-frequency measurements it is necessary to rotate the disk so that a sharply defined boundary is obtained. Where two frequencies are combined such as those used for intermodulation tests, it also becomes necessary to rotate the disk in order to obtain readings.

A wide pattern such as encountered at high modulation levels and low turntable speeds may exceed the limiting conditions considered by Bierl and cause an appreciable error.

V. Conclusions

Considering the results of all of the different methods, FM calibrator, variable speed, photomicrograph, and light pattern, we came out with results remarkably close to each other as illustrated in Fig. 7. Each method offers some advantages. It is not expected that the FM calibrator will always check so well at the higher frequencies due to other factors that enter in, such as width of the burnishing edge and the hardness and cold flow of the medium being cut. If groove burnishing is not necessary and sharp cutting edges are used, then the FM calibrator may prove useful for calibration purposes. The static and variable speed methods are valuable for measurements at low frequencies where the wavelengths are long. Neither are applicable at high frequencies where losses due to yield of material and the physical size of the reproducing tip become appreciable. On the other hand, the photomicrograph method is particularly suited for short wavelengths, the disadvantage being, however, that it requires a stylus with imperfections in the cutting edges and this is contrary to good recording practice. The optical method therefore, appears to be best suited for calibration purposes. It is reliable and not too difficult to apply. Comparative measurements between frequency bands can be made easily and in many cases, the relative response, determined in this manner, is all that is wanted.

Acknowledgment

The authors wish to acknowledge the photographic work of H. N. Collins who produced the prints of Figs. 4 and 9, and many others not included in this paper.

[12] R. Birel, "The error in the light band width of phonograph records," *Akus. Zeit.*, vol. 5, pp. 145–147; May, 1940.

Combined Search and Automatic Frequency Control of Mechanically Tuned Oscillators*

J. GREGG STEPHENSON†, MEMBER, IRE

Summary—**A system is described whereby an oscillator is either swept continuously across its tuning range or is tracked to an incoming signal. The novelty of the system is that it automatically switches from the sweeping regime to the tracking regime when a signal is encountered and returns to the sweeping regime when the signal vanishes.**

The control is applied to a superheterodyne receiver whose intermediate frequency is 30 Mc. In the searching mode, the local oscillator is mechanically swept back and forth across its entire tuning range. When a signal is received in the intermediate-frequency channel, the sensing circuit transfers the system to the tracking regime so that the local oscillator keeps the signal centered on the intermediate-frequency passband. In this application, the system is capable of varying the frequency of a mechanically tuned ultra-high-frequency triode oscillator over a 100-Mc range, or of holding it to within 15 parts per million at 1,300 Mc.

Introduction

THIS PAPER describes a difference-frequency servomechanical method for automatic frequency control of mechanically tuned oscillators, combined with an automatic searching feature that embraces a wide pull-in range. The method is applicable to radar, communication, or other equipments having a fixed intermediate frequency.

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† Airborne Instruments Laboratory, Inc., Mineola, L. I., N. Y.

I. Illustrative Application

The essential features of this method are illustrated in Fig. 1. Output from the local oscillator under control is mixed with the reference signal in a conventional crystal mixer. The resulting intermediate-frequency output is amplified, limited, and connected to the automatic frequency control (AFC) unit, which drives a servo motor to tune the controlled local oscillator. The AFC unit contains one further stage of intermediate-frequency amplification, a discriminator, pulse stretcher, balanced modulator, servo amplifier, and a special sensing circuit that will later be described in detail. In the searching regime, the controlled oscillator frequency is swept continuously back and forth across its entire range until a reference signal appears at the input to the AFC unit. The sensing circuit then automatically changes to normal AFC operation, and the controlled oscillator locks in relation to the reference signal so as to maintain a constant difference frequency as long as the signal is present. Automatic frequency control (AFC) is thus made available over a pull-in range much greater than the discriminator bandwidth.

The circuit was designed for controlling an L-band coaxial-line triode oscillator tuning over a total range of 100 Mc at 1,300 Mc. A 30-Mc intermediate frequency having a bandwidth of 2.5 Mc was employed. Because known electronic tuning techniques were inadequate for

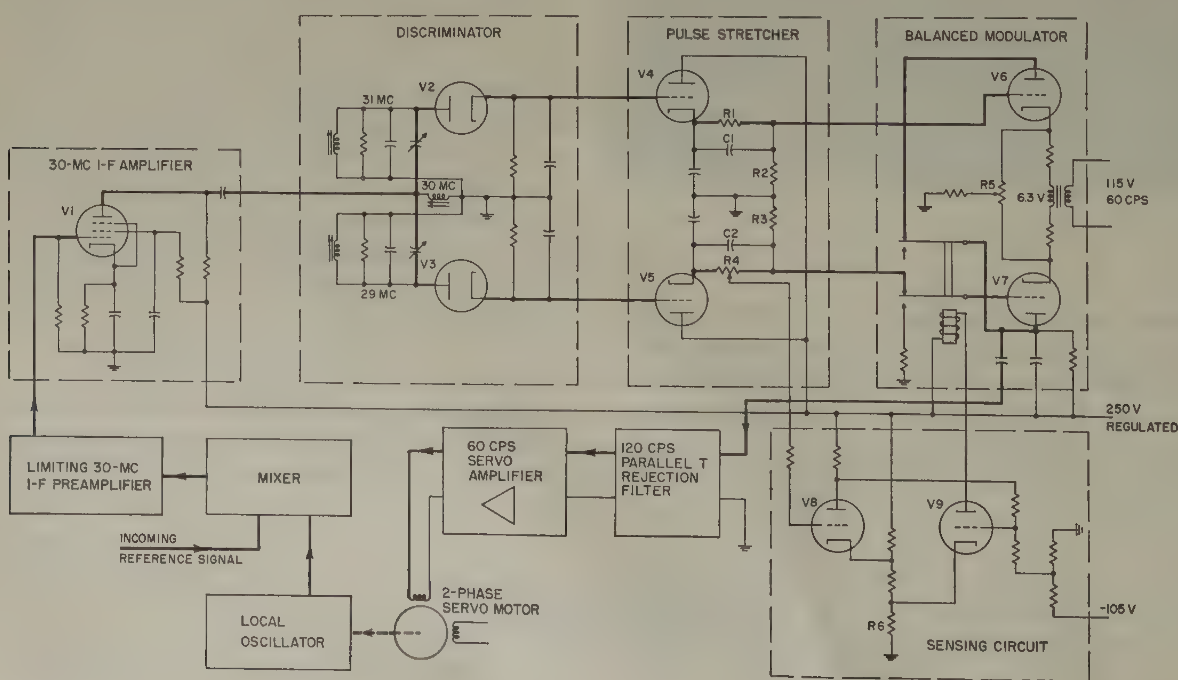

Fig. 1—Combined block diagram and schematic of AFC system.

the relatively wide pull-in range required in this case, mechanical tuning was necessary. A scheme combining an automatic searching function with normal AFC appeared to offer the simplest solution. The problem, therefore, resolved into two parts: (1), design the servomechanical AFC, and (2) design suitable control circuits to provide automatic searching. The first part involves the application of the theory of servomechanisms to AFC systems, and has been covered previously in the literature.[1–3] Principal emphasis in this paper will be on the control circuit requirements of wide-range difference-frequency servomechanical AFC systems.

This AFC method is ordinarily applied to heterodyne systems where the incoming signal is converted to an intermediate frequency. The following discussion will assume that the controlled local oscillator has a tuning range considerably greater than twice the intermediate center frequency, and that continuous rotation of the tuning motor produces cyclical variation of the oscillator frequency throughout the entire tuning range as shown in Fig. 2.

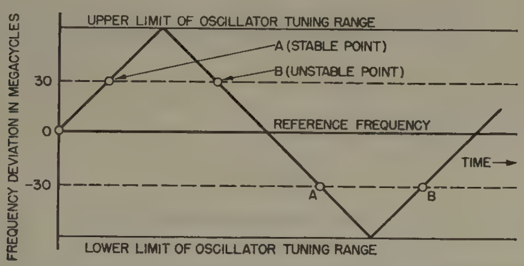

Fig. 2—Tuning characteristics of AFC system.

II. Sensing Circuit

The following considerations entered into the design of the sensing circuit:

1. A positive, rapid transition between the search and normal AFC regimes must be provided.
2. The transition must be controlled by the presence or absence of the desired signal.
3. The circuit must distinguish between stable and unstable operating points, allowing locking only at stable points.

The sensing circuit and its connections in the AFC system are shown in Fig. 1. The switching between the two regimes of operation is accomplished by a small sealed relay under control of the sensing circuit. The double triode flip-flop sensing circuit is similar to one described first by Schmitt,[4] and has been altered to achieve triggering on one of the grids with a positive voltage of about 15 volts. This triggering voltage is derived from one side of the staggered discriminator characteristic (described later). The transition between the relaxed and energized condition of the relay is a rapid snap action resulting from the cross connection of the plate of $V8$ to the grid of $V9$, and the common coupling of the cathode resistor, $R6$. This satisfies the first design consideration.

It will be noted that a positive direct voltage derived from the supply bus is applied to the cathode of $V8$. This, together with potentiometer $R4$ in the pulse stretcher circuit, enables control of the voltage level at which conduction is transferred from $V9$ to $V8$. Common cathode coupling resistance, $R6$ is important in determining the difference between tripping and release levels at the grid of $V8$. For example, when $V9$ is conducting (relay energized), the resulting voltage drop across $R6$ serves to increase the voltage at the cathode of $V8$, and hence raise the triggering level at the grid of $V8$. But when $V8$ is conducting (with $V9$ cut off), less current flows through $R6$ and the release level is therefore lower than the triggering level. These features satisfy the second design consideration, as will be shown in later discussion.

III. Switching Under Stable Conditions

A special problem arises in AFC systems of this type when the local-oscillator frequency may be varied over a range considerably greater than the intermediate frequency. This problem results from the presence of both stable and unstable operating points (insofar as AFC is concerned) where 30 Mc difference frequency signals are produced at the input to the AFC system, and provision must be made to distinguish between them.

Examination of Fig. 2 shows that there are, in general, four points in the oscillator tuning cycle where a signal will appear at the input to the AFC system. These points appear as conjugate pairs, marked A and B in Fig. 2, and are distinguished by the direction of travel of the signal through the intermediate-frequency passband for a given direction of motor rotation. The A points, for example, show traversal from the low- to high-frequency edges of the passband. If the servo connections are correct for stable, null-seeking operation at the points marked A, the system will lock on frequency at either A point, and normal AFC will result. At the conjugate points B, however, the servo connections are effectively reversed, because a given direction of motor rotation produces a frequency change just opposite to that at the stable points. These are unstable points, and the sensing circuit must reject them and allow the local oscillator frequency to progress to the next stable operating point and lock there.

Differentiating between stable and unstable operating points is based on time and voltage relationships, and is accomplished as follows. By arranging the sensing circuit so that it is triggered by a voltage derived from the low-frequency peak of the discriminator, it will be

[1] V. C. Rideout, "Automatic frequency control of microwave oscillators," Proc. I.R.E., vol. 35, pp. 767–771; August, 1947.
[2] F. A. Jenks, "Simplified microwave AFC," Parts I and II, *Electronics*, vol. 20, pp. 120–125, November, 1947; and pp. 132–136, December, 1947.
[3] H. Lauer, R. Lesnick, and L. E. Matson, "Servomechanism Fundamentals," McGraw-Hill Book Co., New York, N. Y.; 1947.
[4] O. H. Schmitt, "A thermionic trigger," *Jour. Sci. Instr.*, vol. 15, pp. 24–26; January, 1938.

tripped early with reference to the intermediate-frequency passband at the stable A points, and late at the unstable B points. At the stable points there is ample time for the discriminator to take control of the tuning and pull the frequency into the proper value, but at unstable points the tripping is delayed past dead center. This causes the motor to continue tuning away from crossover frequency, because the servo connections at the unstable points are opposite to those necessary for stability. The signal is, therefore, tuned out of the passband until the decrease in signal causes the searching operation to be resumed. The choice of polarities throughout the circuit must be co-ordinated properly so that the direction of tuning on search is such as to approach stable points from the low-frequency side of the passband. Reversing the leads to one phase of the motor interchanges the stable and unstable points, but also reverses the direction of tuning in search, and the system still operates normally.

It should be noted that the intentional differential between trip and release points in the operation of the sensing circuit enables control to be retained by the discriminator at stable points for frequencies somewhat beyond the 30-Mc center frequency. The net result of this control action is that the system searches for a reference signal, and upon finding one, stops and locks at any stable point, but tunes through unstable points after only a brief hesitation. This fulfills the third design consideration.

IV. Circuit Details

The system shown in Fig. 1 represents a practical solution to the design requirements. For proper AFC operation the intermediate-frequency preamplifier must deliver a limited output for either continuous wave or pulsed signals, as otherwise the effective gain of the servomechanical loop would be subjected to unpredictable variations and the subsequent control circuits could not be properly adjusted.

Intermediate-frequency signals are further amplified and applied to the primary of a capacitively coupled, stagger-tuned discriminator. For continuous-wave signals, the output of the discriminator diodes may be applied directly to the following stages, but if the system must operate from a pulsed reference signal, pulse stretching is necessary to obtain direct voltage for control of following circuits. If the requirements for the servo tuning system indicate the desirability of error-rate damping, an RC differentiating network may be interposed at this point, as shown by $R1$, $R2$, $C1$, and $R3$, $R4$, $C2$ in Fig. 1.

The output from each discriminator diode (and whatever pulse-stretching and differentiating circuits follow) consists of a positive direct voltage to ground. These voltages in each half of the symmetrical circuits will be equal when a suitable signal is centered at the crossover frequency, as illustrated in the typical discriminator characteristics of Fig. 3. In addition to performing the error sensing function for normal AFC operation, the discriminator output is also utilized to actuate the sensing circuit as will presently be explained.

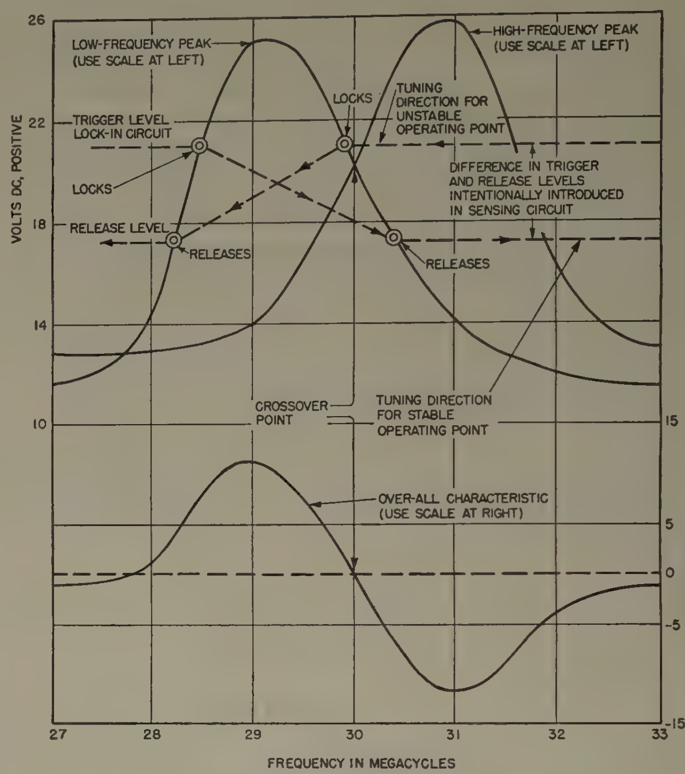

Fig. 3—Typical discriminator characteristics.

For normal AFC operation, the balanced modulator stage serves to convert the direct voltages at the discriminator output into a 60-cps voltage having a magnitude proportional to the frequency error, and with a 180-degree phase reversal occurring on each side of the 30-Mc crossover frequency. Balance potentiometer $R5$ permits the center frequency to be adjusted to compensate for slight variations in discriminator alignment, and to accommodate changes due to tube replacement. Because this stage has a residual 120-cps output at the null point, it is followed by a parallel-T rejection filter tuned to 120 cps. The 60-cps voltage emerging from the balanced modulator and parallel-T filter is then amplified in a conventional servo amplifier that produces enough power to drive the variable phase of the servo motor.

The action of the discriminator on the sensing circuit is shown in Fig. 3. Each of the staggered secondary outputs appears as a positive direct voltage to ground, the system being set up so that the sensing circuit is tripped from the low-frequency peak. Assume first that there is no signal present at the input to the AFC unit. No tripping voltage will appear at the grid of $V8$ so this triode will be cut off. Therefore, $V9$ conducts and energizes the relay. In this condition, the modulator stage is completely unbalanced, causing the servo motor to run continuously in one direction, driving the local-oscillator frequency until a signal is intercepted.

When a signal of sufficient magnitude to exceed the tripping threshold at the grid of $V8$ appears, this tube conducts suddenly and cuts off $V9$, thus de-energizing the relay and connecting the discriminator output directly through to the modulator for normal AFC action. Thus, as long as a sufficiently large signal is present, the normal AFC regime will hold, while in the absence of a signal, the tunable oscillator will search continuously.

In the particular system described, consistent locking of oscillator frequency within about 15 parts per million is possible. Operation is automatic and, when once locked on a reference signal, the 30-Mc difference frequency will be held regareless of drifts in either the reference signal or the local oscillator. The delay in correcting for small frequency changes is well under one-tenth of a second, but because of mechanical inertia this system obviously cannot compare with electronic methods in speed of response. Where short-time stability considerations in the oscillator are important (over periods up to 0.1 second or more), a mechanical AFC system of this type is more advantageous than those electronic methods that cause the oscillator frequency to vary continuously about the desired value. The time required for a complete search cycle depends on many factors such as the frequency range of the oscillator, bandwidth of the discriminator and intermediate-frequency system, inertia of moving parts, and gear ratios. For most applications, the search time will be an appreciable part of a minute.

ACKNOWLEDGMENT

The development of the system described was carried out under the sponsorship of Watson Laboratories of the Air Force. The author wishes to acknowledge the helpful co-operation of a number of his colleagues in making this development possible.

A Test of 450-Megacycle Urban Area Transmission to a Mobile Receiver*

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Summary—**Measurements were made of mobile radio-telephone transmission at 450 Mc in New York City using frequency modulation. Comparison was made with transmission at 150 Mc using identical speech modulation. Effective radiated powers were about equal. Direct comparison tests were made with the receivers installed in a moving automobile. The transmitter and the receiver used at 450 Mc were developed especially for the job. The receivers used at the two frequencies had substantially the same noise figures. The tests permitted estimates of the relative magnitudes of the shadow losses at the two frequencies and included measurements of rf noise. Subjective tests of circuit merit comparing the two frequencies were made by a number of observers.**

THE PROJECT

FROM THEORETICAL considerations it had been predicted that frequencies as high as 450 Mc might be useful for providing mobile telephone service. In order to determine whether such frequencies could be used for telephone transmission to vehicles in urban areas, a series of tests was undertaken with measuring equipment located in a test automobile and with transmitting equipment located atop the Telephone Building at 32 Avenue of the Americas, New York, N. Y.

An FM transmitter operating at 456.090 Mc[1] was located on the roof of the Telephone Building. Bandwidth and frequency deviation of the system were made the same as in the present 150-Mc system (i.e., ±10 radians at the final frequency) wherein the transmitter is also located on the roof of the Telephone Building. Both antennas were about 460 feet above street level. The effective radiated power of the 450-Mc system was equivalent to about 100 watts from a coaxial dipole (24 watts actual power plus 6-db antenna gain). Direct comparison tests were made between the experimental system and one of the channels of the regular 150-Mc system, which uses a coaxial dipole antenna radiating about 200 watts. Effective radiated power was thus about 3 db less at 450 Mc than a 150 Mc.

The experimental 450-Mc receiver consisted of a radio-frequency preamplifier and mixer, the mixer output being fed into a standard Western Electric 38B Mobile Radio Receiver operating at an input frequency of about 160 Mc. A standard 38B Radio Receiver was used directly for the 150-Mc tests so that the bandwidths of the receiving systems, which are determined entirely by the 38B Radio Receivers, were closely comparable for the 450- and 150-Mc tests. The noise figures[2] for both receivers were measured as about 8 db; this value implies good receiver design and alignment.

A survey was made of reception at both frequencies in a car equipped with quarter-wave whip antennas. At many locations in Manhattan, at some points in the

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† Bell Telephone Laboratories, Inc., New York, N. Y.

[1] The actual test frequencies of 456.090 and 152.63 Mc have been rounded off to 450 and 150 Mc for convenience in this discussion.

[2] H. T. Friis "Noise figures of radio receivers," PROC. I.R.E., vol. 32, pp. 419–22; July, 1944.

Bronx, and at a few spots in Westchester County, the strength of the radio-frequency signals across the radio receiver inputs was measured, as well as the audio-frequency signal and noise outputs of the two receivers; the same audio-frequency modulation was used in the two transmitters. On a separate series of test runs, judgments of circuit merit were made at several locations by four different observers. All measurements were made with the car in motion at normal driving speed.

Comparative Results at the Two Frequencies

The radio-frequency signal strengths and circuit merits found in Manhattan are shown on Fig. 1. Signal strengths are given in the form of equal signal contours, expressed in db above 1 microvolt across the radio receiver inputs. Circuit merits[3] are shown by dots with numbers beside them.

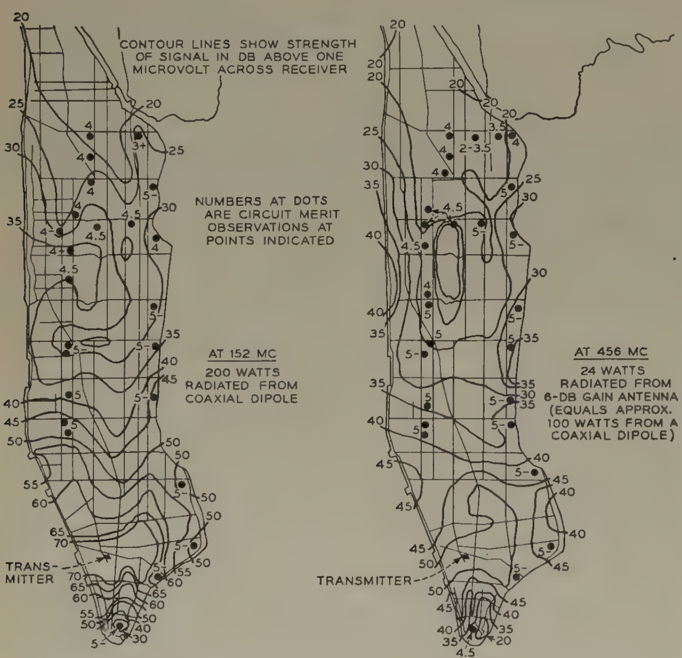

Fig. 1—Transmission on a single FM channel in New York, N. Y.

The signal strength contours must be interpreted in a rather special way. Each value represents an average over a distance of 200 to 300 feet. On the perimeter of Manhattan, the values noted represent the signals received on West Street below 72 Street, on Riverside Drive between 72 Street and 125 Street, on the Henry Hudson Parkway above 125 Street, and on South Street and the East River Drive on the other side of Manhattan. Inland, however, the values represent the signal strengths found on east-west streets in the middle of the blocks. These signal strengths are of interest because they are measured in the spots to which transmission is most likely to be marginal. It was observed that the received signals averaged about 15 db higher at both frequencies on the inland north and south avenues than in adjacent crosstown streets. This latter detail is not shown on the contour maps.

Comparison indicates that the average received signal at the car at 450 Mc is about 4 db weaker than at 150 Mc.[4] Difference in radiated power accounts for 3 of the 4 db. Close correspondence of the average received signals, when taken on an equal radiated power basis, indicates little difference in the net effect of city buildings at the two frequencies.

In tests at 21 suburban locations in the Bronx and Westchester, on parkways and numbered routes, from 13 to 26 miles from the transmitter, the received signal voltage averaged the same at the two frequencies on the basis of equal radiated powers; individual values at 450 Mc ranged from 8 db above to 14 db below corresponding 150-Mc values.

Performance of the radio circuit also depends upon the ambient radio-frequency noise. In many locations in Manhattan, and at all the test points in the Bronx and in Westchester County, measurements were made of the amount of radio-frequency signal required to override ambient noise and produce a specified audio signal-to-noise ratio at the receiver output. On the average, 10 db more radio-frequency signal was required at 150 Mc than at 450 Mc. Test car ignition noise was suppressed.

The factors of signal strength, ambient noise, and signal fluctuations are all combined in the judgment of circuit merit. Judgments of circuit merit were made at several locations in Manhattan. As noted in Fig. 1, the circuit merits at the two frequencies were estimated to be about the same (4 observers). This indicates that for equal transmitted powers, the reception would have been slightly better at 450 Mc than at 150 Mc.

All of the factors taken together tend to confirm the expectation that transmission at the higher frequency would be suitable for mobile telephone coverage in large cities.

Other Results

Comparison with Computed Signal Amplitude

The test results are compared with computed received signal voltage across a 100-ohm receiver in Figs. 2 and 3. The measured values are corrected to a reference of 100 watts of power delivered to a coaxial dipole. The computed values are for 100 watts in a standard dipole 460 feet above ground plane and assume smooth spherical earth.

In Manhattan, the average difference between computed and measured values is about 30 db, practically all the difference lying in the range 20 to 40 db. There is no significant distinction between the two transmitted frequencies, in this regard, nor between receiving locations (all urban) from about 3 to 11 miles from the

[3] The scale of circuit merits is as follows:

Merit	*Description*
5	Excellent
4	Good
3	Just commercial
2	Poor
1	Hopeless

[4] This is an over-all average for urban and suburban locations at distances greater than about 2 miles from the transmitters. At shorter distances, the difference in vertical directivity of the two transmitting antennas tends to depress the 450-Mc signals more than 4 db below the 150-Mc signals.

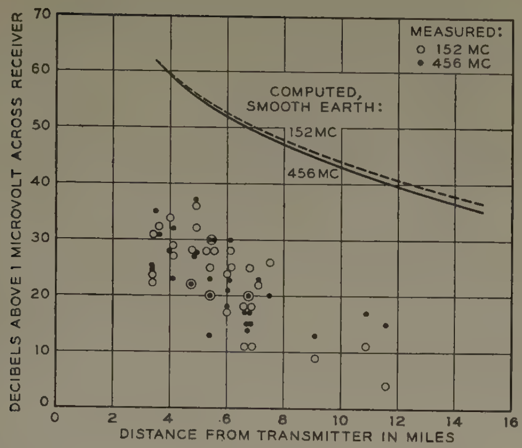

Fig. 2—Received signal strengths in Manhattan.

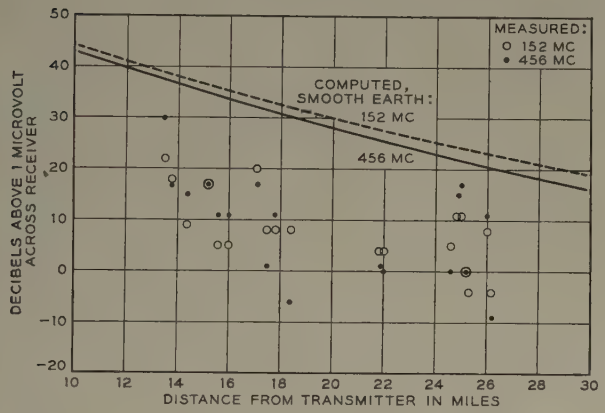

Fig. 3—Received signal strengths in Bronx-Westchester.

transmitter. In considering these values, it will be recalled that they are obtained in a city of skyscrapers, and that a large proportion of them were measured on crosstown streets, the received signals on the north and south avenues averaging some 15 db stronger than on the crosstown streets.

In Bronx-Westchester, on terrain that may roughly be described as suburban rolling country, the average difference between measured values and computed smooth-earth values is about 23 db, all the individual differences being comprised in the range 5 to 40 db; also, there is no significant distinction between the two frequencies in this regard, nor between locations from 13 to 26 miles from the transmitter.

Received Signal Variations in Short Distances

A limted number of measurements at 450 Mc with a recording instrument in the car showed signal variations up to about 15 db in a distance of about one foot of travel (which is about one-half wavelength) on city streets; the average variation per foot of travel was probably closer to 5 db. In a car moving at normal speed, these variations tend to be averaged out. Total variation over distances of 200 to 400 feet was roughly 20 to 25 db, with either a dipole or a gain antenna. On flat open meadows, at 450 Mc the signal variation per foot of travel was around 1 or 2 db, and the total variation in 300 feet was 11 or 12 db.

Noise Versus Received Signal

The *average* audio speech-to-noise ratio plotted vs. received signal amplitude is shown on Fig. 4. These figures represent average conditions in Manhattan and cover regions of lower and higher noise and lower and higher signals.[5] Ignition noise is controlling at both frequencies. Variations of individual locations from the average values shown, for the lower signal values, would cover a range of about ±10 db. Both speech and noise values for the speech-to-noise ratios were measured on an audio noise meter; the speech modulation was normal for the volume-regulated urban mobile radiotelephone system.

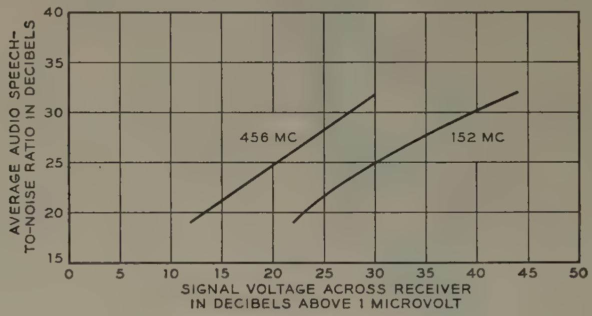

Fig. 4—Average audio speech-to-noise ratio versus radio-frequency signal strength (Manhattan tests).

It will be seen from Fig. 4 that for equal audio speech-to-noise ratios at 450 and 150 Mc, the required radio-frequency signal voltage input to the receiver was, on the average, about 10 db lower at 450 than at 150 Mc. This figure applies to urban reception, ignition noise being controlling, and assumes equal signal frequency-swings at the two frequencies. It assumes set noise low enough (noise figure of about 8 db, and a reasonable input impedance match with a quarter-wave antenna) so that the effect of the ignition noise is permitted to override set noise at both frequencies.

Test Transmitters

Both transmitters were modulated simultaneously by the same speech source and were adjusted for equal phase deviation maxima of about ±10 radians at the final frequency. The speech source was from talkers using standard telephone instruments. The volume was regulated to ±4 vu by regular mobile radiotelephone control terminal equipment. Arrangements were provided for removing modulation during noise measurements.

Acknowledgments

The authors wish to acknowledge the assistance of J. L. Lindner in the construction of the equipment, the help of M. Aruck in conducting tests and analyzing data, and the aid of R. S. Tucker in preparing this paper.

[5] A source of noise, ahead of the radio transmitter, which was peculiar to the particular setup and limited the speech-to-noise ratio to a maximum of about 32 db, has been removed by computation from the values given on Fig. 4.

A Six-System Urban Mobile Telephone Installation with 60-Kilocycle Spacing*

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Summary—**This paper describes a 6-system mobile radiotelephone installation in Chicago, operating in the 152- 162-megacycle band, and using 60-kc spacing of carrier frequencies, rather than the 120-kc spacing of previous practice. The measures required to achieve this frequency saving are described, including filters and special antenna arrangements at the land transmitter, "off-channel squelch" in the land receivers, connection of six land receivers to a common antenna, and other special co-ordinating means.**

Introduction

THE DEMAND for telephone communication to vehicles in some urban areas has far exceeded the capacity of available radio channels. To cope in some measure with this demand, the Bell Telephone Laboratories, in co-operation with the Illinois Bell Telephone Company and the New York Bell Telephone Company, have engineered an installation using a block of six adjacent channels wherein the normal frequency spacing between channels has been halved. Thus, the number of useful channels has been doubled. In the allocation plan of the Federal Communications Commission for the urban frequency range of 152–162 Mc, the designated channel spacing is 60 kc. However, channels have in the past been assigned only every 120 kc within the same city. Field tests have shown that operation with 60-kc spacing, using currently available equipment, is feasible if several special measures, the most important of which is the "co-ordinated operation of land transmitters," are followed.

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† Bell Telephone Laboratories, Inc., New York, N. Y.

The six-system installation shown in Fig. 1 is essentially a group of six single-system systems which are carefully co-ordinated to minimize mutual reactions. The six land transmitting antennas are mounted on a single mast and are arranged so that the coupling between them is a minimum. Separate land receivers at each receiving location are used for each channel and are fed by a single antenna. For reasons of economy, the land and mobile equipments employ the same basic units as a single-channel installation. The frequencies of the land transmitter channels are located near the lower edge of the 152- 162-Mc band and are spaced at intervals of 60 kc. The corresponding mobile transmitter channels are about 5 Mc higher in frequency and are also separated by 60 kc. Standard commercial equipment is used throughout, although in some instances minor modifications have been made to meet the requirements for adjacent channel (60-kc) operation.

The operation of several adjacent channels in the same geographical area creates certain problems that do not occur in the operation of a single isolated channel. Only those problems pertaining to the operation of adjacent channels will be considered here.

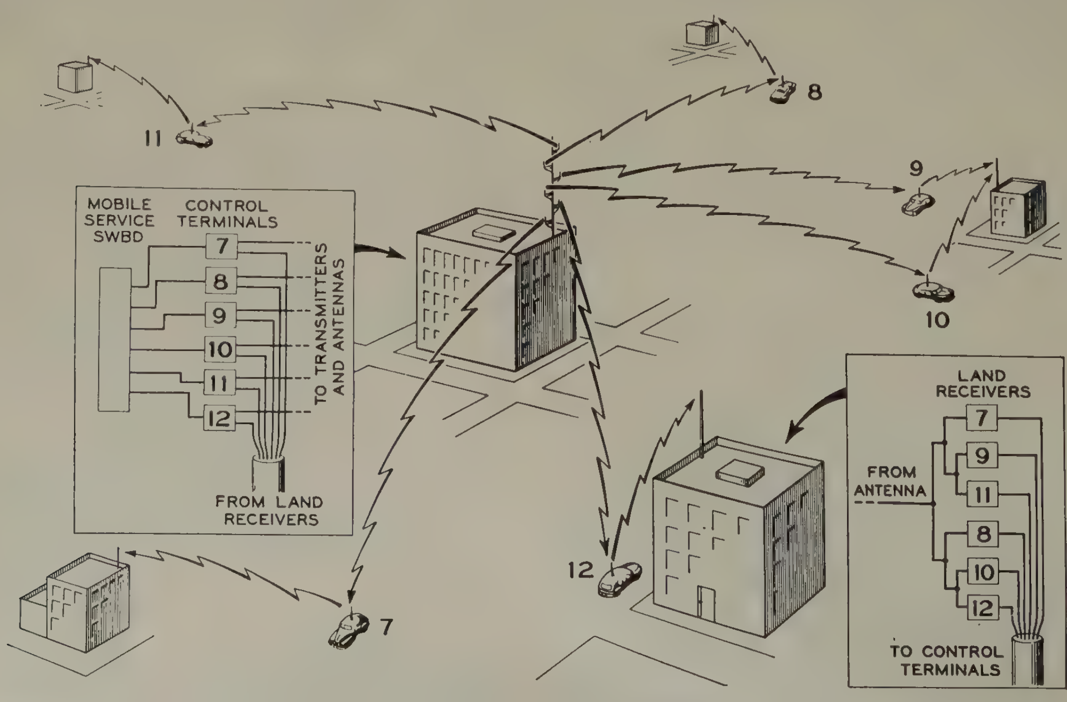

Fig. 1—A six-system urban mobile telephone installation.

Mobile Receiver Selectivity

The selectivity of mobile receivers now in general use is insufficient to permit the audio output of the receiver to be unaffected by a modulated carrier in an adjacent channel under many conditions of reception. When an adjacent channel carrier, but no desired channel carrier is present, a vehicle subscriber, picking up his handset to make a call, might hear distorted modulation and noise. If the desired carrier is present, but an adjacent channel carrier is received in sufficiently greater strength, intolerable cross talk may result. This may occur if the transmitters are widely separated and the vehicle is near the interfering transmitter but distant from its own transmitter.

The situations described above can be avoided if a desired channel carrier is present whenever an adjacent channel carrier is received and is strong enough to override the unwanted signal.[1] To realize this, all land transmitter antennas for a block of adjacent channels should be located at the same point and should be energized simultaneously. This may be termed "co-ordinated operation of transmitters."

The simultaneous activation of all land transmitters in a block of adjacent channels can be easily accomplished by simple relay control circuits. From the viewpoint of field distributions, it might be desirable for all transmitters to radiate from a single antenna so that differences in field strength, due to multiple-path transmissions and shadows, would be minimized. However, it is necessary to introduce appreciable attenuation between power amplifiers of the transmitters to prevent excessive intermodulation. This can be done conveniently by physical separation of the antennas.

Land Transmitter Intermodulation

The problem of intermodulation can be illustrated by considering the operation of two or more land transmitters with their antennas in close proximity. Voltages will be induced in each antenna due to the radiation fields of all other antennas. The resultant antenna voltage will be transmitted to the output stage of the associated transmitter which presents a nonlinear impedance, and thus intermodulation will take place between the various signals present in the plate circuit of the output stage. It can be shown that if p and q represent the output frequencies of two transmitters which are mutually coupled, the major distortion components will have the frequencies $(2p-q)$ and $(2q-p)$. Other terms will also be generated but their levels will be sufficiently low or their frequency so far removed from the useful channels that they can be attenuated by simple filters. The terms produced by six transmitters are shown in Fig. 2. Here, for convenience, channels or frequencies are referred to by numbers. The channels are numbered consecutively and are spaced 60 kc apart in frequency. Channel number 1 is the first (or lowest) channel in the 152- 162-Mc band. Channels 7 through 12 are assumed to be the active channels. Each row shows the frequencies of important intermodulation products generated by a pair of transmitters, and the 15 rows show all possible pairs for the six transmitters. A large proportion of the intermodulation products fall on channels outside the assigned group of six adjacent channels and will be referred to as "extraband radiations."

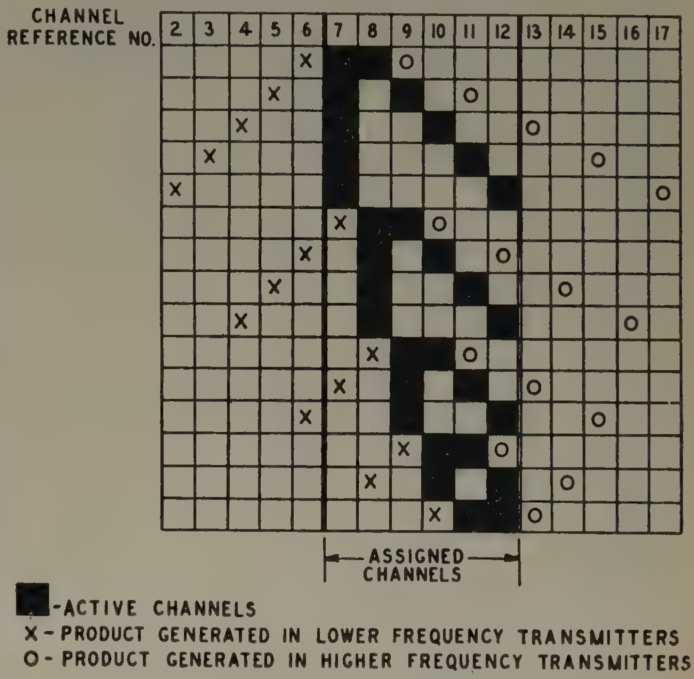

Fig. 2—Third-order intermodulation products generated in six transmitters.

It is desirable to reduce extraband radiation occurring in land transmitters to as low a level as possible. The rules[2] of the Federal Communications Commission, covering the subject of spurious emissions of mobile radio telephone equipment are, for apparatus of this type, that the amplitude of the spurious product shall be at least 70 db below the level of the unmodulated carrier.

In general, distortion products of the form $(2p-q)$ can be reduced by attenuating the product itself with selective circuits or by decreasing the coupling between transmitters. Two methods which appear most feasible are (1) optimum geometrical arrangement of antennas, and (2) frequency-selective circuits located between the output stage of each transmitter and its antenna.

The method of mounting antennas[3] has been studied and an arrangement of staggered antennas adopted. Frequency-selective circuits located in the output transmission lines between each transmitter and its associated antenna can be designed to provide attenuation to both the incoming unwanted signal and to the generated intermodulation products. The selective circuits used in this system were distributed constant

[1] Satisfactory operation has been obtained in some instances when the level of the desired channel carrier was reduced 20 db below its normal value.

[2] *Federal Register*, vol. 14, p. 2315; May 6, 1949.

[3] W. C. Babcock and H. W. Nylund, "Antenna systems for multichannel mobile telephony," Proc. I.R.E., p. 1324, this issue.

circuits of the coaxial-line type supplied by Motorola, Inc.

By measures outlined above, it was possible to reduce the extraband radiation to the desired levels. The unwanted products were, in all cases, more than 70 db below the carrier level, thus meeting the requirements of the Federal Communications Commission.

Mobile Receiver Intermodulation

Some intermodulation may be produced in mobile receivers in channels close to the assigned 6-channel group. When such receivers are operated close to the transmitter site and the desired transmitter is at a considerable distance, these distortion products may override the desired signal. The susceptibility of receivers to this type of difficulty increases as the field strength of the unwanted signals becomes stronger but is a problem only in the immediate vicinity of the unwanted transmitters.

Land Receiver Selectivity

Insufficient selectivity in the land receivers may result in interference from adjacent channel signals when the desired channel is inactive.

The traffic operator in a mobile telephone system is notified of a call from a vehicle by a signaling circuit actuated by a reduction in noise output of the detector of a land receiver. This operation takes place when an on-channel carrier of sufficient strength to give acceptable service is received. However, it is possible for a strong carrier on an adjacent channel, in the absence of a carrier on the desired channel, to reduce the noise sufficiently to cause a false signal to be sent to the traffic operator. To overcome this fault, a relatively simple "off-channel" squelch was developed.

The off-channel squelch is controlled by the second limiter grid voltage. This voltage has to exceed a certain limit in order for the receiver to be enabled. Gain and selectivity of the intermediate-frequency stages of the receiver are such that an adjacent channel carrier will never produce a limiter grid voltage exceeding the threshold value. The off-channel squelch serves only to decrease the susceptibility to interference when no carrier is present in the desired channel. Experience indicates that other types of interference in land receivers are of negligible proportions.

Description of Chicago Six-System Installation

A six-system urban mobile telephone installation has been placed in operation by the Illinois Bell Telephone Company in Chicago, Ill. Following is a description of the essential components of this system.

Land Transmitters

The land station transmitters are operated on channels 7 through 12 inclusive from 152.390 Mc, with 60-kc channel spacing, to 152.690 Mc as assigned by the Federal Communications Commission.[4] Fig. 3 is a functional block diagram of the land transmitting equipment.

One Western Electric Company 540A Radio Transmitting Equipment is used for each of the six urban mobile telephone channels. A seventh serves as a standby in the event of a transmitter failure and to facilitate routine maintenance with a minimum loss of service. A trunk terminating equipment panel provides

[4] Effective July 1, 1949, channels 9 through 14 were assigned for this service.

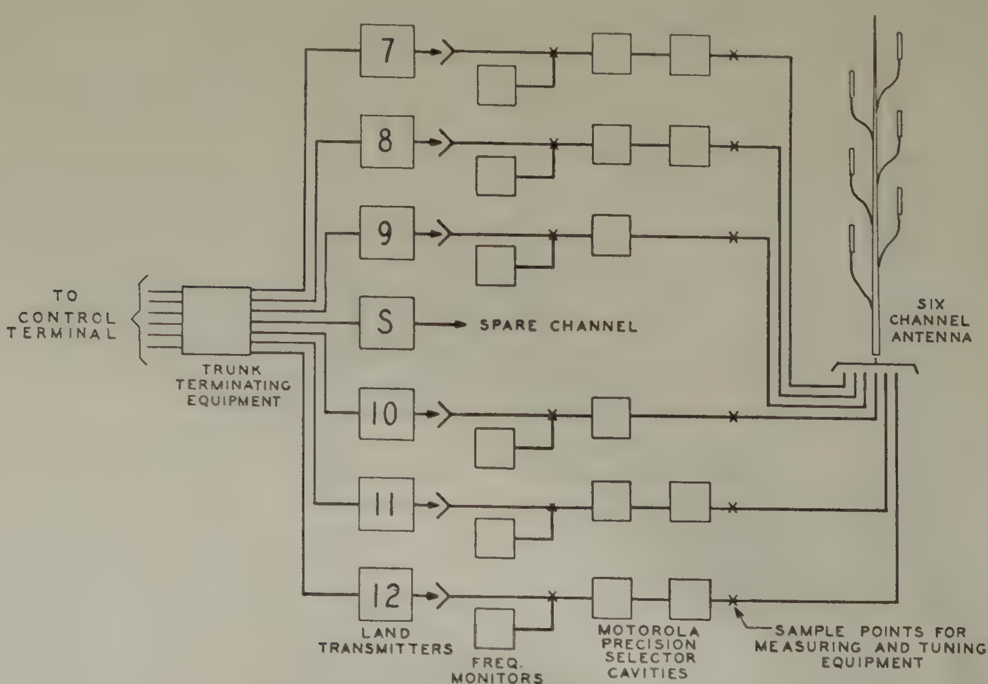

Fig. 3—Block diagram of land transmitter installation.

patching facilities for rapid switching of the audio, remote control, and frequency monitor alarm circuits.

Two cavities in cascade are interposed between each transmitter and its antenna for channels 7, 8, 11, and 12. A single cavity is used in conjunction with the transmitters on channels 9 and 10. Fig. 4 shows the bank of transmitters and high-Q filters.

Fig. 4—Land transmitter installation at Chicago.

Land Receivers

The land receivers used in the Chicago six-system installation are Western Electric 40A and 40B Radio Receivers. These are double-heterodyne FM receivers specifically designed for mobile telephone applications. The receivers are rated at 40-db selectivity to an unmodulated adjacent channel carrier. Each receiver has been modified to incorporate an off-channel squelch circuit.

Each receiver installation, of which there are ten in the Chicago area, consists of six receivers, one for each of the six adjacent channels. The receivers are connected to a single antenna, as shown in Fig. 5, through a coaxial-line bridging network that provides satisfactory impedance matches. The incoming power divides equally among the six receivers, thus giving a bridging loss of about 8 db relative to a single receiver connected to the antenna. For most urban receiver locations, this reduction of input is not important because ambient noise rather than receiver noise is controlling. In a few locations where receiver noise is expected to control, antennas have been installed having gain in the horizontal plane to partially offset the bridging loss.

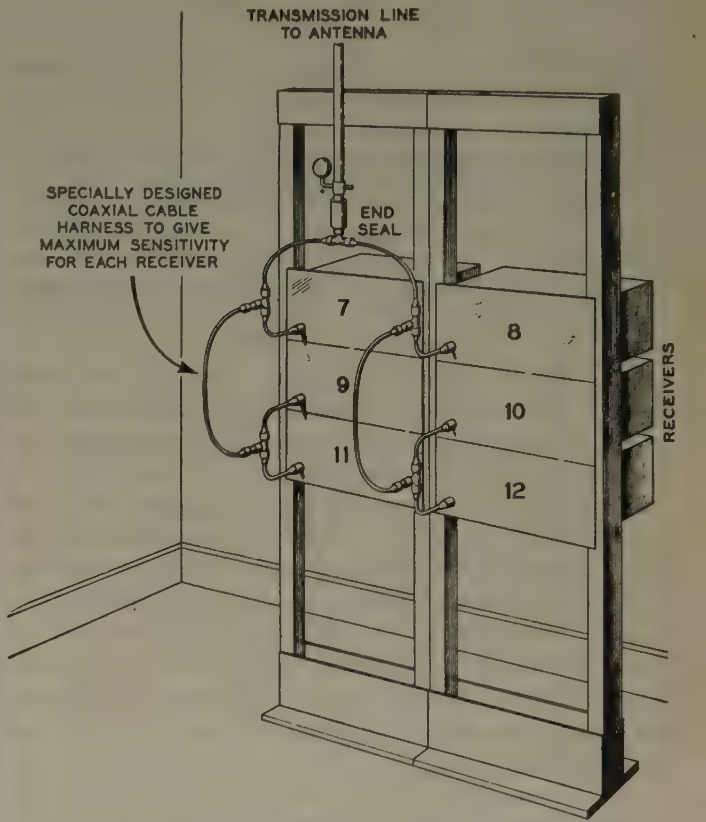

Fig. 5—Land receiver installation.

Mobile Equipment

Western Electric 238B (6-volt) or 238C (12-volt) Mobile Radio Equipments are installed in the subscribers' vehicles. Each installation consists of a receiver, a transmitter, a quarter-wave whip antenna, and a control unit which includes a handset, a power switch, and a bell.

The receiver is a double-heterodyne FM type designed to operate from the car battery. The selectivity is rated at 50 db to an unmodulated adjacent channel carrier. An automatic-gain-control circuit reduces the gain of the radio-frequency amplifier as the first limiter grid voltage increases.

The transmitter is a crystal-controlled phase-modulated transmitter with a frequency multiplication of 36. The radiated power is 20 watts. Modulation limiters are employed to prevent extraband radiation.

Conclusions

This Chicago six-system installation has been in commercial operation since November, 1948. A similar installation in New York City, having three channels with 60-kc spacing, has also been in service for about the same length of time. A highly satisfactory grade of service has been provided by both of these installations. Systems of this type will permit doubling the number of available channels in most of the large cities without any major equipment redesign.

Antenna Systems for Multichannel Mobile Telephony*

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Summary—**This paper describes an arrangement whereby several antennas may be mounted on a single mast at the transmitting site of a multichannel system operating in the 152- 162-megacycle band. The antennas are so disposed as to minimize shadowing effect of the mounting structure, while keeping intertransmitter coupling to a tolerable minimum. Measurements of the electrical characteristics are presented for arrangements of 6 antennas mounted on a 62-foot steel mast. These measurements on a full-scale structure are supplemented by tests at a higher frequency on reduced-scale, simplified models.**

ONE OF THE EARLY problems that had to be solved in the development of multichannel mobile telephone systems was that of interchannel cross talk. This was troublesome because of the close frequency spacing between adjacent channels, which was 60 kilocycles in a frequency-modulated system operating in the 152- 158-Mc band, and because the mobile unit might be considerably closer to the transmitting antenna which served an adjacent channel than to the antenna which served its own channel. This problem was very appreciably reduced by locating all of the transmitting antennas serving a given area at the same site.

When mounted on the same rooftop, however, the proximity of the transmitting antennas, which proved so useful in solving the interchannel cross-talk problem, introduced another of almost equal magnitude. The simultaneous operation of two or more transmitters, serving antennas closely coupled because of their proximity to each other, gives rise to the generation of intermodulation products. These products are radiated, and, if uncontrolled, will appear as interfering signals at unwanted points in the frequency spectrum.

The problem posed, therefore, was to locate as many antennas as possible in as limited space as possible without at the same time permitting the coupling between them to get completely out of hand. To be more specific, system requirements demand that the coupling between most of the antennas be of the order of 40 db or more whereas that between a few of them may be of the order of 30 db. Furthermore, each antenna should radiate a vertically polarized wave having a more or less uniform radiation pattern in the horizontal plane.

For the same coupling two radiators collinearly disposed can be located much closer to one another, as is well known, than when placed broadside. This suggests that a collinear arrangement of radiators might be suitable for our purpose. With one-wavelength spacing an over-all length of eight wavelengths is required to accommodate six half-wave antennas. At urban mobile telephone frequencies, which are around 152 Mc, eight wavelengths are of the order of 50 feet so that a mast about 65 feet in the clear is required if the lowermost antenna is to be elevated some 15 feet above the mounting surface.

Coaxial antennas are currently used at urban land stations and are readily available commercially. Fig. 1 shows schematically two arrangements of six such antennas mounted on brackets attached to a mast. The question now arises: How does the presence of the mast, cross arms, and transmission lines affect the electrical characteristics of the several antennas? A 62-foot unguyed steel mast was erected on Bell Laboratories property at Murray Hill, N. J., to answer that question. This mast, shown in Fig. 2, was designed by J. H. Gray of our Outside Plant Department and fabricated by "The Pole and Tube Works" at Newark, N. J. It is 14 inches in diameter at the base, tapering to $3\frac{1}{2}$ inches at the top. The brackets are telescoping affairs so that the spacing between mast and antennas is variable over the limited range of $3\frac{1}{2}$ to 6 feet. The brackets can also be swung around the pole so that the antennas may be oriented in any desired azimuthal direction.

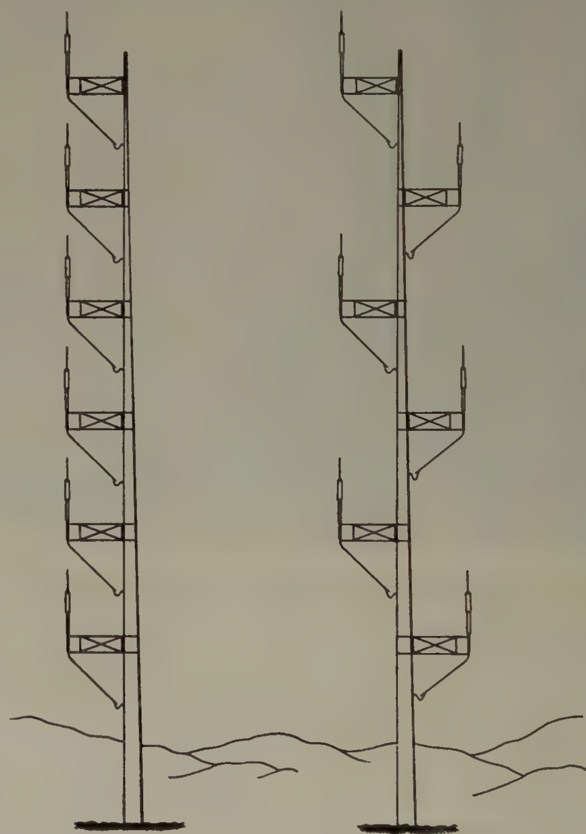

Fig. 1—Two mounting arrangements, six antennas on one mast.

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† Bell Telephone Laboratories, Inc., New York, N. Y.

Fig. 2—The Murray Hill mast.

At the conclusion of experimental work conducted on this Murray Hill mast, a similar structure was erected on the roof of the Morton Building in Chicago where commercial mobile telephone service is now being given on six separate channels. Still more recently, the New York skyline has been modified by the appearance of a somewhat similar structure on the Long Lines Building of the American Telephone and Telegraph Company at 32 Avenue of the Americas. Fig. 3 shows the Morton Building mast with its six urban system antennas and, in addition, a single highway system antenna operating at approximately 35.5 Mc mounted at the very top of the mast.

Objectives

The tests herein described were made with the following objectives in mind:

1. To determine the best antenna arrangement for use in the six-channel system that was later installed in Chicago.
2. To develop a standard multichannel arrangement for general use if such standardization appears desirable and feasible.

Description of Individual Antennas and Their Test Leads

The antennas used in the tests as well as in the Chicago installation are half-wave coaxial-type antennas, which are readily available commercially. Those used in the tests at Murray Hill were provided with a length of RG-12/U coaxial cable which extended six feet below the bottom of the support staff. For most of the tests this cable was connected through type-N weatherproof fittings to another section of similar cable of sufficient length to connect to the test equipment.

In the Chicago installation, however, the length of RG-12/U cable provided with the antenna is connected to an end seal terminating a length of 7/8-inch pressurized cable mounted on the mast. This pressurized cable connects the end seal at the transmitter end of the line to the end seal at the antenna end of the line. Voltage standing-wave measurements were therefore made on a setup simulating this arrangement to insure that the load presented to the transmitter by the antenna and its associated connecting cable and fittings would be satisfactory.

Tests

Three types of tests were made for each of several antenna arrangements. These tests may be described as follows:

1. The voltage standing-wave ratio on the several transmission lines feeding the antennas was determined.
2. The couplings were determined for all antenna combinations.
3. The radiation pattern in the horizontal plane was determined for representative antennas.

Impedance Measurements

Impedance measurements were made on a slotted line using standard techniques. The antennas were measured at different mast spacings and at different positions with respect to one another. The voltage

Fig. 3—The Chicago mast.

standing-wave ratios on a 70-ohm transmission line, which included two end seals and associated N-type connectors, ranged between 1.1 and 1.3 and were considered satisfactory.

Antenna Coupling

The setup used in measuring the antenna coupling is shown in Fig. 4. Referring to this figure, the measuring procedure was as follows: The microvolter was connected initially through the variable attenuator box to the transmission line leading to one of the antennas while another antenna was connected through a fixed 10-db pad to the receiver and associated audio detector. A convenient reading was obtained on the

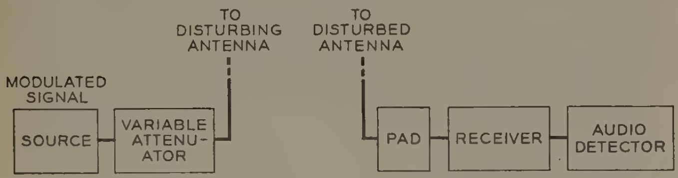

Fig. 4—Setup for measuring antenna coupling.

audio detector with 10 db in the attenuator box to stabilize the impedance seen by the microvolter. To calibrate this reading the microvolter was connected through the variable attenuator box directly into the pad preceding the receiver. The attenuator was then adjusted until the same reading was again obtained on the audio detector. The final setting of the attenuator down 10 db represents the coupling between the two antennas under test plus the line loss in the coaxial cables associated with the test antennas.

Coupling measurements were made for all possible combinations of six antennas on a single mast for the following arrangements:

1. Collinear arrangement of antennas.
   a. 4-foot spacing from center line of mast.
   b. $4\frac{1}{2}$-foot spacing from center line of mast.
   c. 5-foot spacing from center line of mast.
2. Staggered arrangement of antennas such as was used later at Chicago, as shown in Fig. 2.
   a. $4\frac{1}{2}$-foot spacing from center line of mast.

Numbering the antennas in sequence from 1 to 6 inclusive, with the lowest number corresponding to the lowest antenna, the vertical spacing between like points on successive antennas was 9 feet for all arrangements.

With the exception of a few of the nonadjacent combinations, the measured values of antenna coupling in arrangement 1 showed only minor variations with change of spacing between antennas and mast. The measured values of antenna coupling for arrangements 1b and 2a are shown in Table I.

Arrangement 2a represents an effort to reduce the coupling between adjacent antennas by rotating alternate antennas 180 degrees around the mast in such fashion that the mast itself constitutes a shield lying between all antennas having a vertical separation of only 9 feet. The measurements indicate that the coupling between successive antennas was decreased from 3 to 6 db by this rearrangement of alternate antennas.

TABLE I
Measured Values of Antenna Coupling

Collinear antenna arrangement 1b (coupling in db)					
	2	3	4	5	6
1	35	47	57	58	61
2		35	47	48	53
3			34	43	46
4				33	49
5					33
Staggered antenna arrangement 2a (coupling in db)					
	2	3	4	5	6
1	39	45	51	51	66
2		41	42	62	69
3			37	40	46
4				37	38
5					38

Radiation Patterns

It is extremely difficult to obtain reliable radiation patterns of mast-mounted antennas. It is usually impractical to move a test antenna around the fixed mast-mounted antenna, since at any reasonable testing distance there are obstacles between the two antennas in some directions unlike those in others, which would therefore distort the measured pattern. Electrically it would be desirable to locate a test antenna at an optimum testing distance in a direction that is free of obstacles and then rotate the mast and mounting fixtures around an axis that is coincident with the mast-mounted antenna. This is unfortunately not feasible mechanically. The procedure that was employed at Murray Hill involved locating a test antenna at an optimum testing distance in a direction that was free of obstacles and rotating the mast-mounted antenna around the mast. The deviations in the horizontal plane pattern from a true circle may not all be attributed to the mast and associated trappings since a pattern obtained by moving the antenna around the same orbit in the absence of the mast would doubtless exhibit deviations from a true circle. For this reason patterns were also taken on a model basis at a testing frequency of 456 Mc to confirm, if possible, the patterns obtained on the actual mast-mounted antennas. Fig. 5 shows patterns of antenna number 1, the lowermost antenna on the mast, obtained at a frequency of 153 Mc at spacings of 4.0, 4.5, 5.0, 5.5 and 6.0 feet, respectively, with the test antenna located at a point approximately 144 feet west of the mast. Patterns of this and other antennas on the mast were also obtained with the test antenna

located at different points both west, north, and south of the mast. The circular patterns also shown in Fig. 5 represent estimates of the patterns that would have been obtained if the antennas had been located in free space and hence had not been influenced by the presence of the mast and its associated trappings. These idealized patterns were derived by replotting the measured pattern in terms of arbitrary microvolts, integrating the area of the resulting pattern with a planimeter, determining the radius of a circle having an equivalent area, and finally reconverting the microvolts corresponding to that radius back into decibels.

An inspection of Fig. 5 shows that the response of antenna number 1 when spaced 4½ feet from the center line of the pole was about 5 db below normal in some directions and about 2 db above normal in other directions. At all other spacings the loss was 6 db or more in some directions while the gain remained about 2 db in certain other directions. The shadow of the mast is very pronounced at the 4-foot spacing whereas rather deep nulls occur at the sides of the pattern at the higher spacings. These nulls are apparently produced by the wave that is bounced off the mast in such fashion as to partially cancel the direct wave impinging on the antenna.

Pattern measurements (not shown in Fig. 5) were also obtained on antenna number 5 when spaced 4½ feet from the center line of the mast. It is of interest to note that this pattern is essentially a carbon copy of that obtained on antenna number 1 at the same spacing although the diameter of the mast is 13 inches at the point of attachment of antenna number 1 and only 4 inches at the point of attachment of antenna number 5. The similarity of the two patterns is all the more remarkable in that the test antenna was located west of the mast for the pattern measurements on antenna number 1 and south of the mast for those on antenna number 5.

Model Tests

Supplemental studies, using model techniques, were made at a frequency of 456 Mc on a reduced-scale mast and antenna. These studies were made to obtain confirmation of the results already obtained from the full-scale model tests and to obtain whatever additional information some simple tests might yield. These tests were designed to determine the effect upon the pattern of a single antenna of mounting the antenna at various distances from a metal mast.

Setup for Model Tests

The measurements were made at 456 Mc (with a few exceptions noted) which is very nearly three times the normal transmitting frequency. This permitted scaling down the physical dimensions of the mast and antenna to one-third full size, which greatly facilitated setting up the desired test conditions.

The mast and antenna arrangements are shown in Fig. 6. The coaxial antenna used was an early model, urban vehicular type, with whip and skirt shortened to give a good match to a 73-ohm feed line at 456 Mc. Two different masts, one of 2-inch brass pipe and one of

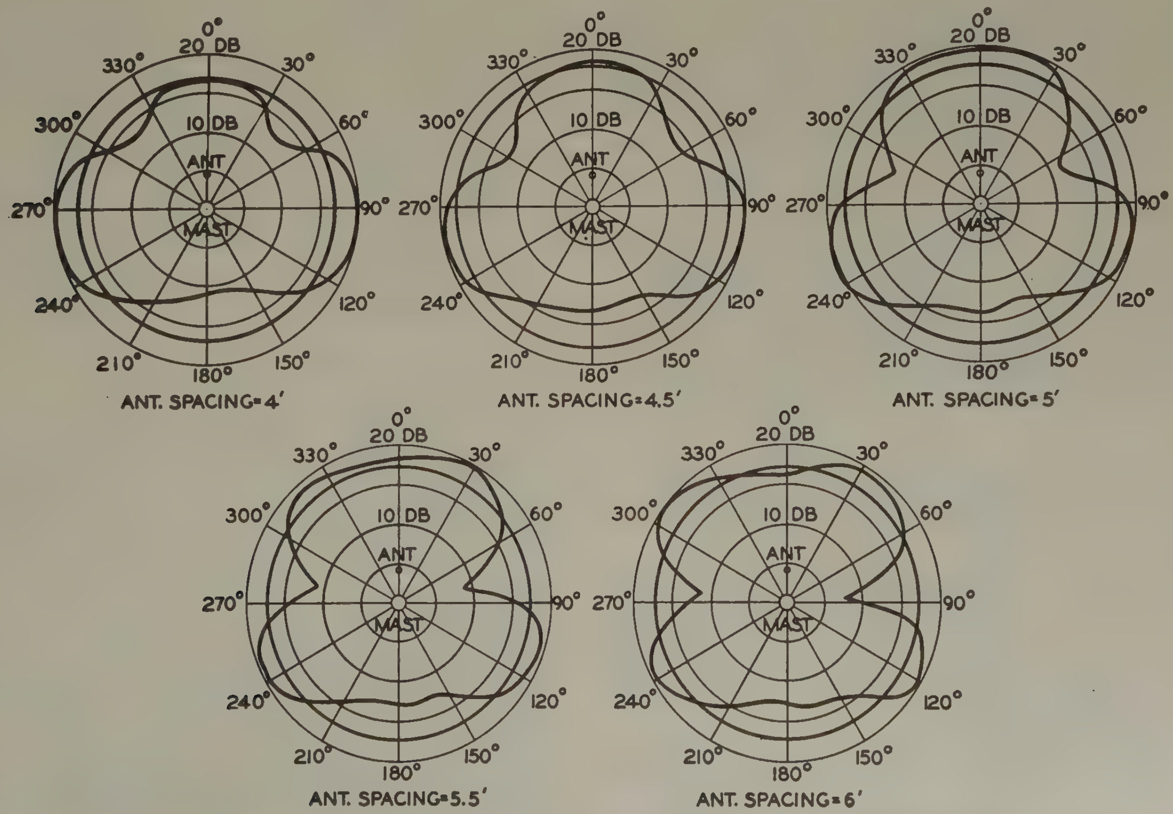

Fig. 5—Antenna patterns, full-scale tests.

4-inch galvanized leader pipe, were used. Both were 12 feet tall above ground, equivalent (at 152 Mc) to a 6- or 12-inch mast 36 feet tall. This is less than the height of the steel mast, but as only one antenna was used in the tests it was felt that mast height was not critical in pat-

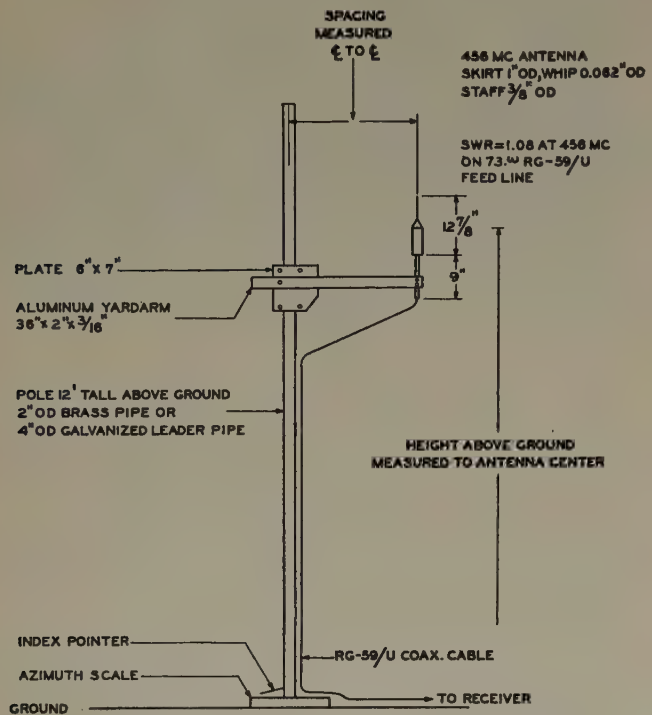

Fig. 6—Mast and antenna arrangements, model tests.

tern measurements. The antenna was mounted on the mast by means of an aluminum bar yardarm, $3/16 \times 2$ inch in cross section, with provision for varying the spacing of the antenna from the mast by 2-inch increments from about 6 to 30 inches. The mast was free to rotate about its vertical axis, and a graduated azimuth scale was provided at the base to measure rotation.

The over-all arrangements for the tests are shown in Fig. 7. The coaxial antenna on the mast was used as a receiving antenna, connected to a receiver through about 30 feet of RG-59/U line with a coaxial attenuator and impedance-stabilizing pad inserted near the receiver. A test signal was provided by a signal generator

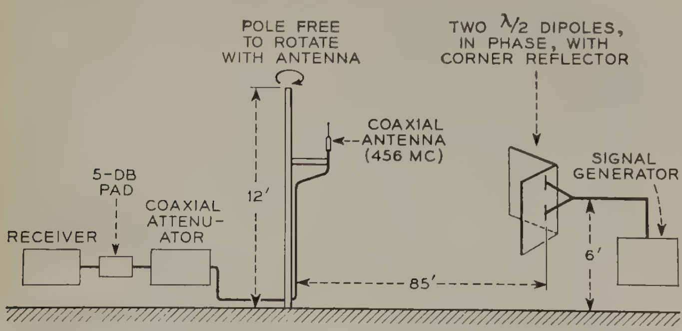

Fig. 7—Setup for measuring antenna patterns, model tests.

connected to a vertically polarized directive antenna, located 6 feet above ground and 85 feet distant from the mast. This produced a substantially uniform field within the area immediately surrounding the mast. Measurements made at various times during the tests indicated that within a 3-foot radius from the pole the signal field at a fixed height varied less than ± 0.8 db, and in the range of heights between 4 and 12 feet the variation was less than ± 2 db.

Method of Measurement

Pattern measurements were made by setting up the desired conditions on the mast, rotating it in small angular increments and noting at each step the attenuator setting required to bring the field-strength meter on the receiver back to an arbitrary reference reading. At intervals, the mast was removed and the antenna held in place on a wooden support to determine the response obtained in the absence of obstructions.

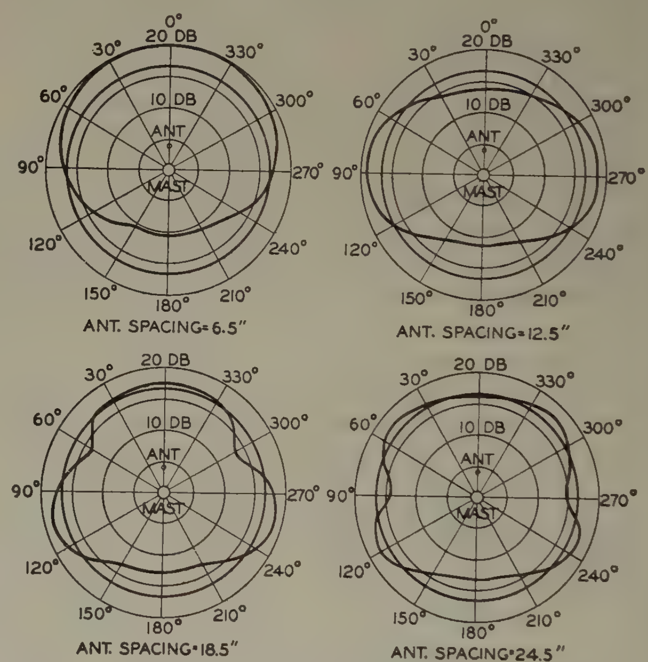

Fig. 8—Antenna patterns, 2-inch mast.

Tests on the Two-Inch Mast

The first series of pattern measurements was made with the antenna mounted on the 2-inch diameter mast, 12 feet tall. The antenna was located with its center 9 feet 4 inches off ground, and its distance from the mast was varied in 2-inch steps from 6.5 to 28.5 inches, as measured between the axis of the mast and the antenna. Typical patterns thus obtained are shown in Fig. 8. The response obtained with the mast removed is shown on each pattern for comparison as a superimposed circle. It is interesting to note that, whereas the shape of the pattern is essentially the same as that obtained on the full-scale mast at a spacing of 0.715 wavelength, the deep

nulls obtained at larger spacings on the full-scale mast are not in evidence on these perhaps oversimplified model measurements.

The curves shown in Fig. 9 portray the effect of antenna spacing from mast on the horizontal-plane pattern distortion introduced by the mast. Curve *A* shows that in the direction of maximum field intensity the field strength has been increased about 2 db regardless of the spacing. This confirms similar results obtained in the tests on the full-scale mast. Curve *B* shows that at 0°

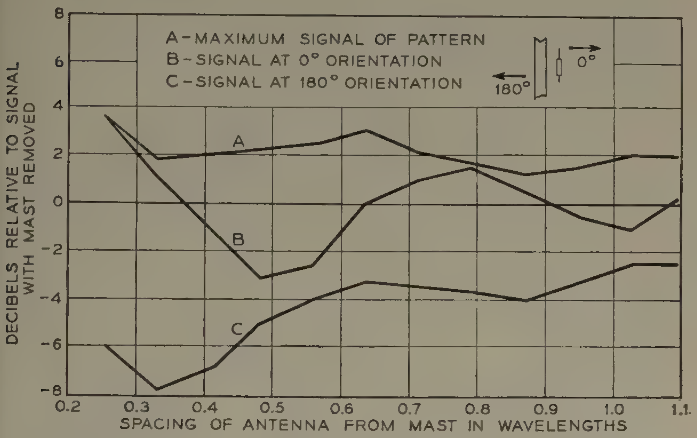

Fig. 9—Horizontal-plane pattern distortion, 2-inch mast.

orientation with the mast acting as a parasitic reflector, reinforcement of signal occurs when the spacing is an odd number of quarter wavelengths and degradation of signal occurs when the spacing is an even number of quarter wavelengths. Curve *C* shows in general that at 180° orientation the shadowing effect of the mast varies inversely with the spacing.

Tests on the Four-Inch Mast

While the model measurements at 456 Mc on the 2-inch mast tended to confirm the full-scale measurements at 153 Mc, there was nevertheless a discrepancy in the pattern measurements at spacings between mast and antenna in excess of 0.75 wavelength. The relatively sharp deep minima observed at these spacings in the full-scale tests were much less prominent in the model tests. It appeared that this discrepancy might be attributable to the difference in mast sizes since the full-scale measurements were made at a point where the mast was 13 inches (0.168 wavelength) in diameter while the model measurements were made on a 2-inch (0.0773 wavelength) mast.

Accordingly, further measurements were made on a 4-inch mast that was 12 feet tall. A pattern obtained at 456 Mc with 23.5-inch (0.91 wavelength) spacing of antenna from mast is shown in Fig. 10. Since this pattern showed no sharp side minima, the frequency was raised to 497 Mc at which frequency the 4-inch mast diameter is 0.168 wavelength (the same as the 13-inch mast diameter at 153 Mc) and further data were taken. Complete patterns were not obtained, but the radiation was measured at 0° and 180° and at those intervening positions which gave a maximum or a minimum reading. The side minima were not pronounced and observation of the signal as the antenna and mast were slowly rotated confirmed the fact that they were not sharp, but rather of a smooth character like those shown in Fig. 10.

At present, therefore, this point of disagreement between the full-scale tests and the model measurements cannot be explained. It may be caused by the presence on the large mast of other antennas than the one being studied, which, with their yardarms, make the entire structure more complex than the model setup.

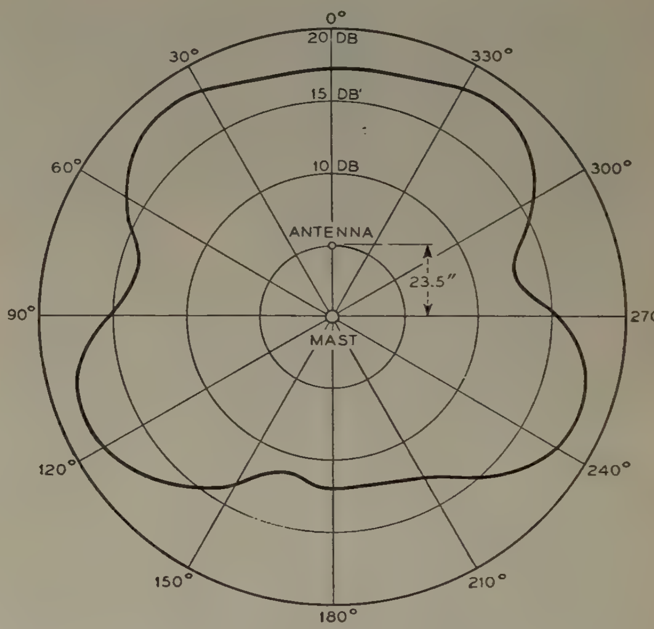

Fig. 10—Antenna pattern, 4-inch mast.

Conclusions

1. A staggered arrangement of antennas which takes advantage of the shielding effect of the mast to reduce the coupling between successive antennas is somewhat preferable to a collinear arrangement of antennas from the standpoint of realizing lower over-all antenna coupling.
2. Horizontal-plane pattern measurements obtained on the full-scale mast at 153 Mc indicated the optimum spacing of antenna from mast center line to be about 4.5 feet (0.70 wavelength). At this spacing the total spread of signal strength between the most favored and least favored directions of transmission was about 7 db.
3. Simplified model measurements gave good over-all confirmation of the results obtained on the full-scale mast; the model tests, however, did not show such pronounced nulls in the patterns as was experienced on the full-scale mast for antenna spacings in excess of 0.75 wavelength.

Cross-Talk Considerations in Time-Division Multiplex Systems*

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Summary—**An experimental study was made of the effects on interchannel cross talk of the bandwidth characteristics of the transmission medium in pulse-time multiplex systems. Pulse-amplitude modulation and pulse-position modulation systems are considered. The effects of various types of high- and low-frequency response are discussed from both experimental and theoretical points of view.**

I. Introduction

One of the significant problems present in the development of pulse-multiplex communication systems is interchannel cross talk. An investigation was conducted to determine the factors that would permit the design of a system combining high interchannel cross-talk ratios with maximum economy of bandwidth. Pulse-position and pulse-amplitude modulation[1,2] were investigated.

Pulse-amplitude modulation is of considerable importance, inasmuch as some form of it is used in many pulse-multiplex systems. In its simplest form, it is derived by sampling a signal at fixed time intervals. These pulse samples comprise the pulse-amplitude-modulated signal. If these modulation samples are transformed to time displacement of the pulses with respect to a fixed time reference (such as a marker pulse), pulse-position modulation results.

One common form of cross talk is caused by carryover of energy from one pulse to the following pulse. Thus, cross talk may occur from one channel to the following channel, decreasing rapidly as the pulses are further separated in time. This cross talk may be expressed as the ratio of the signal output of a given channel under normal modulation to the signal output of the same channel resulting from the modulation of some other channel. This ratio is customarily expressed in decibels.

II. Pulse-Position Modulation

In analyzing the effect of bandwidth on pulse carryover, it is necessary to investigate the effect of the response-frequency characteristic of the transmission system on the shape of the pulses. Two types of high-frequency response will be considered, a slow and an extremely rapid rate of high-frequency cutoff.

A. Slow Rate of High-Frequency Cutoff

A slow rate of cutoff may be obtained by a resistance-capacitance circuit of the type shown in Fig. 1. Such a circuit represents either the standard resistance-coupled amplifier without peaking circuits or its band-pass analogue, the single-tuned circuit.

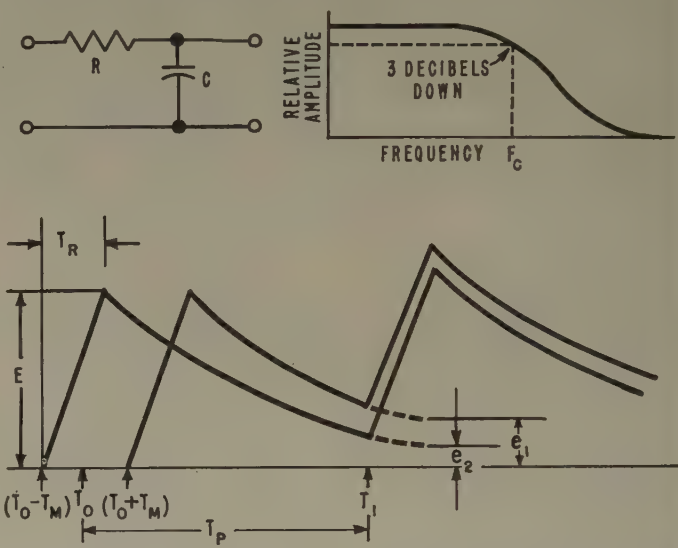

Fig. 1—Effect of high-frequency response on cross talk in pulse-position modulation using a resistance-capacitance filter.

$$\text{Cross-talk ratio} = \frac{\exp\,(T_P/RC)}{\sinh\,(T_M/RC)}\left(\frac{T_M}{T_R}\right).$$

The relative amplitude-frequency response of such a circuit may be expressed as

$$E_0 = E_I \cos\left[\tan^{-1}(F/F_c)\right] \tag{1}$$

where F is the frequency considered and F_c is the cutoff frequency, which is equal to $1/(2\pi RC)$, at which point the relative amplitude drops 3 db. Fig. 1 shows the type of response given by this circuit. The rate of cutoff approaches 6 db per octave.

The effect of such a response on a pulse of voltage may be calculated by simple transient analysis. With a rectangular input pulse, the rise time T_R of the received pulse increases with increasing transmission-circuit time constant RC until a value of T_R equal to the width of the original pulse is obtained. If RC is increased further, T_R remains constant.

The cross talk introduced by transmission of time-modulated pulses through a medium having the type of frequency characteristic described can be calculated as follows.

Referring to Fig. 1, two adjacent pulses of period T_P and rise time T_R are shown. The first pulse is time modulated by a displacement T_M from its resting position

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† Federal Telecommunication Laboratories, Inc., Nutley, N. J.

[1] F. F. Roberts and J. C. Simmonds, "Multichannel communication systems," *Wireless Eng.*, vol. 22, pp. 538–549; November, 1945, and pp. 576–580; December, 1945.

[2] V. D. Landon, "Theoretical analysis of various systems of multiplex transmission," *RCA Rev.*, vol. 9, pp. 287–351, June, 1948; and pp. 438–482, September, 1948.

T_O and therefore moves between T_O+T_M and T_O-T_M, producing carryover e_1 and e_2.

At one extreme, the carryover is

$$e_1 = E \exp -[(T_P - T_M)/RC].$$

At the other extreme, the carryover is

$$e_2 = E \exp -[(T_P + T_M)/RC].$$

The peak-to-peak variation in amplitude of the pulse, because of carryover, is then

$$e_1 - e_2 = E\left\{\exp\left[-\left(\frac{T_P - T_M}{RC}\right)\right] - \exp\left[-\left(\frac{T_P + T_M}{RC}\right)\right]\right\}.$$

This may be rewritten as

$$\begin{aligned} e_1 - e_2 &= E[\exp -(T_P/RC) \exp (T_M/RC) \\ &\quad - \exp -(T_P/RC) \exp -(T_M/RC)] \\ &= E \exp -(T_P/RC)[\exp (T_M/RC) \\ &\quad - \exp -(T_M/RC)]. \end{aligned}$$

This formula may also be expressed in terms of the time constant RC.

$$\text{Cross-talk ratio} = \frac{\exp (T_P/RC)}{\sinh (T_M/RC)}\left(\frac{T_M}{T_R}\right). \quad (2a)$$

To check the results that might be expected from the use of (2), a series of experiments were run using a commercial pulse-position-modulation multiplex system in which

$T_M=1$ microsecond
$T_R=0.3$ microsecond (for low values of F_c)
$T_P=5$ microseconds.

The inherent cross talk in the terminals was 66 db, and this was therefore the highest cross-talk ratio that could be measured. A block diagram of the setup used is shown in Fig. 2. Resistance-capacitance filters of various cutoff frequencies were inserted at the point indicated, and cross talk at a modulating frequency of 1,000 cycles was measured with a General Radio 736A wave analyzer. Only adjacent-channel cross talk was considered.

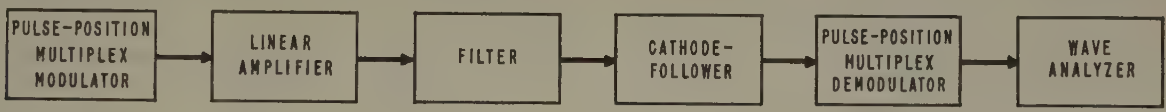


Fig. 2—Equipment used for cross-talk measurements on pulse-position modulation.

The ratio of pulse amplitude to carryover is then

$$\begin{aligned} \frac{E}{e_1-e_2} &= \frac{1}{\exp -(T_P/RC)[\exp (T_M/RC)-\exp -(T_M/RC)]} \\ &= \frac{\exp (T_P/RC)}{\exp (T_M/RC)-\exp -(T_M/RC)} \\ &= \frac{\exp (T_P/RC)}{2 \sinh (T_M/RC)}. \end{aligned}$$

Since $F_c=1/(2\pi\ RC)$,

$$\frac{E}{e_1 - e_2} = \frac{\exp (2\pi F_c T_P)}{2 \sinh (2\pi F_c T_M)}.$$

This cross talk is expressed in terms of amplitude variation. It can be shown[3,4] that in pulse-position modulation there is an improvement factor of $2(T_M/T_R)$ which may be applied to signal-to-noise ratio or to cross talk.

The output cross-talk ratio then becomes

$$\text{cross-talk ratio} = \frac{\exp (2\pi F_c T_P)}{\sinh (2\pi F_c T_M)}\left(\frac{T_M}{T_R}\right). \quad (2)$$

[3] E. M. Deloraine and E. Labin, "Pulse-time modulation," *Elec. Commun.*, vol. 22, pp. 91–98; 1944.

[4] S. Moskowitz and D. D. Grieg, "Noise-suppression characteristics of pulse-time modulation," Proc. I.R.E., vol. 36, pp. 446–450; April, 1948.

The results of these tests, together with the values derived from (2), are plotted in Fig. 3. It will be noted that the experimental results agree closely with theory, except at high cutoff frequencies, where the inherent cross talk in the equipment used becomes the limiting factor.

B. Rapid Rate of High-Frequency Cutoff

The cross talk arising from sharp cutoff of the transmission response was also studied. Such a response is approached in intermediate-frequency amplifiers. If a rectangular pulse is passed through a low-pass system having uniform frequency and linear phase characteristics up to a certain frequency, and zero response above this frequency, the equation of the resulting pulse can be shown to be[5]

$$e = E\{\text{Si}\ [n\pi(t' + \tfrac{1}{2})] - \text{Si}\ [n\pi(t' - \tfrac{1}{2})]\},$$

where

$Si(X) \int_0^x[(\sin u)/u]du$
e = output voltage at any instant
E = input pulse amplitude
$n=2F_cW$

[5] E. A. Guillemin, "Communication Networks," vol. 2, John Wiley and Sons, Inc., New York, N. Y., p. 485; 1935.

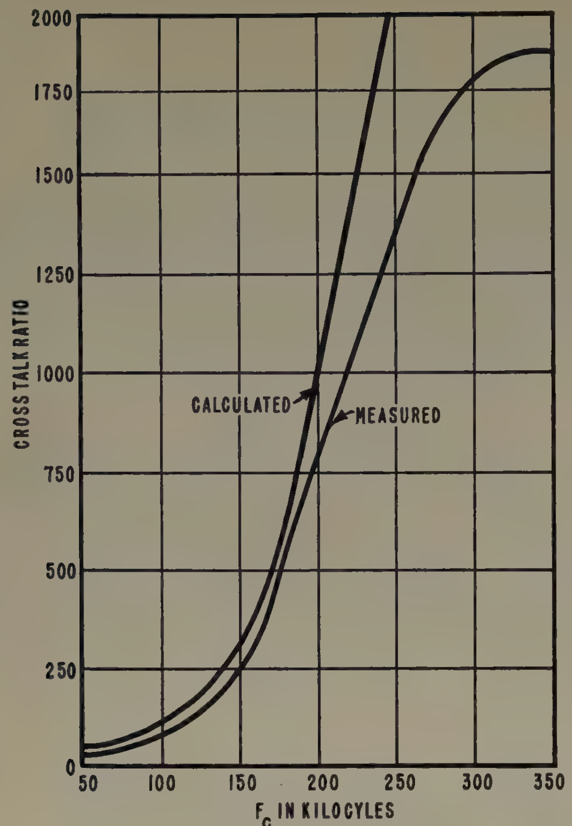


Fig. 3—Cross-talk ratio versus bandwidth for pulse-position modulation. High frequencies have been attenuated by a filter as shown in Fig. 2, where $F_C=1/(2\pi RC)$.

F_c = cutoff frequency
W = width of input pulse
$t' = T_P/W$
T_P = pulse period.

For such a theoretical transmission system, the cross talk may be derived as follows.

The carryover at one extreme of modulation is

$$e_1 = E\left\{\mathrm{Si}\left[n\pi\left(\frac{T_P - T_M}{W} + \frac{1}{2}\right)\right] - \mathrm{Si}\left[n\pi\left(\frac{T_P - T_M}{W} - \frac{1}{2}\right)\right]\right\}$$

and at the other extreme it is

$$e_2 = E\left\{\mathrm{Si}\left[n\pi\left(\frac{T_P + T_M}{W} + \frac{1}{2}\right)\right] - \mathrm{Si}\left[n\pi\left(\frac{T_P + T_M}{W} - \frac{1}{2}\right)\right]\right\}.$$

The peak-to-peak carryover ratio is then

$$\frac{E}{e_1 - e_2} = \left\{\mathrm{Si}\left[n\pi\left(\frac{T_P - T_M}{W} + \frac{1}{2}\right)\right] - \mathrm{Si}\left[n\pi\left(\frac{T_P + T_M}{W} + \frac{1}{2}\right)\right] + \mathrm{Si}\left[n\pi\left(\frac{T_P + T_M}{W} - \frac{1}{2}\right)\right] - \mathrm{Si}\left[n\pi\left(\frac{T_P - T_M}{W} - \frac{1}{2}\right)\right]\right\}^{-1}. \quad (3)$$

The build-up time of the pulse, after transmission through this system, is $1/(2F_c)$ so that, if $E/(e_1-e_2)$ is denoted by ψ, the net cross-talk ratio will be

$$\text{cross-talk ratio} = 4\psi T_M F_c. \quad (4)$$

The characteristics of a filter or transmission medium as given in the above analysis cannot be met in practice. A fair approximation can be made, however, by means of a low-pass π-section filter having a general characteristic as shown in Fig. 4. This filter had a rate of cutoff of approximately 20 db per octave. Tests on the same terminal equipment were made using such filters between modulator and demodulator. The results of these tests are shown in Fig. 5. From these data, substantiated by theory, a high-frequency cutoff of about 500 kilocycles is sufficient to obtain a cross-talk ratio of about 70 db. If a slow rate of cutoff is maintained such as that given by low-Q transmission circuits, transmission lines, or resistance-capacitance video-frequency circuits, a half-power point at about 350 kilocycles would give an equally satisfactory cross-talk ratio.

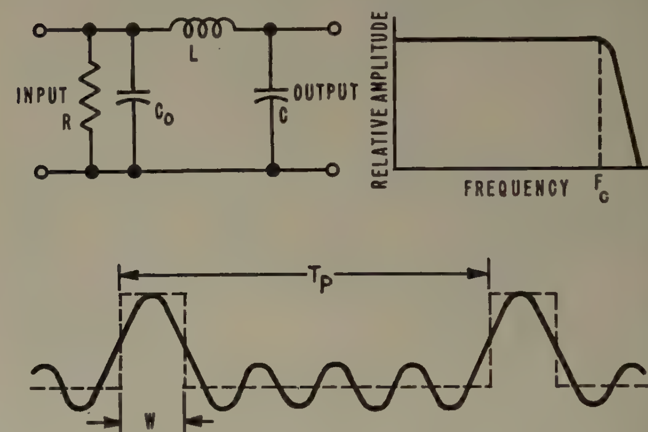


Fig. 4—Effect of high-frequency response on cross-talk in pulse-position modulation using a low-pass filter. For the π network $C_O R C_c = 78{,}900$, $C=2C_O$, $C_O L F_c^2 = 12{,}670$, in micromicrofarads, microhenries, ohms, and megacycles.

C. Low-Frequency Cutoff

The effect of low-frequency cutoff has also been considered. If a slow rate of cutoff such as that given by the resistance-capacitance circuit of Fig. 6 characterizes the transmitting medium, the pulse will be distorted as shown. The response-frequency plot of such a circuit is also shown in this figure and is given by

$$\text{relative amplitude} = \cos \tan^{-1} F_c/F, \quad (5)$$

where $F_c = 1/(2\pi RC)$.

The voltage e at any time $(T-W)$, where W is the width of the pulse, is

$$e=-E[1-\exp-(W/RC)]\exp-[(T-W)/RC].$$

At one extreme of modulation, the carryover to the next pulse is

$$e_1=-E[1-\exp-(W/RC)]\exp-[(T_P-W+T_M)/RC].$$

At the other extreme, the carryover is

$$e_2=-E[1-\exp-(W/RC)]\exp-[(T_P-W-T_M)/RC].$$

The peak-to-peak carryover is then

$$\begin{aligned}e_2-e_1&=-E[1-\exp-(W/RC)]\{\exp-[(T_P-W)/RC]\}\\&\quad[\exp(T_M/RC)-\exp-(T_M/RC)]\\&=-E[1-\exp-(W/RC)]\{\exp-[(T_P-W)/RC]\}\\&\quad[2\sinh(T_M/RC)].\end{aligned}$$

The ratio of pulse amplitude to carryover is

$$\frac{E}{e_2-e_1}=\frac{-\exp[(T_P-W)/RC]}{2[1-\exp-(W/RC)]\sinh(T_M/RC)}.$$

Applying the pulse-position-modulation improvement factor $2T_M/T_R$, the following equation is obtained.

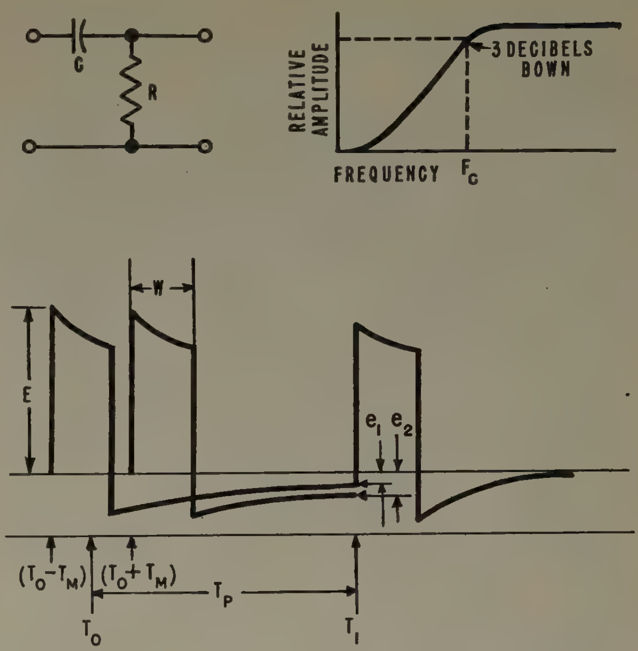


Fig. 6—Effect of low-frequency response on cross talk in pulse-position modulation using a resistance-capacitance filter.

$$\text{Cross-talk ratio}=\frac{-T_M\exp[(T_P-W)/RC]}{T_R[1-\exp-(W/RC)][\sinh(T_M/RC)]}.$$

Output cross-talk ratio

$$=\frac{-T_M\exp[(T_P-W)/RC]}{T_R[1-\exp-(W/RC)][\sinh(T_M/RC)]}. \quad (6)$$

Substituting $F_c=1/(2\pi RC)$,

Output cross-talk ratio

$$=-\frac{T_M\exp[2\pi F_c(T_P-W)]}{T_R[1-\exp(-2\pi F_cW)](\sinh 2\pi F_cT_M)}. \quad (6a)$$

Note that the cross talk introduced by poor low-frequency response is opposite in polarity to that caused by poor high-frequency response. Further inspection of (6a) shows that the cross-talk ratio may be high at two values of F_c. The latter fact is borne out by the experimental results plotted in Fig. 7.

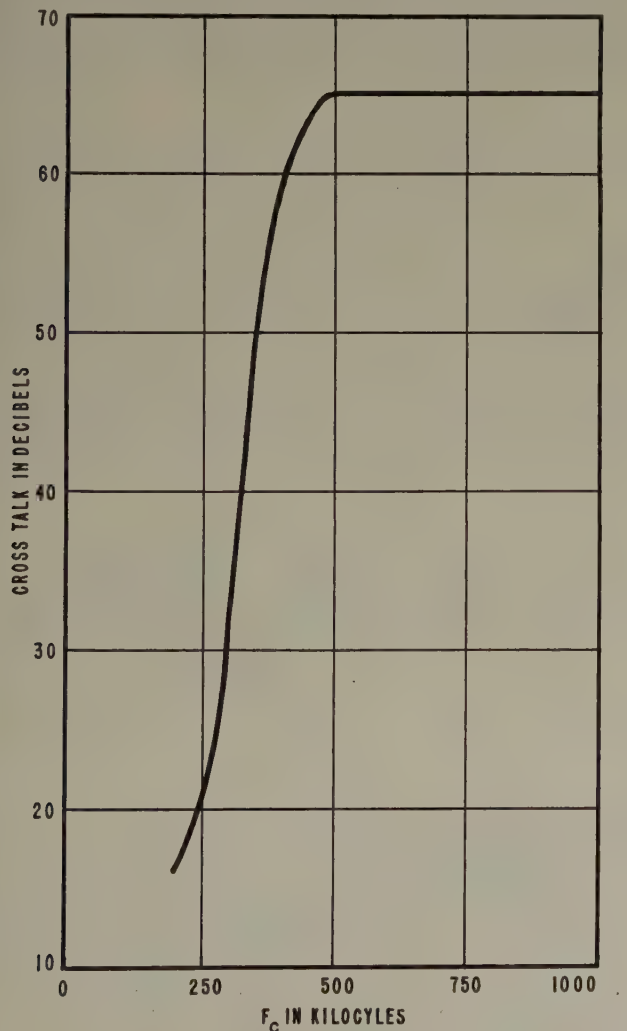


Fig. 5—Cross talk plotted against bandwidth for pulse-position modulation. High frequencies have been attenuated by the π filter shown in Fig. 4.

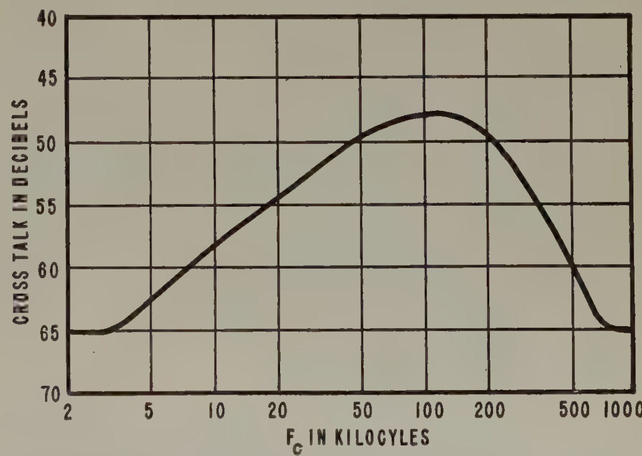


Fig. 7—Cross talk plotted against bandwidth for pulse-position modulation. Low frequencies have been attenuated by a resistance-capacitance filter having a cutoff frequency of $F_c=1/(2\pi RC)$. The high-frequency cutoff is at 3 Mc.

The physical explanation of this curve is simply that, at very low values of cutoff frequency, the pulse is passed by the coupling network without distortion, so that no appreciable carryover occurs. At higher values of cutoff frequency, cross talk is present because of the carryover from channel to channel as shown in Fig. 6. As the cutoff frequency is raised still further, the pulse becomes differentiated and returns to the base line some time before the next channel occurs, again resulting in negligible carryover.

D. Combined High- and Low-Frequency Cutoff

The fact that cross talk due to high-frequency attenuation is of opposite phase from cross talk due to low-frequency attenuation indicates that by suitably choosing the high and low cutoff frequencies some form of cancellation might produce a system featuring high cross-talk ratios with very narrow bandwidth. Results of experiments along this line are plotted in Fig. 8. In each case, the high-frequency response was kept to a value that would not greatly affect cross talk, and the

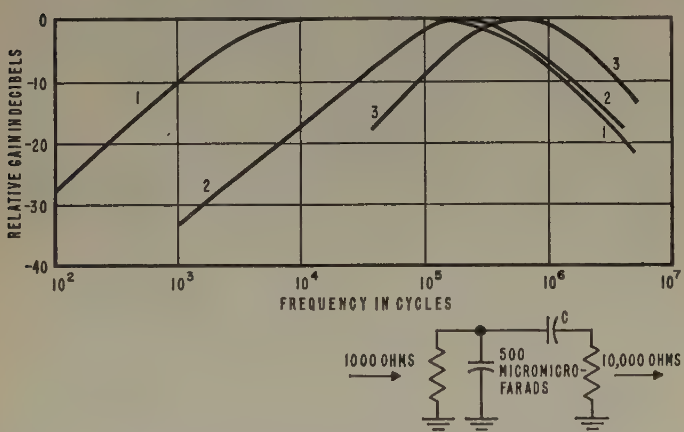


Fig. 8—Relative gain plotted against frequency for pulse-position modulation with filter network shown. Values of C in micromicrofarads and of cross talk in decibels are, respectively, for curve 1, 4,500 and 62; curve 2, 132 and 38; and curve 3, 4.8 and 73.

low-frequency cutoff was varied. Notice that in curve 3, where the over-all bandwidth between the 3-db points was considerably less than 1 Mc, a cross-talk ratio of 73 db was obtained, whereas the inherent cross talk in the system was only 66 db. Thus, some of this inherent cross talk was actually cancelled out. In investigating causes of cross talk in a given system, it is important to remember that a good cross-talk figure may be the result of some form of accidental cancellation of two forms of cross talk arising from different sources.

E. Miscellaneous Considerations

The formula given in Fig. 1 is also useful in determining the spacing between adjacent pulses, and thus the number of channels that may be employed in a given bandwidth. The smaller the pulse spacing, the larger is the required bandwidth for a given deviation and cross-talk figure. In the case of the sharp-cutoff filter, however, this is not necessarily true. Because of the oscillations on the base line resulting from the distortion of the pulse, the pulses may be positioned such that the peaks and troughs of the overshoot can cause cross-talk cancellation. Thus, a position may be found where cross-talk ratio is a maximum, whereas moving the pulse in either direction will cause the cross-talk ratio to decrease. With pulse-position modulation, it will be found that this type of cross talk varies considerably with percentage modulation, depending on the peak deviation used. Such cross talk also may occur accidentally, due to ringing resulting from long inductive leads, insufficient decoupling of circuits containing inductances, and other causes.

III. Pulse-Amplitude Modulation

A. High-Frequency Cutoff

The interchannel cross talk caused by poor high-frequency response may be studied in a manner similar to that used in the above discussion. The expression is derived by the same procedure, with the exception that the displacement of the pulse under modulation is one of amplitude and not a time deviation, and that there is no pulse-position-modulation improvement factor in this case. Referring to Fig. 9, at one extreme of modulation, the carryover is

$$e_1 = E \exp\left[-(T_P/RC)\right](1 + M).$$

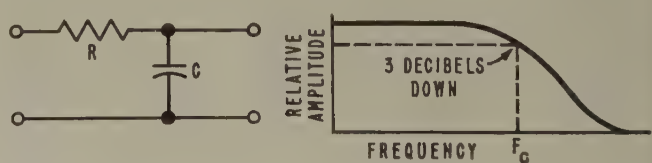
At the other extreme of modulation, the carryover is

$$e_2 = E \exp\left[-(T_P/RC)\right](1 - M).$$

The peak-to-peak amplitude variation of the pulse due to carryover is then

$$e_1 - e_2 = 2ME \exp -(T_P/RC).$$

Since the peak-to-peak output of the channel when modulated in the normal fashion is $2ME$, the cross-talk ratio becomes



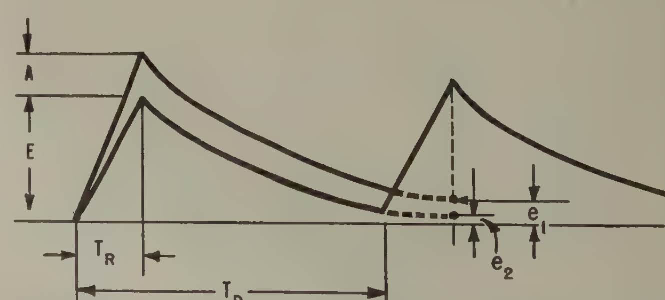


Fig. 9—Effect of high-frequency response on cross talk in pulse-amplitude modulation using a resistance-capacitance filter. $M = A/E$ = modulation factor. Cross-talk ratio = exp (T_P/RC). Cross talk in decibels for 100-per cent modulation = 8.68 T_P/RC.

$$\frac{2ME}{e_1 - e_2} = \frac{2ME}{2ME \exp -(T_P/RC)}.$$

$$\text{Cross-talk ratio} = \exp (T_P/RC). \qquad (7)$$

Taking the logarithm to the base 10 of both sides and multiplying by 20, gives

$$\text{cross-talk ratio in decibels} = \frac{8.68 T_P}{RC}. \qquad (8)$$

B. Low-Frequency Cutoff

Because of the presence of an audio-frequency component in pulse-amplitude modulation, the effect of low-frequency attenuation must be considered from a different standpoint. This audio-frequency component represents a change in the average value of the signal with modulation. When a pulse-amplitude-modulated signal is passed through a coupling network that does not transmit the direct-current component, displacement of the base line occurs. This displacement will vary in accordance with the modulation. The result is that when one channel is modulated, *all other channels* are affected.

Since an exact mathematical analysis is rather involved, an experimental procedure was followed similar to that used in the experiments with pulse-position modulation. A block diagram of the equipment is shown in Fig. 10.

The transmitter consisted of a two-channel pulse-amplitude modulator with a pulse-repetition rate of 8 kilocycles and a pulse width of approximately 3 microseconds. A phasing network was incorporated so that the relative position in time of the two channels could be adjusted at will. Standard resistance-capacitance coupling networks of the type shown in Fig. 6 and having various time constants were inserted at the point marked "transmission network." The desired channel was demodulated and cross talk measured by means of a wave analyzer as previously described.

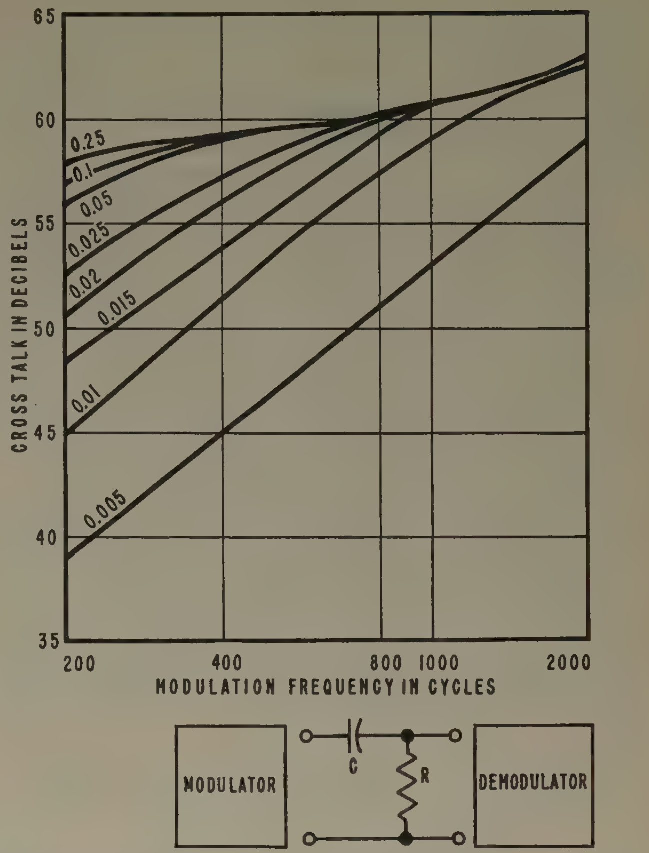


Fig. 11—Cross talk plotted against modulation frequency for 2 pulse-amplitude-modulated channels. The pulse-repetition rate is 8,000 cycles. The designations on the curves correspond to the product of C and R (microfarads × megohms) in the coupling circuit shown.

The results are shown in Fig. 11. The inherent cross talk in the system due to high-frequency carryover was 70 db. It is interesting to note the dependence on modulating frequency, and the extremely large time constants that are required if high cross-talk ratios are to be obtained at low modulating frequencies. For example, to obtain a figure of 56 db with a modulat-

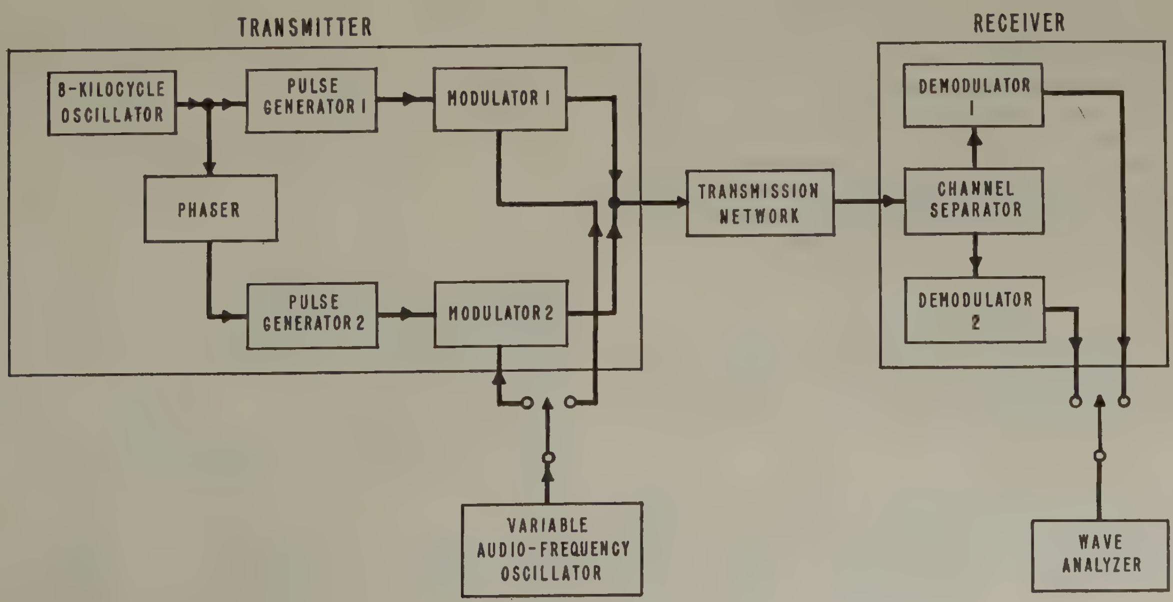


Fig. 10—Equipment used for cross-talk measurements on pulse-amplitude modulation.

ing frequency of 200 cycles, a 0.25-microfarad coupling capacitor and a 1-megohm grid resistor would be required. The cutoff frequency of this network is only 0.6 cycle! This, of course, represents only one coupling network. The requirements become far more severe in a multistage amplifier having several such networks in cascade. The use of such large coupling capacitors introduces new problems, such as the effect on the high-frequency response of capacitance to ground, and changes in the direct-current grid bias of the following tube resulting from leakage currents.

The separation between the two channels was varied and the cross talk at 1,000 cycles measured for two values of transmission time constant. Cross-talk ratio was found to be independent of channel spacing.

The effect of the coupling network may be expressed in several ways: in terms of time constant, cutoff frequency, or in phase shift at a certain frequency. A convenient empirical relation may be derived by taking the data of Fig. 11 and plotting cross talk in decibels against $\log_{10}$ phase shift in degrees, where $\tan\theta = F_c/F$. This is shown in Fig. 12. Since the result is a straight line, the equation of the curve is

$$\text{cross-talk in decibels} = 56.7 - 15.8 \log_{10}\theta. \quad (9)$$

To obtain a cross-talk figure of better than 56.7 db, a phase shift of less than one degree is required. In applications where it may be found impractical to obtain the required low-frequency response by using large coupling capacitors, the usual methods of correcting the low-frequency response may be applied, providing all the stages involved are linear. However, it is advisable to determine the effect on the phase response of slight variations in component tolerances. Adjustment of such compensating networks may be found to be extremely critical. Any other networks that may affect low-frequency response must also be controlled, such as screen-grid and cathode by-pass networks.

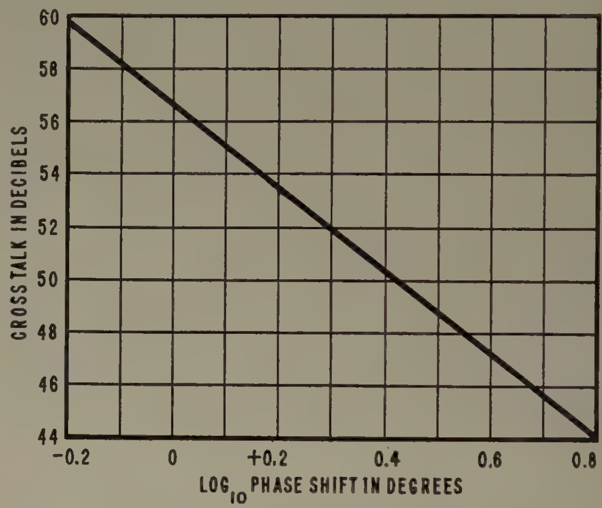


Fig. 12—Cross talk as a function of the logarithm of the phase shift of the coupling network for pulse-amplitude modulation.

The Remainder Theorem and Its Application to Operational Calculus Techniques*

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Summary—**The necessary procedure involved in the transition from the Laplace transformation to the solution of linear differential equations is summarized, and a particular form of partial-fraction expansion which may be advantageous in special cases is noted. The remainder theorem with regard to algebraic polynomials is restated, and it is shown how this theorem may be applied to the evaluation of high-degree polynomials for real and complex numbers. A numerical example is treated to illustrate application to a typical transfer function. Finally, the method is shown to be useful in the evaluation of the frequency response of such a transfer function.**

Introduction

MANY ENGINEERING PROBLEMS, particularly those associated with the design of automatic-control systems, are readily handled by means of the Laplace transform method. Although the method is widely recognized as a powerful tool of analysis, some difficulties are encountered in formulating a numerical solution to any given problem. Accordingly, the following is presented in the interest of those whose have encountered such difficulties.

The Inverse Transform

One of the properties of the Laplace transform is that it reduces a set of linear differential equations in terms of the dependent variables and a real independent variable to a set of linear algebraic equations in terms of the transform of the dependent variables and the complex Laplace variable. This set of equations is usually solved by Cramer's rule and the transformed dependent variables are given by equations of the form

$$L(\theta) = \frac{N(S)}{S^n D(S)} \equiv \frac{a_0 S^r + a_1 S^{r-1} + \cdots + a_{r-1}S + a_r}{S^n(S^q + b_1 S^{q-1} \cdots b_{q-1}S + b_q)} \quad (1)$$

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where S is the Laplace variable, and the degree of the denominator is at least twice greater than the degree of the numerator, i.e., $-1+n+q>r$. Equation (1) may be written alternatively, if the roots of the numerator and denominator are known, as follows:

$$L(\theta) = \frac{a_0(S-S_{1r})(S-S_{2r})\cdots(S-S_{rr})}{S^n(S-S_{1q})(S-S_{2q})\cdots(S-S_{qq})}$$

$$\equiv \frac{a_0\prod_i^r (S-S_{ir})}{S^n\prod_i^q (S-S_{iq})} \tag{2}$$

where the S_{ir} and the S_{iq} are the real and/or complex roots of the numerator and denominator respectively. Note that there are n zero roots in the denominator.

The procedure then followed in the solution of any practical problem at this point is to obtain the inverse transform $L^{-1}(\theta)$. This constitutes a solution to the problem because $L^{-1}(\theta)$ is actually $\theta(t)$ if t is the original independent variable. In general, if there are N dependent variables associated with the problem, there will be N such inverse transforms to determine. The usual manner of obtaining the inverse transform is to expand (2) in partial fractions, evaluate the residues at the poles S_{iq}, and identify the expansion form with inverse transforms which are tabulated.

Evaluating the Residues

Thus the right-hand side (2) may be expanded in partial fractions as follows:

$$\frac{a_0(S-S_{1r})\cdots(S-S_{rr})}{S^n(S-S_{1q})\cdots(S-S_{qq})} \equiv \frac{K_{01}}{S^n}+\frac{K_{02}}{S^{n-1}}+\cdots+\frac{K_{0n}}{S} \tag{3}$$

$$\frac{K_1}{S-S_{1q}}+\frac{K_2}{S-S_{2q}}+\cdots+\frac{K_q}{S-S_{qq}}$$

and the inverse transform of (3) is

$$\theta(t) = \frac{K_{01}}{(n-1)!}t^{n-1}+\frac{K_{02}}{(n-2)!}t^{n-2}+\cdots+K_{0n} \tag{4}$$

$$K_1e^{S_{1q}t}+K_2e^{S_{2q}t}+\cdots+K_qe^{S_{qq}t}.$$

The basic difficulty so often the plague of those attempting a solution in the above manner may now be stated. That is, it is always necessary to obtain the constants (residues) on the right-hand side of (4) such that the initial conditions of the problem are uniquely satisfied. If, for example, at $t=0$, $\theta(0)=0$, the residues must then satisfy the equation

$$\sum_i^q K_i = -K_{0n}. \tag{5}$$

Note that this condition requires a reasonable degree of accuracy in the determination of the various K's. This is especially so, if the K_i's are of the same order of magnitude and if K_{0n} is a relatively small quantity.

We will now review, briefly, the usual procedure of evaluating the residues. Referring to (3), both sides of this identity are multiplied by $S-S_{jq}$ which is a factor in the denominator on the left-hand side. Since the identity is not changed by this operation, S is assigned the value S_{jq}; therefore, all terms on the right-hand side drop out except K_j—corresponding to the root S_{jq}—and the left-hand side becomes a number. Hence K_j is given by the formula

$$K_j = \frac{a_0\prod_i^r (S_{jq}-S_{ir})}{S_{jq}^n\prod_i^q (S_{jq}-S_{iq})_{i\neq j}}. \tag{6}$$

But

$$\prod_i^q (S_{jq}-S_{iq})_{i\neq j} = D'(S_{jq})$$

where $D'(S_{jq})$ is the derivative of the polynomial in the denominator evaluated for $S=S_{jq}$. The K_j is therefore

$$K_j = \left.\frac{N(s)}{S^nD'(S)}\right|_{S=S_{jq}}. \tag{7}$$

Another Form of Expansion

It will now be shown that the same result will be obtained for the K_j when one considers an alternative form of partial fraction expansion for the right-hand side of (3). The form of the expansion will be chosen in such a manner that the initial condition $\theta(0)=0$ at $t=0$ is satisfied by the inverse transform—thus obviating the conditional (5). Alternatively, (3) may be written

$$a_0\frac{N(S)}{S^nD(S)} \equiv \frac{K_{01}}{S^n}+\frac{K_{02}}{S^{n-1}}+\cdots+\frac{K_{0n-1}}{S^2}$$
$$\frac{K_1S_{1q}}{S(S-S_{1q})}+\frac{K_2S_{2q}}{S(S-S_{2q})}+\cdots+\frac{K_qS_{qq}}{S(S-S_{qq})}. \tag{8}$$

Following the familiar procedure of multiplying both sides of the identity by $S-S_{jq}$ and letting $S=S_{jq}$, the K_j are given by

$$K_j = a_0\left.\frac{(S-S_{ir})\cdots(S-S_{rr})(S-S_{jq})}{S^n(S-S_{1q})\cdots(S-S_{jq})\cdots(S-S_{qq})}\right|_{S=S_{jq}} \tag{9}$$

$$K_j = \frac{a_0\prod_i^r (S_{jq}-S_{ir})}{S_{jq}{}^n\prod_i^q (S_{jq}-S_{iq})_{i\neq j}} \tag{10}$$

which is the result obtained previously. We note that

$$L^{-1}\left[\frac{S_{iq}}{S(S-S_{iq})}\right] = [e^{-S_{iq}t}-1]$$

and hence the condition $\theta(0)=0$ is satisfied uniquely by this form of expansion.

The Remainder Theorem

The remainder theorem states that if $Q(S)$ is the quotient obtained by dividing a polynomial $P(S)$ by $f(S)$, and $R(S)$ is the remainder after the division, then the following relationship exists:

$$P(S) \equiv f(S)Q(S) + R(S) \tag{11}$$

in which P, Q, f, and R may all be polynomial functions of S. If $f(S)$ is a linear factor of form $(S-r)$, the remainder $R(S)$ will be a constant, and we may deduce the following:

$$\left.\begin{aligned} \lim_{s\to r} P(S) &\equiv \lim_{s\to r} (S-r)Q(S) + R_0 \\ P(r) &= R_0 \end{aligned}\right\}. \tag{12}$$

Thus if we wish to evaluate a polynomial $P(S)$ for $S=r$ we divide $P(S)$ by $S-r$ and note the remainder. A convenient and rapid way of doing this is by synthetic division.

If r is of complex form $re^{j\alpha}$, it is more convenient to treat $f(S)$ as a quadratic factor in which case the remainder is a linear function of S. We may perform the same limiting process as before, and show that

$$\left.\begin{aligned} \lim_{s\to re^{j\alpha}} P(S) &= \lim_{s\to re^{j\alpha}} [S^2 - 2r\cos\alpha S + r^2]Q(S) \\ &\quad + \lim_{s\to re^{j\alpha}} [R_1 S] + R_0 \end{aligned}\right\} \tag{13}$$

$$P(re^{j\alpha}) = R_1[re^{j\alpha}] + R_0.$$

Thus if we wish to evaluate a polynomial for $S=re^{j\alpha}$, we divide $P(S)$ by the quadratic factor associated with $re^{j\alpha}$ and note the remainder. In particular we see that if r is a root of $P(S)=0$ the remainder is zero. Also we may show that an extension of the above considerations yields the result

$$\left.\begin{aligned} \lim_{s\to r} \frac{dP(S)}{dS} &= \lim_{s\to r} f'(S)Q(S) + \lim_{s\to r} f(S)Q'(S) \\ &\quad + \lim_{s\to r} R'(S) \end{aligned}\right\} \tag{14}$$

$$P'(r) = Q(r)$$

if $f(S)$ is taken to be a linear function of S.

For the vast majority of problems that are solved by the present method, (12), (13), and (14) will provide a convenient and easy means of evaluating the residues of all rational functions that occur in those problems.

Numerical Example

We now consider the evaluation of a typical transfer function. In this regard we express the transfer function as a rational function of S and consider only those residues which serve to illustrate the present method. For real values the division is performed synthetically.

Partial Fraction Expansion

$$\text{T.F.} = \frac{1573.34S^3 + 887.048S^2 + 36.0956S - 0.39513}{S(S+0.0125)^2(S+0.284718)(S^2+0.0789479S+0.00181788)} = \frac{K_1}{S} + \frac{K_2}{(S+0.0125)^2} + \frac{K_3}{S+0.0125}$$
$$+ \frac{K_4}{S+0.284718} + \frac{K_5}{S+0.0394739-0.0161148j} + \frac{K_6}{S+0.0394739+0.0161148j}.$$

Evaluation of Residues

K_2—Real

$$(\text{Residue at } S = r) = \frac{N(r)}{rD'(r)}$$

K_2—Numerator: $N(r)$

$$\begin{array}{rl} 1573.34+887.048+36.0956-0.395130 & \lfloor -0.0125 \\ -\ 19.666-10.8422-0.315667 & \\ \hline 1573.34+867.382+25.2534-\underline{\underline{0.710797}} & \text{Remainder} \end{array}$$

K_2—Denominator: $D'(r)$

$$\begin{array}{rl} 1+0.363666\ +0.0242958+0.000517583 & \lfloor -0.0125 \\ 1-0.0125000-0.0043895-0.000248828 & \\ \hline +0.351166\ +0.0199063+\underline{\underline{0.000268755}} & \text{Remainder} \end{array}$$

$$K_2 = \frac{-0.710797}{-0.0125\times 0.000268755} = 2.115826\times 10^5$$

By continuation of the above method we may determine all the remaining residues.

Summarizing we have:

	Pole	Residue$\times 10^{-5}$
1	S	-48.8585
2	$(S+0.0125)^2$	2.11582
3	$S+0.0125$	2.24833
4	$S+0.284718$	-0.195556
5	$S+0.0394739-0.0161147j$	$23.4025+9.17831j$
6	$S+0.0394739+0.0161147j$	$23.4025-9.17831j$

Frequency Response

We recall that the procedure followed in evaluating the frequency response of a system is to substitute $j\omega$ for S in the system transfer function and evaluate the resulting expression throughout the frequency range of interest. For example, we would be interested in evaluating the expression $N(j\omega)/(j\omega)^n D(j\omega)$ for a selected range of ω.

It is not difficult to show, with the help of the remainder theorem, that we may evaluate the poly-

nomials individually if we divide each by $S^2-\omega^2$ and note the remainder. This is a considerable simplification over the usual method of substituting $j\omega$ directly, separating real and imaginary parts and then carrying out the calculations for all ω of interest. In addition we may show that the artifice of synthetic division may be applied if we skip every other coefficient in the polynomial.

As an illustration we consider the same transfer function as before, confining our attention to an evaluation of the numerator because nothing new will be learned by continuing the calculation further than this.

Frequency Response: Numerator: $N(j\omega)$

$$\omega = 1 \text{ rad/sec}$$

$$\begin{array}{l} 1573.34+887.048+\ 36.0956-\ 0.39513 \quad \underline{|-1} \\ \qquad\qquad\qquad -1573.34 \quad -887.048 \\ \hline 1573.34+887.048-\underline{\underline{1537.25j\ -887.443}} \end{array}$$

$$\text{Remainder} = N(j)$$

$$= -887.443-1537.25j.$$

$$\omega = \sqrt{2} \text{ rad/sec}$$

$$\begin{array}{l} 1573.34+887.048+\ 36.0956-\ 0.39513 \quad \underline{|-2} \\ \qquad\qquad\qquad -3146.68 \quad -1774.096 \\ \hline 1573.34+887.048-\underline{\underline{3110.59\sqrt{2}j-1774.491}} \end{array}$$

$$\text{Remainder} = N(\sqrt{2}j)$$

$$= -1774.491-4398.8j.$$

Thus we see from the above results that the frequency response method may be considerably augmented by the use of the remainder theorem. We note that a polynomial of very high degree, say eight or ten, may be evaluated with little additional labor. Synthetic division is easy to apply, and a whole range of ω may be investigated in relatively short order. It should be mentioned however, that since we are effectively dividing by $S^2-\omega^2$ the remainder will always be of the form $A\omega j+B$.

Conclusions

We have shown that Laplace transform methods of analysis may be simplified somewhat by the use of the remainder theorem as applied to the numerical evaluation of polynomials of high degree.

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Band-Pass Low-Pass Transformation in Variable Networks*

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Summary—**An extension of the band-pass low-pass transformation to linear varying-parameter systems is developed. It is shown that this transformation in conjunction with the use of frequency analysis techniques can be applied with advantage to the analysis of a superregenerator operating in the linear mode.**

Introduction

IN RECENT YEARS the technique of band-pass low-pass transformation has gained wide recognition among communication engineers as a valuable tool in the analysis of various types of carrier transmission systems.[1] This technique has proved particularly useful in connection with the determination of the envelope response of band-pass systems and also, though to a lesser degree, in the analysis of assymetrical carrier systems.

Heretofore, the use of band-pass low-pass transformation has been limited to fixed networks. The purpose of the present paper is to show that the technique of band-pass low-pass transformation may be extended to linear varying-parameter networks[2] and used in the case of the latter in much the same manner as in the case of fixed networks. In particular, it will be shown that this technique may be used with advantage in the analysis of a superregenerator operated in the linear mode. An interesting aspect of the extended form of the transformation and its possible generalizations is that it suggests an unorthodox mathematical technique for solving certain forms of linear differential equations with time-dependent coefficients.

General Theory

A linear varying-parameter network is essentially a linear system in which one or more parameter values are functions of time. The external behavior of such a

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[1] P. R. Aigrain, B. R. Teare, Jr., and E. M. Williams, "Generalized theory of the band-pass low-pass analogy," PROC. I.R.E., vol. 37, pp. 1152–1155; October, 1949. Additional references may be found in the text of this paper.

[2] To simplify the terminology, such networks will frequently be referred to in the present paper as variable networks.

network is usually described by a linear differential equation of the form

$$[a_n(t)p^n + \cdots + a_1(t)p + a_0(t)]v(t) = [b_m(t)p^m + \cdots + b_1(t)p + b_0(t)]u(t) \quad (1)$$

where $p=d/dt$, the a's and b's are known functions of time, and $u(t)$ and $v(t)$ represent respectively the input (voltage or current) and the output (voltage or current) of the network. For convenience (1) may be written in a compact form

$$L(p;\, t)v(t) = K(p;\, t)u(t) \quad (2)$$

where $L(p;\, t)$ and $K(p;\, t)$ denote respectively the left-hand and right-hand operators in (1).

The behavior of a variable network may be described explicitly by means of the system function[3] of the network. This function is denoted by the symbol $H(j\omega;\, t)$ and is defined by the relation

$$H(j\omega;\, t) = \left.\frac{v(t)}{u(t)}\right|_{u(t)=e^{j\omega t}}. \quad (3)$$

In other words, $H(j\omega;\, t)$ is a function of $j\omega$ and t such that $H(j\omega;\, t)e^{j\omega t}$ represents the response of the system to an exponential input $u(t)=e^{j\omega t}$. It may readily be shown[3] that the determination of $H(j\omega;\, t)$ requires in general the solution of the differential equation

$$L(p + j\omega;\, t)H(j\omega;\, t) = K(j\omega;\, t). \quad (4)$$

The problem of solving this equation for $H(j\omega;\, t)$ is discussed in footnote reference 3.

The system function of a variable network has the same basic properties as the system function of a fixed network. Thus, for example, using operational notation the response $v(t)$ to an arbitrary input $u(t)$ may be expressed as

$$v(t) = H(p;\, t)u(t) \quad (5)$$

where $H(p;\, t)$ should be treated as a usual Heaviside operator; that is, the variable t in $H(p;\, t)$ should be treated as a fixed parameter. The physical significance of $H(j\omega;\, t)$ may assume various forms. For instance, when $u(t)$ and $v(t)$ represent respectively the terminal current and voltage of a two-terminal network N, $H(j\omega;\, t)$ may be called the *instantaneous input impedance* of N and accordingly might be denoted as $Z(j\omega;\, t)$. In other cases, $H(j\omega;\, t)$ may represent the instantaneous gain, instantaneous admittance, and so forth, depending on the physical significance of $u(t)$ and $v(t)$.

The notion of the system function of a variable network provides the necessary basis for the extension to linear varying-parameter networks of the technique of band-pass low-pass transformation. As a preliminary, it will be recalled that, in general, any real signal $f(t)$ may be written as

$$f(t) = Re\{F(t)e^{j\omega_0 t}\} \quad (6)$$

where Re means "real part of," ω_0 is an arbitrary frequency, and $F(t)$ is a complex function of time usually referred to as the *complex envelope* of the signal relative to a carrier of frequency ω_0. It is important to note that ω_0 is completely arbitrary. Thus when $f(t)$ is actually an amplitude modulated carrier of frequency ω_c, the frequency ω_0 need not be equal to ω_c.

Expressing $u(t)$ and $v(t)$ in the above form and representing their complex envelopes by $U(t)$ and $V(t)$ respectively, (5) becomes, dropping Re,

$$V(t)e^{j\omega_0 t} = H(p;\, t)\{U(t)e^{j\omega_0 t}\} \quad (7)$$

or, equivalently

$$V(t) = H(p + j\omega_0;\, t)U(t). \quad (8)$$

Equation (8) shows that the complex envelopes of the input and the output may be regarded as being themselves the input and the output of a network whose system function is $H(p+j\omega_0;\, t)$. Under general conditions this point of view is of little practical value, for in the first place $H(p+j\omega_0;\, t)$ is not a real function of p and hence does not represent a physical system. In the second place, there is no reason to assume that $H(p+j\omega_0;\, t)$ would, in general, be simpler than $H(p;\, t)$ and hence there would be little, if any, advantage in using (8) in place of (5).

In the case of band-pass systems, however, it frequently happens that for ω_0 equal to the midband frequency of the system, $H(p+j\omega_0;\, t)$ is very nearly a real function of p and t (for small values of p) and furthermore $H(p+j\omega_0;\, t)$ is simpler than $H(p;\, t)$. It is only in such cases that the process of passing from $H(p;\, t)$ to $H(p+j\omega_0;\, t)$, which involves essentially nothing more than a translation in the frequency domain, is given the distinctive name of the band-pass low-pass transformation. More precisely, assume that there is a frequency ω_0 and a frequency band Δ $(\Delta \ll \omega_0)$, such that for $|p| < \Delta$, $H(p+j\omega_0;\, t)$ is very nearly a real function of p and t, i.e.,

$$H(p + j\omega_0;\, t) \simeq H_L(p;\, t), \quad (|p| < \Delta) \quad (9)$$

where $H_L(p;\, t)$ is a real function of p and t resulting from neglecting small terms in $H(p+j\omega_0;\, t)$. If these conditions are satisfied, then it will be said that the given network N whose system function is $H(p;\, t)$ possesses a low-pass analogue N_L whose system function is $H_L(p;\, t)$. The complex envelopes of the input and output of N may be regarded as being themselves the input and output of N_L. This is expressed by the relation

$$V(t) = H_L(p;\, t)U(t) \quad (10)$$

which follows directly from (8) and (9).

The definition given above is not the most general one possible but is sufficiently general to be useful in many practical applications. The usefulness of the band-pass low-pass transformation in the case of variable net-

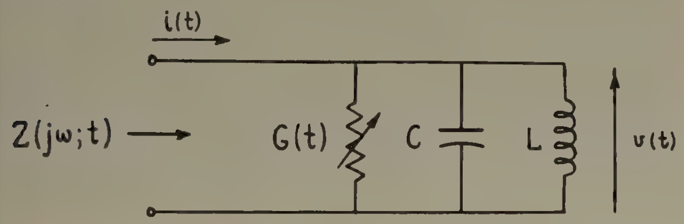
[3] L. A. Zadeh, "Frequency analysis of variable networks," Proc. I.R.E., vol. 38, pp. 291–299; March, 1950.

works is due primarily to the fact that the differential equation satisfied by $H_L(j\omega; t)$, which is (see (4))

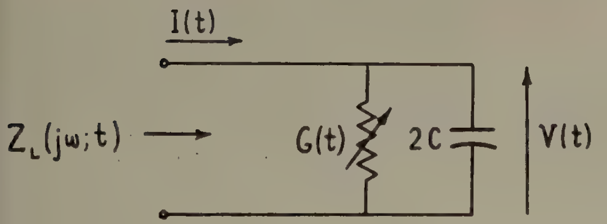
$$L(p + j\omega + j\omega_0; t)H_L(j\omega; t) = K(j\omega + j\omega_0; t), \quad (11)$$

is, in general, much easier to solve than the differential equation satisfied by $H(j\omega; t)$. In other words, it is simpler in most practical cases to determine the system function of the low-pass analogue of a band-pass variable network than it is to determine the system function of the network itself.

As an illustration of the use of the above relations, consider the case of a superregenerator operating in the linear node.[4,5] The equivalent circuit of the superregenerator is shown in Fig. 1(a); it consists of a tank circuit in parallel with a periodically varying conductance $G(t)$. The input to the circuit is $i(t)$ and the output is $v(t)$. The problem is to find the response $v(t)$ to a given input $i(t)$.



(a)



(b)

Fig. 1—The equivalent circuit of the superregenerator and its low-pass analogue.

Since the system function connecting $v(t)$ amd $i(t)$ is the instantaneous input impedance of the superregenerator, it follows that the problem is essentially that of determining the instantaneous input impedance of the circuit shown in Fig. 1(a). It is evident, however, that by using the band-pass low-pass transformation the same result may be achieved in conjunction with (6) and (8) by finding the instantaneous input impedance of the low-pass analogue of the superregenerator. This, of course, should be a much simpler problem.

The differential equation connecting $v(t)$ and $i(t)$ is

$$[Cp^2 + G(t)p + \dot{G}(t) + C\omega_0^2]v(t) = pi(t) \quad (12)$$

where the dot represents differentiation with respect to time and $\omega_0^2 = (LC)^{-1}$. Forming the differential equation satisfied by the instantaneous input impedance of the low-pass analogue (11) and neglecting the small terms in this equation the following relation is obtained:

$$[2Cp + 2Cj\omega + G(t)]Z_L(j\omega; t) = 1 \quad (13)$$

where $Z_L(j\omega; t)$ denotes the instantaneous input impedance of the low-pass analogue, the equivalent circuit of which is shown in Fig. 1(b). Since (13) is a first-order differential equation its solution and hence the expression for $Z_L(j\omega; t)$ is readily found to be

$$Z_L(j\omega; t) = \frac{1}{2C}\int_{-\infty}^{t} \exp\left[-j\omega(t-\tau) - \frac{1}{2C}\int_{\tau}^{t} G(t)dt\right] d\tau. \quad (14)$$

From the general relation (10) it follows that the complex envelopes of the input and the output of the superregenerator are related to each other by the operational relation

$$V(t) = Z_L(p; t)I(t) \quad (15)$$

where $V(t)$ and $I(t)$ represent the complex envelopes of $v(t)$ and $i(t)$, respectively, and the variable t in $Z_L(p; t)$ should be treated as if it were a fixed parameter. Equation (15), in conjunction with (14) and (6), provides the desired solution of the problem.

It should be remarked that what is usually referred to as the "superregenerator selectivity" is in fact the function

$$F(\omega) = |Z_L(j\omega; t_0)| \quad (16)$$

where t_0 is the instant at which the conductance $G(t)$ reaches zero and is in the process of becoming positive. In a similar manner, other parameters and functions usually associated with a superregenerator operating in the linear mode may be interpreted in terms of the properties of the instantaneous input impedance of the low-pass analogue of the superregenerator.

Concluding Remarks

The extended form of the band-pass low-pass transformation described in this paper is useful in the analysis of band-pass linear varying-parameter networks having fixed midband frequency. It is possible to broaden the applicability of this transformation by relaxing the requirement that the system function of the low-pass analogue be a purely real function of p and t. It is also possible to use a more general form of transformation than a simple translation in the frequency domain. On the whole, it appears that the band-pass low-pass transformation considered in this paper is just one of the many commonly used frequency transformations that may usefully be extended to linear varying-parameter networks. The extension of these transformations to such networks should provide a fruitful field for further study and investigation.

[4] H. A. Wheeler, "A simple theory and design formulas for superregenerative receivers," *Wheeler Monographs*, no. 3; June, 1948.

[5] H. A. Glucksman, "Superregeneration—an analysis of the linear mode," Proc. I.R.E., vol. 37, pp. 500–504; May, 1949. Additional references may be found in the text of this paper.

Correlation Functions and Power Spectra in Variable Networks*

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Summary—The problem considered in this paper is that of establishing a relation between the correlation functions and also the power spectra of the input and output of a linear varying-parameter network (variable network) whose transmission characteristics are random-periodic functions of time. The notion of the correlation function of such a network is introduced and the following theorem is established:

The correlation functions of the input and output of a variable network N may formally be regarded as the input and output of a variable network N^* whose system function is the correlation function of the system function of N.

This theorem has many practical applications, particularly in connection with the determination of the correlation functions and power spectra of various random-periodic wave forms.

Introduction

ONE OF THE basic problems in the analysis of signal transmission systems is that of relating the statistical parameters of the output signal to those of the input signal and the transmission characteristics of the system. The purpose of the present paper is to establish such a relation between the correlation functions and also the power spectra of the input and the output of a linear transmission system in which one or more parameter-values are functions of time. Such systems are usually referred to as linear varying-parameter systems or, more simply but less precisely, as variable networks. Familiar examples of systems of this type are the various kinds of modulators, dynamic amplifiers, and—as a limiting case—the ordinary fixed linear networks.

On the basis of the obtained results it appears that the above problem and its solution can be formulated best in terms of the notion of the *system function*[1] of a variable network. Briefly, the system function of a variable network N is defined as a function $H(j\omega;\,t)$ such that $H(j\omega;\,t)e^{j\omega t}$ is the response of N to the exponential input $e^{j\omega t}$. Thus denoting the input and output of N by the symbols $e_1(t)$ and $e_2(t)$, respectively, the definition of $H(j\omega;\,t)$ reads

$$H(j\omega;\,t) = \left.\frac{e_2(t)}{e_1(t)}\right]_{e_1(t)=e^{j\omega t}}. \tag{1}$$

This definition is of the same form as the conventional definition of the system function of a fixed network and, as is shown in the literature,[1] $H(j\omega;\,t)$ has the same basic properties as the system function of a fixed network. The physical significance of $H(j\omega;\,t)$ depends on the meaning attached to $e_1(t)$ and $e_2(t)$. Thus, for example, when N is a two-terminal network and $e_1(t)$ and $e_2(t)$ represent respectively the terminal current and voltage of N, $H(j\omega;\,t)$ may be interpreted as the *instantaneous* input impedance of N and might appropriately be denoted as $Z(j\omega;\,t)$. In other cases $H(j\omega;\,t)$ may be interpreted as the instantaneous transfer impedance, instantaneous admittance, and so forth, depending on the physical significance of $e_1(t)$ and $e_2(t)$.

The most important property of the system function of a variable network N is that it relates the output and input signal of N in much the same manner as the system function of a fixed network relates its output and input signals. The relation between the output and input of N may be expressed in several different, but equivalent forms which are given below. In all of the following relations the variable t in the expression for the system function should be treated as if it were a constant parameter.

1. *Contour integral form.*

$$e_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega;\,t)E_1(j\omega)e^{j\omega t}d\omega \tag{2}$$

where $E_1(j\omega)$ is the Fourier transform of $e_1(t)$.

2. *Fourier transform form.*

$$e_2(t) = F^{-1}\{H(j\omega;\,t)E_1(j\omega)\}, \tag{3}$$

where F^{-1} represents the operation of inverse Fourier transformation.

3. *Laplace transform form.*

$$e_2(t) = L^{-1}\{H(s;\,t)E_1(s)\}, \tag{4}$$

where $E_1(s)$ is the Laplace transform of $e_1(t)$ and L^{-1} represents the operation of inverse Laplace transformation.

4. *Operational form.*

$$e_2(t) = H(p;\,t)e_1(t) \tag{5}$$

where p is the usual differential operator.

A simple illustration of the use of the above relations is provided by a delay network whose delay Δ is a periodic function of time, say

$$\Delta = a + b\cos t. \tag{6}$$

The system function of this network is (in operational form):

$$H(p;\,t) = e^{-p(a+b\cos t)}. \tag{7}$$

Using (5) the response of the network to a signal $e_1(t)$ may be written as

$$e_2(t) = e^{-p(a+b\cos t)}e_1(t). \tag{8}$$

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[1] L. A. Zadeh, "Frequency analysis of variable networks," PROC. I.R.E., vol. 38, pp. 291–299; March, 1950.

Since the variable t in $e^{-p(a+b\cos t)}$ should be treated as a constant, therefore

$$e_2(t) = e_1(t - a - b\cos t) \tag{9}$$

which is the desired result.

The properties of the system function as outlined above will now be used to establish a general relation between the correlation functions of the output and input of a variable network.

General Theory

It will be recalled that the correlation function[2] of a signal $e(t)$ is defined by the relation

$$\psi(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T e(t)e(t+\tau)dt \tag{10}$$

where $\psi(\tau)$ denotes the correlation function of $e(t)$. This definition applies to any random-periodic signal—that is, a signal whose amplitude distribution at any one instant is a periodic function of time. In particular, it applies to the two extreme forms of random-periodic signals, namely, periodic signals and stationary signals. The latter are signals in which the amplitude distribution is independent of time. For stationary signals the correlation function may be written as

$$\psi(\tau) = \overline{e(t)e(t+\tau)}, \tag{11}$$

where the bar represents the operation of averaging over the ensemble.

As is well known, the correlation function and the power spectrum of a signal are Fourier transforms of each other. Thus, denoting the power spectrum[3] of a signal by the symbol $S(\omega)$, the relation between $S(\omega)$ and $\psi(\tau)$ reads

$$S(\omega) = \int_{-\infty}^{\infty} \psi(\tau)e^{-j\omega t}d\tau \tag{12}$$

and

$$\psi(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{j\omega t}d\omega. \tag{13}$$

It is evident that $S(\omega)$ may be found from the knowledge of $\psi(\tau)$ and vice versa. In general, the determination of the correlation function is simpler than that of the power spectrum. In many cases, knowledge of the correlation function alone is sufficient. In cases in which the power spectrum is required, its indirect determination through the use of the correlation function and (12) is frequently the most convenient procedure.

Turning to the problem under consideration, it will be assumed that the transmission characteristics of a network N vary with time in a random-periodic manner. In terms of the system function of N this means that $H(j\omega; t)$ is a random-periodic function of time involving $j\omega$ as a parameter. The input to N is assumed to be a random-periodic function of time which is uncorrelated with $H(j\omega; t)$. The correlation functions of the input and output of N will be denoted by the symbol $\psi_1(\tau)$ and $\psi_2(\tau)$, respectively, The problem is to establish a relation between $\psi_1(\tau)$, $\psi_2(\tau)$, and a function describing the pertinent statistical properties of $H(j\omega; t)$.

It will be seen later that the pertinent statistical properties of N are represented by a function which in view of its form might appropriately be called the *correlation function of the system function of the network*, or, more simply, the correlation function of N. This function is denoted by the symbol $\Psi_H(\tau; \omega)$ and is defined by the relation

$$\psi_H(\tau; \omega) = \lim_{T\to\infty} \frac{1}{T}\int_0^T H(j\omega; t)H(-j\omega; t+\tau)dt, \tag{14}$$

which in the case of stationary system functions is equivalent to

$$\psi_H(\tau; \omega) = \overline{H(j\omega; t)H(-j\omega; t+\tau)}. \tag{15}$$

It will be demonstrated later that $\psi_H(\tau; \omega)$ has a remarkable property which is that $\psi_H(\tau; \omega)$ may be regarded as the system function of a variable network N^* such that the correlation functions $\psi_1(\tau)$ and $\psi_2(\tau)$ are the input and output of this network, respectively. This result, which is essentially the main result of this paper, is expressed by the following relation

$$\psi_2(\tau) = F^{-1}\{\psi_H(\tau; \omega)S_1(\omega)\}, \tag{16}$$

where $S_1(\omega)$ is the power spectrum of the input (the Fourier transform of $\psi_1(\tau)$), $\psi_H(\tau; \omega)$ is the correlation function of $H(j\omega; t)$, and F^{-1} represents the operation of inverse Fourier transformation.[4]

Comparison of (16) with the relation connecting the output and input of a variable network

$$e_2(t) = F^{-1}\{H(j\omega; t)E_1(j\omega)\} \tag{3}$$

leads to the interpretation[5] of (16) stated above. The result expressed by (16) may conveniently be stated in the form of a theorem, as follows:

The correlation functions of the input and output of a variable network N may formally be regarded as the input and output of a variable network N whose system function is the correlation function of the system function of N.*

To simplify the proof of the theorem it will be assumed that the input $e_1(t)$ and the system function $H(j\omega; t)$ are stationary functions of time.[6] With this

[2] A comprehensive discussion of the properties of random signals may be found in S. O. Rice, "Mathematical analysis of random noise," *Bell Sys. Tech. Jour.*, vol. 23, pp. 282–332; July, 1944. Also vol. 24, pp. 46–156; January, 1945.

[3] Contrary to the usual practice, the power spectrum $S(\omega)$ is assumed to extend over both the negative and positive frequencies, i.e., $S(-\omega) = S(\omega)$. This assumption facilitates the mathematical analysis of the problem.

[4] It should be remembered that in the process of evaluating the inverse Fourier transform of $\psi_H(\tau; \omega)\ S_1(\omega)$, the variable τ in $\psi_H(\tau; \omega)$ should be treated as if it were a constant parameter.

[5] It will be recognized, of course, that N^* is not a physical system since its system function $\psi_H(\tau; \omega)$ is an even function of ω.

[6] For the general case where the input and the system function are uncorrelated random-periodic functions of time, the proof is similar but somewhat more involved.

assumption, the theorem may be proved as follows.[7]

Use of (2) gives

$$e_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega; t)E_1(j\omega)e^{j\omega t}d\omega \tag{17}$$

and

$$e_2(t+\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega; t+\tau)E_1(j\omega)e^{j\omega(t+\tau)}d\omega. \tag{18}$$

Changing the variable of integration in (18) from ω to ω' and forming the product of (17) and (18), the expression for $e_2(t)e_2(t+\tau)$ reads

$$e_2(t)e_2(t+\tau) = (2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(j\omega; t)H(j\omega'; t+\tau) \cdot E_1(j\omega)\cdot E_1(j\omega')e^{j\omega t}e^{j\omega'(t+\tau)}d\omega d\omega'. \tag{19}$$

The averaging of both sides of (19) with respect to random variations in $e_1(t)$ yields

$$\overline{e_2(t)e_2(t+\tau)} = (2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(j\omega; t)H(j\omega'; t+\tau) \cdot \overline{E_1(j\omega)E_1(j\omega')}e^{j\omega t}e^{j\omega'(t+\tau)}d\omega d\omega'. \tag{20}$$

Now it can readily be shown that for any stationary signal such as $e_1(t)$

$$\overline{E_1(j\omega)E_1(j\omega')} = S_1(\omega)\delta(\omega - \omega') \tag{21}$$

where $\delta(\omega-\omega')$ denotes a unit impulse in the frequency domain.

In view of this relation, it is evident that the integrand of (20) is zero unless $\omega+\omega'=0$. Therefore (20) reduces to

$$\overline{e_2(t)e_2(t+\tau)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega; t)H(-j\omega; t+\tau)S_1(\omega)e^{-j\omega\tau}d\omega. \tag{22}$$

To obtain the correlation function of $e_2(t)$ it is necessary to perform additional averaging of both sides of (22)—this time with respect to random variations in $H(j\omega; t)$. The result of this averaging is

$$\psi_2(\tau) = \overline{\overline{e_2(t)e_2(t+\tau)}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{H(j\omega; t)H(-j\omega; t+\tau)}S_1(\omega)e^{-j\omega\tau}d\omega. \tag{23}$$

In view of the definition of the correlation function of N, (23) may be written as

$$\psi_2(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_H(\tau; \omega)S_1(\omega)e^{j\omega\tau}d\omega \tag{24}$$

or

$$\psi_2(\tau) = F^{-1}\{\psi_H(\tau; \omega)S_1(\omega)\}, \tag{16}$$

[7] The symbols $e_1(t)$, $e_2(t)$, and $H(j\omega; t)$ appearing in this proof represent functions of time which are equal, respectively, to the input, output and the system function of N for $0<t<T$ (a long interval of time) and are equal to zero outside of this interval.

which in conjunction with (3) completes the proof of the theorem.

The fact that $\psi_1(\tau)$ and $\psi_2(\tau)$ are the input and output, respectively, of a variable network whose system function is $\psi_H(\tau; \omega)$ may be used with advantage to obtain a direct relation between the power spectra of the input and output of N. Thus, in footnote reference 1 it is shown that the Fourier transforms of the input and output of a variable network are related to each other by

$$E_2(j\omega) = \int_{-\infty}^{\infty} \Gamma(j\omega'; j\omega)E_1(j\omega')d\omega' \tag{25}$$

where the so-called *bifrequency system function*, $\Gamma(j\omega'; j\omega)$, is the Fourier transform of $H(j\omega'; t)e^{j\omega' t}$. Now, since the power spectra $S_1(\omega)$ and $S_2(\omega)$ are the Fourier transforms of $\psi_1(\tau)$ and $\psi_2(\tau)$, it follows from (25) that

$$S_2(\omega) = \int_{-\infty}^{\infty} \Gamma^*(j\omega'; j\omega)S_1(\omega')d\omega', \tag{26}$$

where $\Gamma^*(j\omega'; j\omega)$ is the Fourier transform of $\psi_H(\tau; \omega')e^{j\omega' t}$. This expression provides the desired relation between the power spectra of the input and output of N.

The theorem established above regarding the relation connecting $\psi_1(\tau)$, $\psi_2(\tau)$, and $\psi_H(\tau; \omega)$, has many practical applications. The use of this theorem will be illustrated below by its application to the two degenerate forms of variable networks, namely, fixed networks and amplitude modulating networks. In addition, one other simple example will be considered.

Special Cases

A. Fixed Networks.

The system function of a fixed network is independent of time, i.e., it is of the form

$$H(j\omega; t) = H(j\omega). \tag{27}$$

The expression for the correlation function of the system function reduces in this case to:

$$\psi_H(\tau; \omega) = |H(j\omega)|^2, \tag{28}$$

which is a function of frequency only. From (16) it follows that

$$\psi_2(\tau) = F^{-1}\{|H(j\omega)|^2 S_1(\omega)\} \tag{29}$$

and hence

$$S_2(\omega) = |H(j\omega)|^2 S_1(\omega), \tag{30}$$

which is the well-known relation connecting the power spectra of the output and input of a fixed network.

B. Amplitude Modulating Networks.

In this cases the system function is independent of frequency, i.e., it is of the form

$$H(j\omega; t) = H(t). \tag{31}$$

The expression for the correlation function of the system function in this case becomes

$$\psi_H(\tau;\omega) = \psi_H(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T H(t)H(t+\tau)dt \quad (32)$$

which is a function of τ only. From (16) it follows that

$$\psi_2(\tau) = \psi_H(\tau)\psi_1(\tau). \quad (33)$$

In other words, *the correlation function of the output of an amplitude modulating network is the product of the correlation function of the input and the correlation function of the system function of the network.*

C. Illustrative Example.

An amplitude modulator whose system function is

$$H(t) = 1 + m\cos\omega_0 t \quad (34)$$

will be considered. It is assumed that the power spectrum of the input is

$$S_1(\omega) = \frac{2a}{a^2+\omega^2} \quad (35)$$

and correspondingly

$$\psi_1(\tau) = e^{-a|\tau|}. \quad (36)$$

The correlation function of $H(t)$ is

$$\psi_H(\tau) = 1 + \frac{m^2}{2}\cos\omega_0\tau. \quad (37)$$

From (33) it follows that the correlation function of the output is given by

$$\psi_2(\tau) = \left(1 + \frac{m^2}{2}\cos\omega_0\tau\right)e^{-a|\tau|}. \quad (38)$$

The corresponding power spectrum is readily found to be

$$S_2(\omega) = \frac{2a}{a^2+\omega^2} + \frac{am^2}{2}\left[\frac{1}{(\omega+\omega_0)^2+a^2} + \frac{1}{(\omega-\omega_0)^2+a^2}\right]. \quad (39)$$

Further applications of the results obtained in this paper will be given in a forthcoming paper.

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Nyquist Diagrams and the Routh-Hurwitz Stability Criterion*

FRANK E. BOTHWELL†

Summary—**The Nyquist and the Routh-Hurwitz stability criteria as methods of locating regions of stability of dynamical and electrical systems are described and compared. The superiority of the Routh-Hurwitz method in some applications is demonstrated by two examples, the first a two-loop, three-stage, feedback amplifier of Llewellyn, and the second a multiloop servo system.**

I. Introduction

THE APPEARANCE in 1932 of the famous paper of Nyquist[1] marked the beginning of a new era in the analysis of stability of electric circuits. Prior to this time stability investigations were usually effected by direct solutions for the complex roots of the characteristic equation of the system, a process generally believed to be considerably more difficult than the calculation of a Nyquist diagram. Consequently, the Nyquist method was hailed as a tremendous advance in the art of stability analysis. It has grown steadily in popularity in the electrical engineering profession in this country, particularly in the fields of communications and servomechanisms wherein it has been given special emphasis.

The growth in popularity of the Nyquist criterion is little short of amazing when it is considered that there had existed for fifty-five years prior to the work of Nyquist a method (originally due to Routh[2,3]) simpler than that of Nyquist and, in some respects, more powerful. It shall be the purpose in this paper to describe each method and to demonstrate by numerical examples the possible superiority, in some cases, of the lesser known method of Routh.

* Decimal classification: R140. Original manuscript received by the Institute, November 18, 1949; revised manuscript received, May 26, 1950.

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[1] H. Nyquist, "Regeneration theory," *Bell Sys. Tech. Jour.*, vol. 11, pp. 126–147; January, 1932.

[2] E. J. Routh, "Stability of a Given State of Motion," Adams Prize Essay, Macmillan, London, England; 1877.

[3] E. J. Routh, "Dynamics of a System of Rigid Bodies," Macmillan, London, England, part II, chap. VI; 1905.

II. The Nyquist Criterion

The Nyquist criterion in its most general form as it is used today is not easily stated. It was originally derived for a single-loop feedback circuit and involves the numerical calculation of the complex open-loop transfer characteristic of the network. The question of stability is decided by whether or not the transfer characteristic encloses the critical point (1, 0). In case the system contains several feedback loops it is necessary to calculate the open-loop characteristic for each loop and to record the algebraic number of counterclockwise encirclements of the critical points. Stability of the over-all circuit is subsequently decided by the sum of these encirclements. This procedure is confusing to the uninitiated and to the engineer not familiar with the mathematical logic behind these rules, and has consequently led to many misconceptions concerning the interpretation of the Nyquist diagrams. Moreover, there are complicated circuits in which it is difficult or impossible to decide on the number and location of these loops.

For the reasons just outlined, the Nyquist criterion will be stated as a condition on the characteristic equation of the system under investigation. In any linear dynamical or electrical system, the determination of the characteristic equation is straightforward and usually a simple matter. Let the characteristic equation (always an algebraic equation) be

$$f(p) = a_0 p^n + a_1 p^{n-1} + \cdots + a_n = 0. \tag{1}$$

The question of stability is decided by the signs of the real parts of the roots of (1). In fact the necessary and sufficient condition that the system be stable is that all n roots of (1) have negative real parts. That is, all roots of (1) must lie to the left of the axis of imaginaries in the p plane. Let the imaginary axis, $p = i\omega$, be mapped on the $f(p)$ plane. If, as ω is increased from negative to positive infinity, the origin of the f plane lies to the left of the curve $f(i\omega)$, the roots of $f(p)$ all lie in the negative half p plane, and consequently, the system is stable.

III. The Routh-Hurwitz Criterion

The Routh criterion may be stated as follows.[4,5] Form the equation

$$\begin{vmatrix} a_0 & a_{-1} & a_{-2} & a_{-3} & \cdots & a_{-n} \\ a_2 & a_1 & a_0 & a_{-1} & \cdots & a_{2-n} \\ a_4 & a_3 & a_2 & a_1 & \cdots & a_{4-n} \\ a_6 & a_5 & a_4 & a_3 & \cdots & a_{6-n} \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ a_{2n} & a_{2n-1} & a_{2n-2} & a_{2n-3} & \cdots & a_n \end{vmatrix} = \Delta \tag{2}$$

from the coefficients of the characteristic equation (1). All elements with subscripts greater than n or less than zero are set equal to zero. For example, a characteristic equation which is a quartic yields the expression

$$\Delta = a_0[(a_1a_2 - a_0a_3)a_3 - a_1^2a_4]a_4.$$

The stability boundary of the system whose characteristic equation is (1) consists of certain branches of the equation $\Delta = 0$. In general, not all branches are boundaries, but the equation does contain the entire boundary. In order to establish the boundary, it is necessary to check each branch by a numerical computation.

The coefficients a_0 and a_n are factors of Δ. When Δ becomes zero through a_n changing sign, the corresponding unstable root of (1) is real and represents a nonoscillatory case, whereas if a_n does not pass through zero as Δ passes through zero, the corresponding unstable root of (1) produces oscillations.

By means of certain transformations of (1), additional information concerning the location of its roots may be obtained[6,7] and in fact either the Nyquist or Routh method, together with these transformations, may be used to obtain the roots with any degree of accuracy.[8] For example, if either criterion is applied successfully to the polynomial

$$\begin{aligned}(p - b) &= a_0(p - b)^n + a_1(p - b)^{n-1} + \cdots + a_0 \\ &= \bar{a}_0 p^n + \bar{a}_1 p^{n-1} + \cdots + \bar{a}_n = 0,\end{aligned} \tag{3}$$

the real part of each root of $f(p)$ must be less than $-b$.

The history of the development of the Routh criterion is interesting. The initial work was due to Routh in 1877.[2] In 1895, Hurwitz, who apparently was unfamiliar with the work of Routh, independently derived the conditions for stability in the form of determinants.[4] In 1929 Frazer and Duncan, apparently unfamiliar with the work of Hurwitz, and starting with the results of Routh, deduced exactly the criterion as presented by Hurwitz.[9] Finally in 1945, Wall (however, familiar with the work of each) deduced the results of Hurwitz by an entirely different method, namely, by means of a partial fraction expansion.[10] Most British texts on advanced dynamics refer to Routh but never to Hurwitz, while the German authors usually refer to Hurwitz and not Routh. In this country the Routh-Hurwitz criterion has received a great deal of attention

[4] A. Hurwitz, "Ueber die Bedingungen, unter welchen eine Gleichung nur Wurzeln mit negativen reelen Theilen besitzt," *Math. Ann.*, vol. 46, pp. 273–284; 1895.

[5] P. A. Samuelson, "Foundations of Economic Analysis," Harvard University Press, Cambridge, Mass., p. 435; 1948.

[6] P. A. Samuelson, "Conditions that the roots of a polynomial be less than unity in absolute value," *Ann. Math. Statistics*, vol. 12, pp. 360–364; September, 1941.

[7] A. Vazsonyi, "A generalization of Nyquist's stability criteria," *Jour. Appl. Phys.*, vol. 20, pp. 863–867; September, 1949.

[8] E. Frank, "The location of the zeros of polynomials with complex coefficients," *Bull. Amer. Math. Soc.*, vol. 52, pp. 890–898; October, 1946.

[9] R. A. Frazer and W. J. Duncan, "On the criteria of the stability of small motions," *Proc. Roy. Soc.* (London), series A, vol. 124, p. 642; July, 1929.

[10] H. S. Wall, "Polynomials whose zeros have negative real parts," *Amer. Math.*, vol. 52, pp. 308–322; June–July, 1945.

recently among mathematicians[11] and engineers.[12,13]

It is interesting that Nyquist, Routh, and Hurwitz all employed the same mapping of the axis of imaginaries of the p plane. Nyquist stopped at this point and stated his criterion, whereas Routh and Hurwitz went beyond to deduce analytical relationships for the stability boundary.

IV. Examples

As an example of the use of the Routh-Hurwitz criterion, consider the three-stage, two-loop, shunt feedback amplifier of Fig. 1, initially described by Llewellyn.[14] For simplicity, all the admittances have been

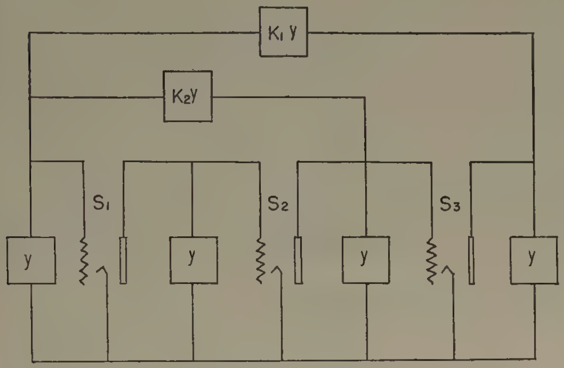

Fig. 1—Illustrative circuit, shunt feedback amplifier.

taken equal. By setting up the network equations on the node basis, the characteristic equation of the network under the assumptions

$$K_1 \leqq \frac{1}{10}, \qquad K_2 \leqq \frac{1}{10}, \qquad S_3 \geqq 10 \tag{4}$$

is

$$y(y^3 + K_1K_2S_3y^2 - K_2S_1S_2y + K_1S_1S_2S_3) = 0. \tag{5}$$

Some indication of the application of the Nyquist criterion to this circuit under the assumptions of (4) is given by Bode.[15]

Let us investigate the case in which the admittance y is composed of a resistance and capacitance in parallel, that is,

$$y = Cp + \frac{1}{R} \cdot \tag{6}$$

Of course, one root of the characteristic equation always occurs at $y=0$, that is, at $p=-1/RC$. Let

$$p = \frac{1}{RC}[(1-b)x - b], \quad b < 1, \tag{7}$$

and substitute (7) in (5). The result is the equation

$$(x+1)(a_0x^3 + a_1x^2 + a_2x + a_3) = 0 \tag{8}$$

in which

$$\begin{aligned} a_0 &= 1 \\ a_1 &= 3 + \alpha_1\alpha_2 \\ a_2 &= 3 + 2\alpha_1\alpha_2 - \alpha_1\alpha_3 \\ a_3 &= 1 + \alpha_1\alpha_2 - \alpha_1\alpha_3 + \alpha_2\alpha_3 \end{aligned} \tag{9}$$

where

$$\begin{aligned} \alpha_1 &= K_2 \\ \alpha_2 &= \frac{K_1RS_3}{1-b} \\ \alpha_3 &= \frac{R^2S_1S_2}{(1-b)^2} \cdot \end{aligned} \tag{10}$$

Now the condition that the real parts of the roots of (8) be negative insures that the real parts of the roots of (5) be less than $-(b/RC)$. In fact, the stability boundary of (8) is the locus of points such that the maximum time constant of the system is just equal to RC/b. The stability region of (8) according to the Routh-Hurwitz criterion is given by the inequalities (since $a_0=1>0$)

$$\begin{aligned} a_3 &> 0 \\ a_1a_2 - a_0a_3 &> 0. \end{aligned} \tag{11}$$

Substituting (9) into (11), it is found that the stability conditions are

$$\begin{aligned} \alpha_3 &< \frac{1}{\alpha_1 - \alpha_2} && \text{for} \quad \alpha_2 < \frac{2}{3}\alpha_1 \\ &< \frac{2(2+\alpha_1\alpha_2)^2}{2\alpha_1 + \alpha_2} && \text{for} \quad \alpha_2 > \frac{2}{3}\alpha_1. \end{aligned} \tag{12}$$

It is noted that the time constants of the circuit are determined by the three parameters, K_2, K_1RS_3, and $R^2S_1S_2$. If the inequality signs in (12) are replaced by equality signs, the result is the locus of parameter values for which the maximum time constant of the system is just equal to RC/b. This locus is shown in Fig. 2. The only restriction on b is that it be less than unity; it may be positive or negative. For the case $b=0$, the curves of Fig. 2 are the stability boundaries of the circuit of Fig. 1. To the left of the peak, the root with maximum time constant is real, whereas to the right of the peak, the root is complex, corresponding to an oscillatory term.

A considerable amount of information can be gathered from Fig. 2. Suppose that for $b=0$ and $K_2=10^{-2}$, the operating point is located at A in Fig. 2. The circuit is stable, but if tube 3 fails, S_3 is reduced to zero

[11] E. Frank, "On the zeros of polynomials with complex coefficients," *Bull. Amer. Math. Soc.*, vol. 52, pp. 144–157; February, 1946.

[12] M. F. Gardner and J. L. Barnes, "Transients in Linear Systems," John Wiley and Sons, Inc., New York, N. Y., pp. 197–201; 1942.

[13] E. A. Guillemin, "The Mathematics of Circuit Analysis," John Wiley and Sons, Inc., New York, N. Y., pp. 395–409; 1949.

[14] F. B. Llewellyn, U. S. Patent No. 2,245,598, June 17, 1941.

[15] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., Inc., New York, N. Y., pp. 158–162; 1945.

and the operating point moves across the stability boundary as indicated by the arrow. However, failure of either tube S_1 or S_2 never causes instability. Notice that as S_3 is increased from zero, the circuit may pass from an unstable situation successively through stability, then instability, and finally to a stable situation. Notice also that the direction of movement of the stability boundaries as K_2 is increased from zero depends markedly on the value of the parameter K_1RS_3.

An advantage of the Routh-Hurwitz method over the Nyquist method is clearly illustrated by this example. The former yields the single analytical expressions (12) for the stability boundaries or maximum time constants. A complete description of the situation is given by graphs in Fig. 2.

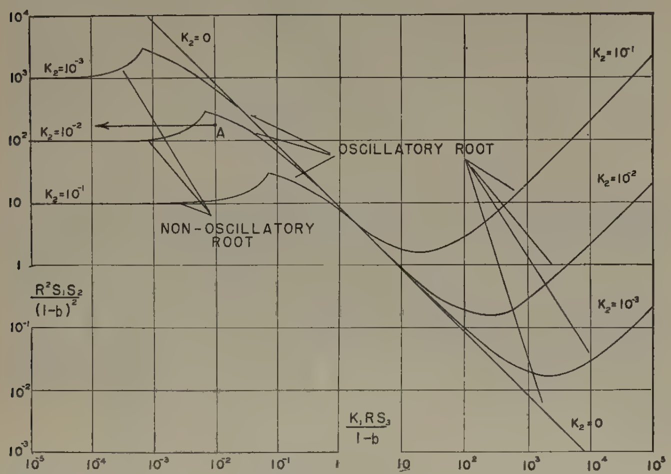

Fig. 2—Locus of points for which the maximum time constant of the circuit of Fig. 1 is RC/b.

On the other hand, the Nyquist method is essentially a numerical process, and the problem of locating stability boundaries is a trial-and-error procedure. One may start with (8), but for each choice of the parameters α_1, α_2, and α_3, it is necessary to make a Nyquist diagram. One then varies one parameter, say α_3, recalculating the diagram after each choice, until the boundary is located. If one has no idea of the order of magnitude of α_3 on the boundary, the actual location of the boundary is tedious. The location of the peaks in Fig. 2 by the Nyquist method is particularly difficult.

A second example, suggested by James, Nichols, and Phillips,[16] is afforded by the multiloop servo system

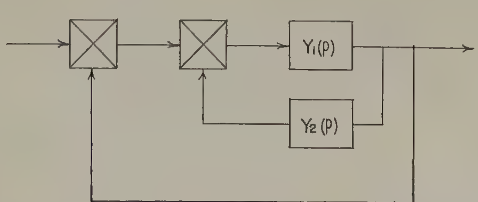

Fig. 3—Illustrative circuit, multiloop servomechanism.

[16] H. M. James, N. B. Nichols, and R. S. Phillips, "The Theory of Servomechanisms," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 74–75; 1947.

shown in Fig. 3. The circuit consists of an amplifier and motor with transfer characteristic

$$Y_1(p) = \frac{K_2}{p(T_m p + 1)} \tag{13}$$

and a tachometer and filter combination with characteristic

$$Y_2(p) = K_1 p\left(\frac{T_0 p}{T_0 p + 1}\right)^3. \tag{14}$$

The stability boundaries are shown in Fig. 4.

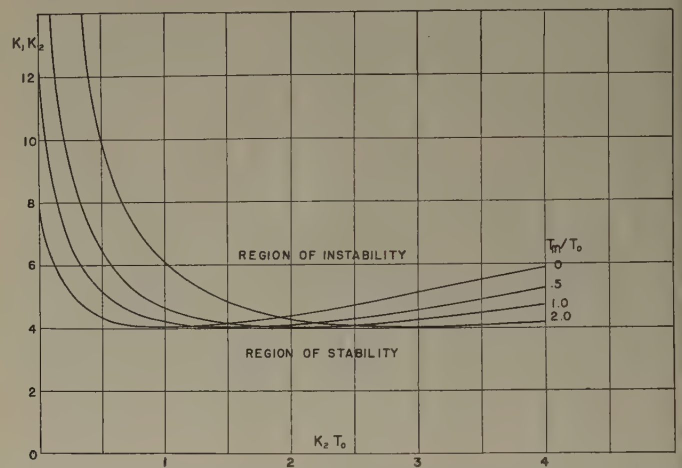

Fig. 4—Stability boundaries for the multiloop servo system of Fig. 3 and equations (13) and (14).

Expressions relating the parameters α_1, α_2, α_3 to the maximum time constant may be obtained in much the same manner as in the first example. However, they are somewhat involved and will not be presented here.

V. Concluding Remarks

A few final remarks are worthy of note. One might be somewhat dismayed at the size of the Hurwitz determinant in certain involved cases in which the characteristic equation is of high order. However, it is usually found that the expanded determinantal equation (always an algebraic equation in the circuit parameters) is of low degree in at least one of the parameters under consideration, so that it is a simple matter to compute the stability boundary. In the last example, although the characteristic equation is of fifth degree, the stability boundary is a second-degree equation in the parameter K_1K_2.

For a fair comparison of Nyquist and Routh methods it must be mentioned that the former permits stability investigation from experimentally determined transfer functions not in analytical form, whereas the latter does not. Also, in cases in which the expansion of the Hurwitz determinant is too laborious or does not lead to a low-order equation in any parameter, a numerical procedure must be followed.

On the Approach to Steady State of a Linear Variable Network Containing One Reactance*

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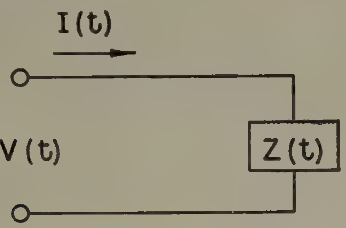
Summary—It is often desirable to determine the manner in which a circuit approaches steady-state operation after a periodic input is applied. More precisely, an answer is sought to the question: "After how many cycles of input may the voltages and currents within the circuit be considered periodic, to within a certain error?"

If the circuit is linear, an answer can always be obtained by the ordinary methods of mathematical attack. All too often, however, such procedures are little more than tours de force of tedious calculations. This is particularly true when some of the circuit component values are variable with time, as is sometimes the case when relays, tubes, switches, and the like are present.

In this paper a method will be developed for answering the question posed above for a restricted, but still quite general, class of linear circuits. Practically speaking, the main restriction is that there be only one reactive component in the circuit. The analysis proceeds by development of a linear, first-order difference equation, whose solution characterizes the circuit's transition from quiescence to transient to steady operation.

Discussion

THE CIRCUIT



if (1) it is linear,
(2) it can be reduced to a circuit having only one reactive component,
(3) all variables involved possess Laplace transforms,
(4) all variable circuit components (resistances and reactances) are periodic in value and all have the same over-all period as
(5) $V(t)$, which is periodic,

will, with time, gradually approach steady-state operation, once the input has been applied. The condition of steady state is distinguished from transient operation by the fact that in the former the circuit voltages and currents during one cycle of input are virtually identical with the same quantities during any succeeding cycle. From the practical point of view, it is often of interest to know how a circuit such as the above approaches steady state, and in how short a time the transients become of negligible value.

Let us ignore for a moment the precise wave forms in the circuit. Consider the value of the voltage across the reactance at the beginning of a certain input cycle. If this value is the same as that of the same reactance voltage at the beginning of the previous input cycle, then the circuit has obviously entered upon a closed, periodic (i.e. steady-state) operation. If the two values of reactance voltages mentioned are "almost the same," then the circuit is "almost at steady state."

We shall proceed to derive a general expression for the current $I(t)$, in terms of the circuit components and $V(t)$.

Symbolism

$i(p)$, $v(p)$, $r_k(p)$, $x(p)$ are the Laplace transforms of the quantities $I(t)$, $V(t)$, $R_k(t)$, $Z(t)$.

$X(t)$ and $R_k(t)$ are, respectively, the unique reactive and the various resistive components of the circuit. As indicated, they may all or severally vary with time. They must, however, be independent in value of any other quantity, since the circuit is linear. Further, by the restrictions before listed, if they are periodic they must have the same period as $V(t)$.

$E(t)$ is the voltage across the reactance. In particular, $E(O)$ is the reactance voltage at the beginning of transform time. The choice of $E(O)$ is not restrictive, since the circuit admits of no two independent parameters of state.

Now the circuit has a governing equation in the transform variable p, which may be found by common methods of analysis:[1]

$$i(p) = F[p; v(p); r_1(p); r_2(p) \cdots ; x(p); E(O)] \equiv F(p) \quad (1)$$

and which can be solved for $I(t)$ by means of the Mellin Inversion Theorem.[2]

$$I(t) = \frac{1}{2\pi j}\int_{B_r} i(\lambda)e^{\lambda t}d\lambda = \frac{1}{2\pi j}\int_{B_r} F(\lambda)e^{\lambda t}d\lambda \quad (2)$$

where

$$\int_{B_r}(\cdots)d\lambda \equiv \lim_{\zeta\to\infty}\int_{\gamma-i\zeta}^{\gamma+i\zeta}(\cdots)d\lambda$$

γ, ζ being real; and $\gamma >$ (real part of any singular point of the integrand).[3]

Having found $I(t)$ as in (2), let us trace the circuit throughout its first cycle of input. At the beginning of this cycle the reactance voltage is $E(O)$, by definition. At any point during this cycle the reactance voltage $E(t)$, can be expressed in terms of $I(t)$ and $E(O)$

$$E(t) = G[t, I(t), E(O)] \quad (3)$$

and $I(t)$ can be eliminated from the expression by using

* Decimal classification: R140. Original manuscript received by the Institute, November 2, 1949; revised manuscript received, April 20, 1950.
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[1] Ernest A. Guillemin, "Communication Networks," John Wiley and Sons, New York, N. Y.; 1931. Also, M. F. Gardner and J. L. Barnes, "Transients in Linear Systems," vol. I, John Wiley and Sons; 1942.

[2] H. S. Carslaw and J. C. Jaeger, "Operational Methods in Applied Mathematics," Oxford University Press, chap. IV; 1941.

[3] The symbol "$\int_{B_r} \ldots d\lambda$" is chosen because of the intimate relation between equation (2) and Bromwich's contours.

(2), giving

$$E(t) = H[t, E(O)]. \quad (4)$$

We have, in (4), an expression of the reactance voltage at any time during the first input cycle. If the period of $V(t)$ is T, then at the end of this first cycle the reactance voltage has reached the value

$$E(T) = H[T, E(O)]. \quad (4a)$$

The equation (4a) is vital to further development. It expresses the reactance voltage at the end of the first input cycle in terms of the initial reactance voltage and time, and (implicitly) the circuit parameters. From the very nature of the circuit under consideration—in particular its linearity—$E(O)$ can enter into (4a) only linearly, and so we can rewrite (4a) as

$$E(T) = \Gamma_1 E(O) + \Gamma_2, \quad (5)$$

where Γ_1, Γ_2 are parameters which characterize the circuit and its input, but do not depend on $E(O)$.

It is quite apparent that though (5) was derived through an argument relating to the *first* cycle of input, the restriction is unessential. In fact the succession of equations

$$\begin{array}{l} (1) \rightarrow (2) \searrow \\ \qquad\qquad (4) \rightarrow (4a) \rightarrow (5) \\ (3) \nearrow \end{array}$$

is valid for any cycle of input, if $E(O)$ and $E(T)$ are reinterpreted as the reactance voltage at the beginning and end, respectively, of that particular cycle. If we introduce the symbol

$E_N \equiv$ reactance voltage at the end of the Nth cycle,

then we can write

$$E_N = \Gamma_1 E_{N-1} + \Gamma_2. \quad (6)$$

Equation (6) is a linear, first-order difference equation with constant coefficients (Γ_1, Γ_2 are calculable from (4)) and applies to the circuit for all cycles of input. Being of the first order, (6) has a general solution containing one arbitrary constant, C:

$$E_N = C\Gamma_1{}^N + \frac{\Gamma_2}{1 - \Gamma_1}. \quad (7)$$

C may be calculated by setting $N=0$, and realizing that the symbol E_0 is to be interpreted as the reactance voltage at the beginning of the first input cycle (which was originally symbolized $E(O)$). Then

$$C = E_0 - \frac{\Gamma_2}{1 - \Gamma_1}$$

and

$$E_N = \left(E_0 - \frac{\Gamma_2}{1 - \Gamma_1}\right)\Gamma_1{}^N + \frac{\Gamma_2}{1 - \Gamma_1}. \quad (8)^4$$

[4] It is interesting to note the implications of (8):
(1) $|\Gamma_1| \leqq 1$ (for practical circuits)
(2) if $\Gamma_1 = 1$, then $\Gamma_2 = 0$, and steady state exists immediately.
(3) if $E_0 = \Gamma_2/1 - \Gamma_1$, no true transients arise.
(4) $\lim\limits_{N\to\infty} E_N = \begin{cases} E_0 & \text{if } \Gamma_1 = 1 \\ \Gamma_2/1-\Gamma_1 & \text{if } \Gamma_1 \neq 1. \end{cases}$

From (8), which determines the reactance voltage at the end of the Nth cycle in terms of the initial value of this voltage and the circuit parameters Γ_1, Γ_2, we may proceed to consider in more detail the transition from transient to steady-state operations.

Such an inquiry presupposes a precise definition of the terms used. We shall adopt the following:

Definition

Transient conditions exist from the moment of application of input until steady state is reached. Steady-state conditions obtain from the Sth cycle on, if, for some arbitrary $0 < \delta \ll 1$, and all positive $\bar{S}$:

$$\left| E_{S+\bar{S}} - E_0 \right| \geqq (1 - \delta)\left| \frac{\Gamma_2}{1 - \Gamma_1} - E_0 \right|. \quad (9)$$

Thus each determination of steady state must be accompanied by a value δ. δ plays the role of setting a maximum error to the evaluation.

The above defines steady state as obtaining after S cycles, and S is determined so that E_N, in changing from E_0 to E_∞ has, at E_S, reached a value from which it will not thereafter change by more than $100\delta/1-\delta$ per cent.

If (9) is solved for S (E_N is a known function of N), we have

$$S \geqq \log \delta / \log \Gamma_1. \quad (10)$$

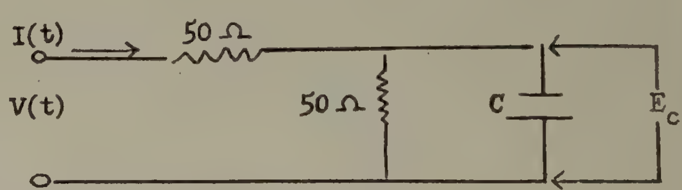
(Note that this relation is independent of E_0 and Γ_2.)

Therefore, the circuit will reach steady state (as determined by δ) in

$\log \delta / \log \Gamma_1$ cycles of input, or

$T \log \delta / \log \Gamma_1$ seconds.

As an example of the use of this method, the circuit below will be analyzed:



$V(t)$ consists of a series of rectangular pulses of amplitude 100 volts, duration 1 millisecond, and repetition rate 400 per second.

C has the value $\begin{cases} 200\ \mu\text{f during an input pulse} \\ \ 50\ \mu\text{f during the inter-pulse periods.} \end{cases}$

C is a potential E_0 at the beginning of the first pulse ($t=0$).

Then the general lines of the argument established previously may be followed to deduce:

1. During the first pulse ($0 \leqq t \leqq 10^{-3}$)

$$E_c(p) = \left(\frac{50}{p} + \frac{E_0 - 50}{p + 200}\right)$$

or $E_c(t) = 50 + (E_0 - 50)e^{-200t}$ and $E_c(10^{-3}) = 9.070 + 0.8186\ E_0$.

2. During the first discharge period ($10^{-3} \leq t \leq 2.5 \times 10^{-3}$)

$$E_c(t) = (9.070 + 0.8186\, E_0)e^{-400(t-10^{-3})}$$

and $E_c(2.5 \times 10^{-3}) = 4.997 + .4492\ E_0$. The difference equation is therefore,

$$E_{c_N} = 4.977 + 0.4492 E_0$$

and has the solution

$$E_{c_N} = (E_0 - 9.036)(0.4492)^N + 9.036.$$

Steady-state condition—for any value of δ—will occur within

$$S = \frac{\log \delta}{\log \Gamma_1} = 2.877(\log \delta) \text{ cycles}$$

or, equivalently, within

$$T_s = 2.5 \times 10^{-3}\, S = -\ 7.192(\log \delta) \text{ milliseconds.}$$

For example, if $\delta = 2$ per cent, $S = 5$ cycles, and the circuit reaches steady state, on this basis, in 12.5 milliseconds.

It is quite simple at this point to proceed a bit further and find the precise wave form. For if we designate the reactance voltage at the end of the Nth input pulse by E_{c_N}' (occurring in time between $E_{c_{N-1}}$ and E_{c_N}), then

$$E_{c_N}' = E_{c_N} e^{0.6} = 1.822 E_{c_N}.$$

With the additional information the accompanying graph, Fig. 1 (for which E_0 is taken equal to 100 volts), may be plotted in its entirety. Points E_{c_N} and E_{c_N}' for $0 \leq N \leq 12$ are located, and then connected with exponential arcs of the appropriate time constant.

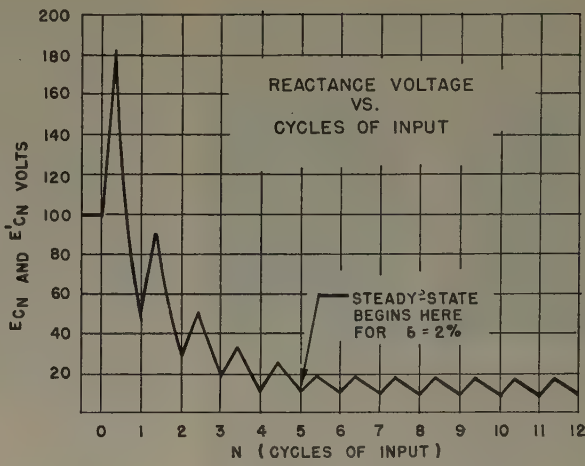


Fig. 1

Conclusion

A method has been presented herein which is quite suitable for the analysis of linear variable circuits containing but one reactance. It is most readily applicable where the variation in circuit components takes the form of abrupt level changes, but can be adapted, though less handily, to circuits containing continuously varying parameters.

Circuits fitting the restrictions listed in the first part of this paper are often encountered in many applications of counting and switching considerations, as well as in simple networks depending on an arc or glow-discharge operation. The presence of a linear voltage or current source does not prevent the use of the method.

A procedure analogous to the one set forth above may be developed for circuits of the same type, but containing several distinct reactances. However, a system of several difference equations in several variables results, and may be extremely difficult of solution.

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In 1922 he joined the engineering staff of the Pacific Telephone and Telegraph Company, and transferred in 1926 to the department of development and research of the American Telephone and Telegraph Company. This department was consolidated with Bell Telephone Laboratories in 1934, where Mr. Aikens is still employed. His work has been largely concerned with noise problems in both wire and radio communication.

❖

D. R. Andrews (M'47) was born on July 28, 1907, in Indiana. He was graduated from the Ball State Teachers College, in Muncie, Ind., and was then engaged in radio service work. He studied radio engineering at Purdue University Extension. During World War II, he was employed by the Allison Division of the General Motors Company, in charge of maintenance of electrical instruments. For the past six years Mr. Andrews has been an engineer in the advanced development group of the RCA Victor Company, in Camden, N. J. His work there is almost entirely devoted to recording. Mr. Andrews is a member of subcommittee 19.1 of the IRE, which is now writing standards for magnetic recording.

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Wallace C. Babcock (A'45–M'45) was born at Northport, L. I., N. Y., on January 16, 1897. After graduating from Harvard with the degree of B.S. in communication engineering in 1922, he joined the development and research department of the American Telephone and Telegraph Company, which was consolidated with Bell Telephone Laboratories in 1934. His work has been largely concerned with inductive interference problems between open-wire telephone circuits. During the war he was engaged on radio countermeasure problems for the National Defense Research Committee. Since that time he has been working on the development of mobile radio systems.

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During the period from 1941 to 1945, Dr. Bothwell was a staff member at the Radiation Laboratory at MIT, and from 1945 to 1947 he was a research engineer on the Project Meteor at the same institution. Since 1947 he has been a senior mathematician at the University of Chicago and a lecturer in mathematics at Northwestern University.

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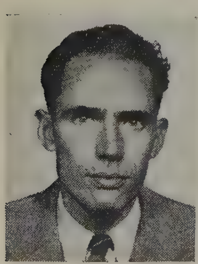
H. N. Christopher, a member of the technical staff of Bell Telephone Laboratories, is at present engaged in studies concerned with the subjective evaluation of television picture impairments. A member of the Installation Department of the New York Telephone Company in 1923, he transferred to the development and research department of the American Telephone and Telegraph Company as a technical assistant in 1925, and subsequently joined the Laboratories with that department in 1934.

H. N. Christopher

Prior to World War II, Mr. Christopher was engaged in development and research on problems relating to the inductive co-ordination of power and telephone systems. His principal contributions in that field included the development of filters for reducing wave-shape distortion caused by the operation of power rectifiers and special testing equipment including harmonic analyzers, noise meters, and carrier frequency analyzers. During the war, he took an active part in special projects for the Armed Services and for NDRC.

❖

Paul V. Dimock (SM'48) was born at East Smithfield, Pa., on July 14, 1915. He received the B.S. degree in electrical engineering from the Pennsylvania State College in 1937. After graduation, he became a member of the technical staff of the Bell Telephone Laboratories, where he engaged in fundamental studies of transmission rating problems, including methods for calculating the loudness and intelligibility of speech as affected by telephone circuits. During the period from 1942 to 1945 he was on leave of absence at Columbia University, where he worked on various projects with the Underwater Sound Laboratory and Airborne Instruments Laboratory.



Paul V. Dimock

Since 1945 he has been associated with the radio transmission engineering department of Bell Telephone Laboratories, and has been concerned with transmission problems related to mobile radiotelephony and microwave relay systems.

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Liscum Diven

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Louis Feit (A'50) was born in Union City, N. J., on May 11, 1921. He received the B.A. degree from New York University in 1943, majoring in physics and mathematics, and the M.S. degree in 1948. He served in the United States Army from June to October, 1943, following which he taught physics at the College of the City of New York.

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Mr. Feit joined the engineering staff of Federal Telecommunication Laboratories in 1944. He has done development work on pulse-multiplex communication systems and is now concerned with selenium-rectifier engineering.

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❖

For a biography and photograph of Ralph I. Cole, see page 822, of the July, 1950, issue of the Proceedings of the I.R.E.

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A. D. Fowler was born on June 10, 1901, in Hopkinsville, Ky. He attended Cornell University for one year and transferred to the Massachusetts Institute of Technology. Interrupting his studies for a period of four years to engage in hydro-electric engineering at Muscle Shoals, Ala., he returned to MIT, where he received the degree of S.B. in 1928. Immediately after graduation he joined the department of development and research of the American Telephone and Telegraph Company, and with that department, transferred to the Bell Telephone Laboratories, Inc., in 1934. He is presently engaged in studies concerned with the subjective evaluation of television picture impairments.

A. D. Fowler

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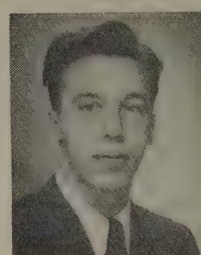

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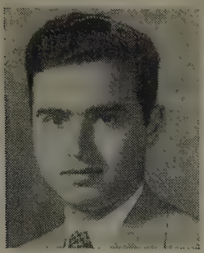
Walter C. Hunter (M'47) was born in Helena, Okla., on July 1, 1913. He received the A.B. degree in physics and the M.Ed. degree in administration from Phillips University, Enid., Okla., in 1936 and 1937, respectively.

Walter C. Hunter

He was an instructor in the Enid secondary schools from 1936 to 1942, and an instructor in the electrical engineering department of Oklahoma Agricultural and Mechanical College, Stillwater, Okla., from 1942 to 1944.

In 1944, Mr. Hunter joined the Western Electric Company as a radar field engineer. In 1945, he transferred to the Bell Telephone Laboratories as a member of the technical staff, and has since been engaged in the development of railroad and mobile radiotelephone equipment.

Lawrence Lutzker (S'47–A'48) was born on March 13, 1927, in New York, N. Y. He received part of his engineering training at the Cooper Union School of Engineering and the remainder at the University of Michigan, graduating from the latter in 1947 with B.Sc. degree in electrical engineering. During the war, he served in the Navy as an electronic technician. After leaving Michigan, he was employed as a quality control engineer by the Allen B. DuMont Laboratories, Inc., where he introduced statistical methods to deal with problems of inspection and manufacturing. Since then he has taken an active part in the promotion of statistical quality control within the company.



L. Lutzker

Mr. Lutzker is a Senior Member of the American Society for Quality Control and a member of Tau Beta Pi and Eta Kappa Nu.

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Pierre Mertz (SM'44–F'50) was born in France on April 2, 1897. He received the A.B. degree in 1918, and the Ph.D. degree in 1926, both from Cornell University. In 1919 he entered the department of development and research of the American Telephone and Telegraph Company, where he served until 1935, with some interruption, when this department was merged with the Bell Telephone Laboratories, Inc. Mr. Mertz is now in the transmission engineering department at Bell, where his principal work has dealt with problems in television and telephotograph transmission.

Pierre Mertz

During World War II he served as section chief and division member with the National Defense Research Council, and also as investigator for TIIC. In 1948 he received the Army-Navy Certificate of Appreciation.

❖

Sidney Moskowitz (M'45) was born in Brooklyn, N. Y., on February 23, 1919. In 1940, he received the B.E.E. degree from the College of the City of New York. From 1941 to 1945 he was a member of the evening-session staff at that college. While teaching evenings, he was also engaged in the development of electronic apparatus for the Industrial Scientific Corporation in New York. In 1943, Mr. Moskowitz joined Federal Telecommunication Laboratories, where he has been active in the design and development of pulse-time-modulation systems. He is serving also as an adjunct assistant professor in the graduate school of engineering of New York University.

S. Moskowitz

Mr. Moskowitz is a member of Tau Beta Pi.

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R. C. Moyer was born in Boston, Mass., on June 24, 1909. He received the B.A. degree from Bowdoin College in 1932, majoring in mathematics. After a brief association with Radio Television Industries Inc., he joined the RCA Victor Distributors in Cambridge, Mass., where he worked in both engineering and sales in the commercial sound department. From 1942 to 1945 he was on the staff of the Radio Research Laboratory at Harvard University as a special research associate.

R. C. Moyer

Since 1945 Mr. Moyer has been associated with RCA Victor as a supervisor in the engineering section of the record department.

❖

Harry W. Nylund was born near Altoona, Iowa, on September 20, 1903. He received the B.S. degree in electrical engineering from Iowa State College in 1925, and joined the Bell Telephone Laboratories staff immediately thereafter. He spent some years in field engineering work, and during the war was engaged in various underwater sound projects and in antenna studies. His work recently has been concerned with development of antennas and related devices for the mobile radio services.

Harry W. Nylund

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Bernard M. Oliver (S'40–A'40–M'46) was born on May 27, 1916, at Santa Cruz, Calif. He received the B.A. degree in electrical engineering from Stanford University in 1935, and the M.S. degree from the California Institute of Technology in 1936. Following a year of study in Germany from 1936 to 1937 under an exchange scholarship, he returned to the California Institute of Technology and in 1940 received the Ph.D. degree. Since that time he has been employed as a research engineer with the Bell Telephone Laboratories. During the war he was active in the development of Army and Navy automatic tracking radar equipment.

B. M. Oliver

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For a photograph and biography of L. Y. Lacy, see page 312 of the March, 1950, issue of the Proceedings of the I.R.E.

Albert S. Richardson, Jr., was born on August 7, 1924 in Canton, Ohio. He received the B.S. degree in aeronautical engineering in 1947 from the Massachusetts Institute of Technology.

As an analytical engineer during 1948 with United Aircraft in East Hartford, Conn., he conducted theoretical flutter and vibration studies of a new type of supersonic propellor and investigated effects of power plant gyroscopic coupling on turbo jet-powered aircraft. From 1948 to 1949 he was a development engineer with the Goodyear Aircraft Corporation, where he participated in the development of automatic control systems for aircraft. Since December, 1949, Mr. Richardson has been engaged, as a research engineer, in conducting theoretical studies of dynamic response of elastic aircraft at the Aero-Elastic and Structures Research Laboratory at MIT.

A. Richardson, Jr.

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J. Gregg Stephenson (A'45–M'47) was born in Kansas City, Mo., on September 21, 1917. He was awarded the B.E. degree from Yale University in 1939, and the Engineer Degree from Stanford University in 1941. He was employed as assistant laboratory engineer at the Ohio Brass Company, in Barberton, Ohio, during 1941 and 1942, engaged in high-voltage testing and measurements. For the next two years he worked as a research associate in the Radio Research Laboratory of Harvard University, engaged in the development of ultra-high-frequency transmitters. This was followed by a year at the American British Laboratory in 1945, where he was concerned with radar countermeasures for the United States Navy and the 8th Air Force and as a Technical Observer in the Mediterranean Area. Mr. Stephenson was assigned as RCM consultant to the Operational Analysis Section of the 8th Air Force Headquarters for four months in 1945.

J. G. Stephenson

Since December, 1945, Mr. Stephenson has been in the receiver group at the Airborne Instruments Laboratory, in Mineola, L. I., New York, engaged in uhf receiver development. He is a member of Sigma Xi, Tau Beta Pi, the American Institute of Electrical Engineers, and the Yale Engineering Association.

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For a photograph and biography of H. E. Roys, see page 313 of the March, 1950, issue of the Proceedings of the I.R.E.

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For a photograph and biography of Lotfi A. Zadeh, see page 314 of the March, 1950, issue of the Proceedings of the I.R.E.

Robert C. Shaw was born on September 8, 1902, at Templeton, Mass. He received the B.S. degree in electrical engineering from Michigan University in 1926.

Robert C Shaw

From 1927 to 1945 he was a member of the technical staff of the Bell Telephone Laboratories, where he was engaged in radio research. During 1944 and 1945 he was on leave of absence to the National Defense Research Committe in the Office of Scientific Research and Development and was Chairman of the Antenna Co-ordinating Committee. From 1945 to 1946 he was a partner in the firm of McKey and Shaw, consulting radio engineers.

He returned to the Bell Laboratories in 1947 where he has been associated with the radio transmission engineering department, and concerned with problems related to mobile radiotelephony.

❖

Walter Strack, Jr., was born in Astoria, N. Y., on January 4, 1925. He received the B.S. degree in electrical engineering in 1944 from the Polytechnic Institute of Brooklyn.

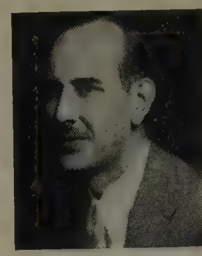
W. Strack, Jr.

During World War II, Mr. Strack served with the Navy at the Naval Ordnance Laboratory in Washington, D. C., where he worked on the development of influence detectors for underwater explosives.

In November, 1945, he joined the Bell Telephone Laboratories where he has been concerned with the development and testing of mobile radio equipment. He is a member of Eta Kappa Nu.

❖

W. T. Wintringham (A'26–SM'45) was born in Brooklyn, N. Y., on January 18, 1904. He received the B.S. degree in electrical communication engineering from Harvard University in 1924, and immediately joined the American Telephone and Telegraph Company in the department of development and research. In 1935 he transferred to the Bell Telephone Laboratories.



W.T.Wintringham

Before World War II, Mr. Wintringham was associated closely with the development of radio telephone systems for spanning natural barriers; the long-wave transatlantic telephone; the short-wave MUSA receiving system; and the multiplex telephone system used between the Virginia Capes and operated in the vhf region. During the war he worked on a number of classified projects. Recently his work has been in the television field, with particular emphasis on color.

Correspondence

Fourier Transforms in the Theory of Inhomogeneous Transmission Lines*

It is well known that the Fourier Integral Theorem may be used in different technical fields. Thus, for instance, it can be used in the theory of communication[1] and in the antenna theory.[2] Now it also appears to be applicable in the theory of inhomogeneous transmission lines. Starting with the differential equations of an inhomogeneous line,

$$\begin{cases} \dfrac{d^2V}{dx^2} - \dfrac{d \ln Z_0}{dx} \cdot \dfrac{dV}{dx} - \gamma^2 V = 0 & (1) \\ \dfrac{d^2I}{dx^2} + \dfrac{d \ln Z_0}{dx} \cdot \dfrac{dI}{dx} - \gamma^2 I = 0 & (2) \end{cases}$$

where

V = voltage
I = current, and
Z_0 = characteristic impedance

are all functions of the length x and γ = the propagation constant, in this note assumed to be constant $= j\beta$, it is possible, after some substitutions, to deduce a Riceati differential equation for the reflection coefficient ρ at the sending end, when the receiving end is matched:

$$\frac{j}{\beta} \cdot \left[2 \cdot \frac{d\rho}{dx} + \frac{d \ln Z_0}{dx} (1 - \rho^2) \right] + 4\rho = 0. \quad (3)$$

Assuming ρ^2 to be small compared to 1, one obtains the approximate solution

$$|\rho| = \left| \frac{1}{2} \int_{-\infty}^{+\infty} \frac{d \ln Z_0}{dx} \cdot e^{-j2\beta x} dx \right|. \quad (4)$$

This is a Fourier integral. The Fourier transforms will be ρ and $d \ln Z_0/dx$ multiplied by some constant factors, one of which includes Z_2/Z_1, where Z_1 = impedance at the sending end and Z_2 = impedance at the receiving end of the inhomogeneous line. With the integral above, it is possible to examine and deduce the approximate solutions already made by other authors. With the Fourier Integral Theorem it is possible not only to determine $|\rho|$ as a function of (l/λ) (l = the length of the line, λ = wavelength) for an arbitrary $Z_0(x)$, but also, on the contrary, to determine $Z_0(x)$ from the $|\rho|$-diagram. As a special example, there is shown in Fig. 1 the behavior of an inhomogeneous line, whose $d \ln Z_0/dx$-function is triangular. In Fig. 1 (a) the impedances Z_1 and Z_2 are chosen to be 50 and 100 ohms, respectively.

The analogy of treatment in other fields is obvious: Thus, for instance, the problem of constructing an optimized transmission line transformer will be analogous to the problem of finding the antenna distribution that will give the best diffraction pattern with respect to sidelobes and width of the main lobe.

The way of treatment described above may be applied to waves in general, and can be extended to other fields; for instance, to the acoustical.

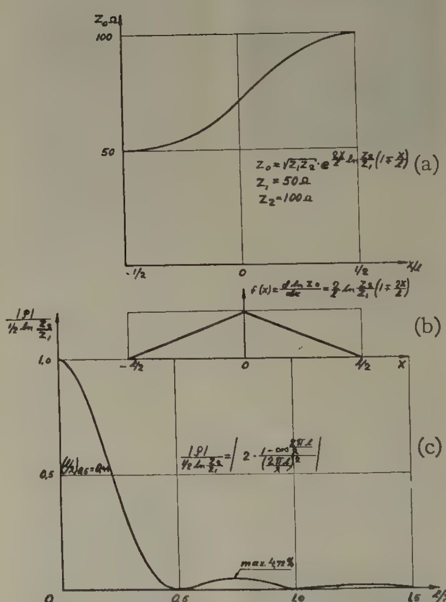


Fig. 1—Diagrams for an inhomogeneous transmission line, whose $d \ln z_0/dx$ function is triangular.

Further details of this work will be published later.

Folke Bolinder
The Research Institute of National Defense
117 Valhallavägen
Stockholm
Sweden

* Received by the Institute, June 19, 1950.

[1] S. Goldman, "Frequency Analysis, Modulation and Noise," McGraw-Hill Book Co., New York, N. Y.; 1948.

[2] J. F. Ramsay, "Fourier transforms in aerial theory," *Marconi Rev.*, 6 parts in vol. IX: 4, X: 1–4 and XI: 2; 1946–1948.

Institute News and Radio Notes

Technical Committee and Professional Group Notes

The **Executive Committee** appointed the following persons to serve as **IRE representatives on various ASA Sectional Committees**: A. F. Pomeroy, ASA Drawings and Correlating Committee; J. R. Steen, C67, Preferred Voltages-100 Volts and Under; and A. G. Clavier, Z10, Letter Symbols and Abbreviations. Mr. Clavier has been appointed as alternate to Austin Bailey, IRE Representative on Z14, Standards for Drawing and Drafting Room. Mr. Clavier has also been selected as the IRE Representative on Z15, Standards for Graphical Presentation. A. F. Pomeroy will be the alternate on this Committee. . . . The **Second High-Frequency Measurements Conference sponsored jointly by IRE/AIEE and the National Bureau of Standards** will be held at the Hotel Statler in Washington, D. C., on **January 10 to 12, 1951.** This will be the first scientific gathering of national scope to be brought to Washington in 1951 in celebration of the semicentennial of the National Bureau of Standards. This Conference is under the general direction of Professor Ernst Weber, Chairman of the IRE Measurements and Instrumentation Committee. Technical Sessions will be held in the spacious auditorium of the Department of the Interior. The program will include about twenty-five outstanding technical papers, an evening demonstration of a spectacular nature, a luncheon, and conducted inspection tours of selected local and nearby institutions. Chartered buses will be provided for transportation to the auditorium, and for the inspection tour. . . . **A two-day conference on Electron Tubes for Computers,** under the sponsorship of The Institute of Radio Engineers and the American Institute of Electrical Engineers, in collaboration with the Panel on Electron Tubes, is being held on December 11–12, 1950, at the Haddon Hall Hotel in Atlantic City, N. J. The emphasis of the Conference will be on reliability of tubes for use in digital computers, with some attention to be devoted to special tubes developed specifically for computers. Further information may be obtained by writing to the Conference Secretary, A. Lederman, Panel on Electron Tubes, Room 601, 139 Centre Street, New York 13, N. Y. . . . The **Audio Techniques Committee** held a meeting on August 16, under the Chairmanship of R. A. Miller. . . . The **Electronic Computers Committee** held a meeting on September 8 in Washington, D. C., under the Chairmanship of J. W. Forrester. . . . A meeting of the **Circuits Committee** was held on September 15, with W. N. Tuttle, Chairman, presiding. . . . A National Meeting of the **Professional Group on Vehicular and Railroad Radio Communications** will be held on November 3, 1950, in Detroit, Mich.

Broadcast Transmission Group

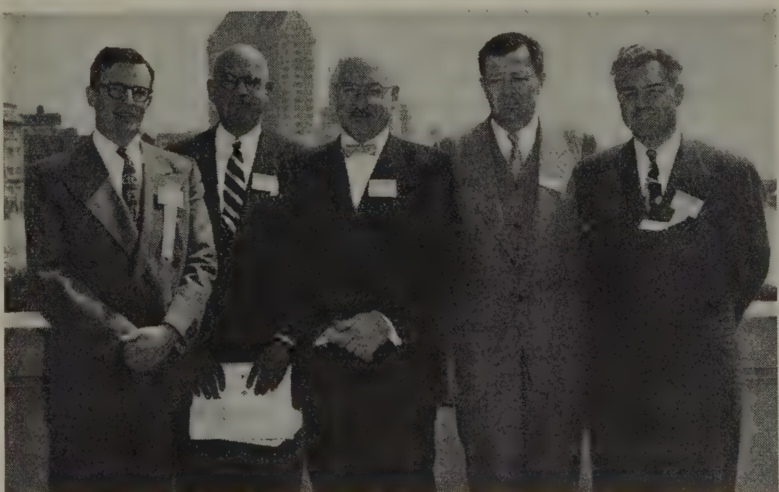
The IRE Professional Group on Broadcast Transmission Systems is tentatively planning to hold a "broadcast day" on Tuesday, March 20, during the 1951 IRE National Convention. Eight papers will be presented during the day.

Plans for "broadcast day" were discussed at the Group Administrative Committee meeting on September 29. Plans were also discussed for sending letters to chief engineers of broadcast stations and to manufacturers of broadcast equipment in order to increase the Group membership and to organize groups locally, as has been done in Boston and Cleveland.

IRE West Coast Convention Held In Fall Is Successful

The annual IRE West Coast Convention held September 13–15 at the Municipal Auditorium, Long Beach, Calif., was the largest convention of its kind in the West Coast history of The Institute of Radio Engineers. Approximately 1,400 registrations were recorded for the technical sessions, and 159 booth spaces were sold to 138 manufacturers. The program was sponsored jointly by the Los Angeles Section of the IRE and the West Coast Electronics Manufacturers Association.

The 1951 West Coast Convention has been scheduled for August 29–31 at San Francisco.



Shown at the IRE West Coast Convention from left to right are: R. L. Sink, Chairman of the Los Angeles Section; G. W. Bailey, IRE Executive Secretary; R. F. Guy, IRE President; F. E. Terman, Past Director of Region 7; and S. F. Johnson, Convention Chairman.

Vacancies at Indiana Navy Station

Vacancies have been announced by the Navy Department at the U. S. Naval Ordnance Plant, Indianapolis 6, Ind., for mechanical, electronic, electrical and ordnance engineers. These positions are at various grade levels under Federal Civil Service at annual salaries ranging from $3,825 to $7,400. The engineering development, research, and testing program at this station is rapidly expanding and qualified personnel in these classifications is urgently needed.

Details concerning employment at this station may be obtained from Commander M. K. Coleman, USNR, Industrial Relations Officer.

Oak Ridge Institute to Give Courses on Radioisotopes

The eighteenth, nineteenth, and twentieth courses in the techniques of using radioisotopes in research will be given by the Special Training Division of the Oak Ridge Institute of Nuclear Studies, Oak Ridge, Tenn., during the winter and spring of 1951. Dates for the courses are as follows: January 8–February 2, February 19–March 16, and April 16–May 11.

The courses are designed to acquaint research workers with the safe and efficient use of radioisotopes in research. The course work consists of laboratory work, lectures on laboratory experiments, general background lectures, and special-topic seminars. Experiments are conducted covering the use and calibration of instruments, the purification and separation of radioactive materials from inert and other radioactive materials, measurement and use of Carbon-14, pile activations, radioautographs, and the like. Seminars include such topics as the use of radioisotopes in animal experimentation, use of radioisotopes in humans, principles and practice of health physics, design of radiochemical laboratories, effect of radiation on cells, and similar topics.

The Special Training Division can accommodate thirty-two participants at each of the three courses. A registration fee of $25 is charged, and participants will bear their own living and traveling expenses.

Additional information and application blanks may be obtained from Dr. Ralph T. Overman, Chairman, Special Training Division, Oak Ridge Institute of Nuclear Studies, P.O. Box 117, Oak Ridge, Tenn.

Calendar of COMING EVENTS

National Conference of the IRE Professional Group On Vehicular Communications, Detroit, Mich., November 3

UHF-Microwave Conference, IRE Kansas City Section, Kansas City, Mo., November 3–4

IRE-AIEE Conference on Electron Tubes for Computers, Atlantic City, N. J., December 11–12.

AAAS Annual Meeting, Cleveland, Ohio, December 26–30

AIEE-IRE-NBS High-Frequency Measurements Conference, Hotel Statler, Washington, D. C., January 10–12, 1951

1951 IRE National Convention, Waldorf-Astoria Hotel and Grand Central Palace, New York, N. Y., March 19–22, 1951

IRE Southwestern Conference, Dallas, Texas, April 20–21, 1951

1951 Annual Meeting of the Engineering Institute of Canada, Mount Royal Hotel, Montreal, May 9–11, 1951

National Bureau of Standards Offers Calibration Services

The National Bureau of Standards is now offering a calibration service for field-intensity meters at all radio frequencies of broadcast and commercial importance up to 300 megacycles. Of special interest are the new standards and methods which have been developed at the Bureau for calibrating field-intensity meters in the very-high frequency region from 30 to 300 megacycles. The new standards were developed to meet a need for an improvement in the available accuracy of field-intensity measurements required because of the greatly increased use of vhf bands by FM and TV stations. Prior calibration service for field-intensity meters had already accommodated meters operating in the range from 10 kilocycles to 30 megacycles.

The extended field-intensity-meter calibration service necessitated the development of new and accurate field-intensity standards. The vhf standards are similar to those already employed at lower frequencies, but several special techniques, particularly in the measurement of antenna current and voltage, have developed to meet the peculiarities of vhf calibration work.

Two distinct experimental methods are used in the Bureau's field-intensity standardization work: the standard-antenna method and the standard-field method. Both methods have been employed in establishing the standards used in calibrating commercial field-intensity meters. It has been found most practical to use the standard-antenna method for frequencies greater than 30 megacycles, and the standard-field method for lower frequencies.

High-Frequency Measurements Conference Slated in January

The second High-Frequency Measurements Conference sponsored jointly by the American Institute of Electrical Engineers, The Institute of Radio Engineers, and the National Bureau of Standards will be held in Washington, D. C., on January 10 to 12, 1951. Conference Headquarters will be at the Hotel Statler with the technical session held in the spacious auditorium of the Department of the Interior. The Conference program will include about 25 important technical papers, an outstanding evening demonstration, a luncheon, and conducted inspection tours of selected local and nearby scientific institutions. Chartered buses will provide transportation to the auditorium and for the inspection tour.

The conference will be a forum at which the nation's leading engineers from educational, governmental, and industrial laboratories will exchange information on progress since the successful 1949 conference. While most of the papers will deal with measurements in the hf through ehf frequency regions, some interesting video measuring techniques will be disclosed. Among the technical sessions being organized are those on Frequency and Time, Transmissions and Reception, Impedance, and Power and Attenuation. Preprints and Proceedings are being dispensed with to encourage active workers to report their latest findings. Brief abstracts will probably appear in *Electrical Engineering* and Proceedings of the I.R.E. following the meeting.

The Conference is under the general direction of the Joint AIEE-IRE Committee on High-Frequency Measurements, of which Professor Ernst Weber of the Microwave Research Institute of the Polytechnic Institute of Brooklyn is Chairman. Harold Lyons of the National Bureau of Standards is Chairman of the Local Arrangements Committee, and Frank Gaffney of the Polytechnic Research and Development Company is Chairman of the Technical Program Committee. Finances are being handled by Ivan Easton of General Radio Company, and publicity by E. P. Felch of the Bell Telephone Laboratories.

It will be the first scientific gathering of national scope to be brought to Washington in 1951 in celebration of the Semi-Centennial of the National Bureau of Standards.

William A. Wildhack Will Head Basic Instrumentation Office

William A. Wildhack, formerly chief of the Missile Instrumentation Section of the National Bureau of Standards, will head the Office of Basic Instrumentation just established at the Bureau. The new office will co-ordinate a program of evaluation and improvement of instruments for measuring basic physical quantities which has been initiated in co-operation with the Department of Defense. Such instruments are vital to all advances in science and technology.

Mr. Wildhack has been with the National Bureau of Standards since 1935, working in the fields of aeronautical, mechanical, and electronic instruments. He received the B.S. degree in electrical engineering and the M.S. in physics at the University of Colorado.

LAST CALL!

AUTHORS FOR IRE NATIONAL CONVENTION!

E. Weber, Chairman of the Technical Program Committee for the 1951 IRE National Convention, requests that prospective authors submit the following information:

1. Name and address of author
2. Title of paper
3. A 100-word abstract and additional information up to 500 words (both in triplicate) to permit an accurate evaluation of the paper for inclusion in the Technical Program.

Please address all material to E. Weber, Microwave Research Institute, Polytechnic Institute of Brooklyn, 55 Johnson Street, Brooklyn 1, N. Y. The deadline for acceptance is November 20, 1950. Your prompt submissions will be appreciated.

AAAS Annual Meeting in Cleveland, December 26–30

The 117th annual meeting of the American Association for the Advancement of Science, scheduled for December 26–30 at Cleveland, Ohio, will feature programs in every field of science from astronomy and botany to, and including, zoology. All 17 of the Association's sections and subsections, and more than 40 participating societies and organizations, are completing plans for an aggregate of more than 200 sessions. An extensive series of tours to museums, laboratories, and industrial plants of the Cleveland area has been planned.

The Annual Science Exposition in the arena of Cleveland's Public Auditorium will be almost twice as large as the New York meeting, and will display about 150 booths. In the Exposition, publishers, supply houses, microscope manufacturers, instrument makers, and industrial concerns will exhibit their latest products and portray their technical accomplishments.

The program for the Section on Engineering will include: (a) joint program with the Cleveland Engineering Society of Mechanical Engineers, December 26; (b) joint program with the Cleveland Engineering Society, December 27; (c) joint program with Case Institute of Technology, December 28; (d) three-day session symposium on "Partnership of Industry and Science in Research," jointly sponsored by the Illinois Institute of Technology, Armour Research Foundation, Mellon Institute of Industrial Research, and the Battelle Memorial Institute, December 29.

IRE People

Lloyd T. De Vore (A'42–SM'44) has been appointed manager of Electronic Laboratory at the General Electric Electronics Park, according to an announcement by **I. J. Kaar** (J'22–A'24–M'29–F'41), manager of engineering for the Electronics Department which is responsible for much of the company's advanced development in the field of electronics. He assumed his new duties on July 1.

L. T. De Vore

Mr. De Vore has been a member of the staff of the electrical engineering department at the University of Illinois, Urbana, Ill., since 1946. He has also been Chairman of the Research Committee and Coordinator of Research for the University's Electrical Engineering Department.

A native of Monongahela, Pa., he received the B.S., M.S., and Ph.D. degrees in physics from Pennsylvania State College in 1930, 1931, and 1933, respectively. He held a research fellowship under a grant from the National Research Council from 1930 to 1934, following which he was appointed to the staff of the physics department at Pennsylvania State College.

In 1943 he went to Wright Field, Dayton, Ohio, as a radio engineer with the Aircraft Radio Laboratories of the Army Air Force. He served there also as chief engineer of the Research Division, and as chief engineer of the Special Projects Laboratory. He resigned his War Department appointment in 1946 to go to the University of Illinois.

Dr. De Vore is a member of the American Physical Society, Sigma Xi, Phi Kappa Phi, Sigma Pi Sigma, Pi Mu Epsilon, and Tau Kappa Epsilon. He holds the War Department Exceptional Civilian Service Medal.

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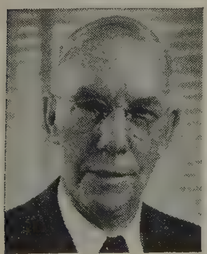
H. J. MacLeod (M'41–SM'43), professor and head of the department of mechanical and electrical engineering at the University of British Columbia, will be the new Dean of Applied Science.



H. J. MacLeod

Dr. MacLeod has combined a distinguished academic career with an extraordinary interest in undergraduate problems and activities that has won him many student friends. Twice he has been honorary president of graduating classes and last year he was awarded a gold medal by the Students' Literary and Scientific Executive for outstanding work with student organizations.

Born on Prince Edward Island, Dr. MacLeod started his academic career by winning the British Association Medal upon graduation from McGill in 1914. He was graduated from Harvard in 1921 with the Ph.D. degree.

Prior to his association at UBS, where he took over as head of electrical and mechanical engineering in 1936, he taught at the University of Alberta.

Dr. MacLeod has served with distinction in two wars; first as Captain and Major in the Canadian Expeditionary Force in France and Belgium during World War I. Following this he commanded the 196th Western Universities Battalion. Here for the first time he met student representatives of UBC in the contingent sent by this University. In the recent war he received the Order of the British Empire for his work on ship protection against magnetic mines for the National Research Council.

He has acted as consultant on many occasions, and from 1939 to 1944 was technical advisor to the Public Utilities Commission of British Columbia. For his distinguished work in the field of electrical communications he was awarded a Fellowship in the AIEE in 1946.

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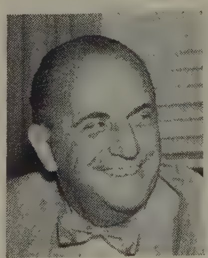
Royal V. Howard (M'41–SM'43) has recently become owner and president of radio station KIKI, Honolulu. The station, which went on the air September 1, operates on 860 kc and employs a General Electric 250-watt transmitter and 285-foot tower to cover the Island of Oahua and adjacent islands. Programming will consist chiefly of news, sports, and music.



R. V. Howard

Mr. Howard is well known in the broadcasting field. He was director of the Department of Engineering of the National Association of Broadcasters, Washington, D. C., from 1947 until his resignation a few months ago. From 1933 to 1947 he was vice-president in charge of engineering for Associated Broadcasters, Inc., (KSFO and KPIX), San Francisco. During the war, he served as director of special combat scientist group for the Office of Scientific Research and Development, U. S. Army Headquarters, European Theatre of Operations.

Active in all phases of broadcasting since 1922, Mr. Howard is also a member of the Institute of Electrical Engineers, and the American Academy for the Advancement of Science, among others. He has been an expert member of the United States delegations to world conferences on telecommunications held at Atlantic City, Havana, Mexico City, Washington, and Montreal.

Edwin L. White (A'24–M'31–SM'43) has been appointed chief of the new Safety and Special Radio Services Bureau of the Federal Communications Commission. He is also chief of the Aviation Division, Bureau of Engineering. He has been associated with the Commission since the latter's creation, and, before that, with its predecessor, the Federal Radio Commission.

Mr. White, who was born at Valley City, N. D., on July 5, 1896, began his radio career as an amateur in 1912 and, professionally, with the Naval Research Laboratory in 1922, where he served as a research engineer and was active in designing equipment, both transmitting and receiving, for various naval operations. In 1926 he joined the staff of the Signal Corps, and while a civilian engineer in the Office of the Department Signal Officer at Fort Shafter, Hawaii, was in charge of the technical maintenance and operation of the radiocommunication system of the Hawaiian Department of the Army.

In June, 1930, Mr. White joined the Federal Radio Commission and, except for periods of military duty, has been continuously associated with radio regulation. In the days of the FRC he specialized in television. Previous to World War II he was active in police, fire, and experimental radio supervision; since the war he has been concerned mainly with aviation radio.

Mr. White served as a commissioned officer in both World Wars. He left the University of North Dakota in his junior year for Navy duty. Later he went with the Signal Corps Reserve and entered the late war as a lieutenant colonel. Shortly thereafter he transferred to the Air Corps, and the major part of his war service was as Communications Officer in the Air Transport Command. He served in the Pacific Theater for nine months; for 15 months in the China-Burma-India Theater, and three months with the Military Government in Germany.

He holds a reserve commission as a colonel in the Air Force.

❖

Fred E. Osgood (A'45) has been advanced to the position of technical supervisor of WBZ-TV, succeeding **Sidney V. Stadig** (M'50) who has been recalled to active duty with the United States Navy.

Mr. Stadig, who will serve as a communications technician, spent three years in the Navy during World War II in the European Theater, during which service he received a letter of commendation. He joined WBZ in 1940 as a transmitter technician at Hull. He has been in charge of WBZ-TV technical facilities since the inauguration of video in Boston.

Mr. Osgood has been supervisor of WBZ's Hull transmitting facilities since December, 1943. He has been with the station for twenty years.

Paul Rosenberg (A'43) has been elected president of the Institute of Navigation for the academic year 1950–1951 and will succeed Rear Admiral Gordon McLintock, USMS. Dr. Rosenberg has been elected also as the General Chairman of the joint meeting and the symposium, which will be held by the Institute of Navigation, the Radio Technical Commission for Aeronautics, as well as the Radio Technical Commission for Marine in New York, N. Y., on September 19, 20, and 21. The general theme of his three-day meeting will be "The Application of Electronics to the Related Problems of Air, Marine, and Land Navigation."

Paul Rosenberg

He is also president of Paul Rosenberg Associates, consulting physicist, in New York, N. Y., a firm which he has headed since its founding in 1945. He directs the firm's work of consultation, research, and development in the industrial and military applications of physics and related sciences.

During the war Dr. Rosenberg was a staff member of the NDRC Radiation Laboratory at the Massachusetts Institute of Technology, where he did work in radar, synthetic training devices, and ultrasonics. Before the war he was lecturer in physics at Columbia University, doing research in molecular beams. He was also instructor of physics at Hunter College.

He served for three years as technical advisor to the president in the Institute of Navigation. He was co-chairman of the National Committee on Upper Atmosphere and Interplanetary Navigation.

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Glenn H. Browning (A'24-M'28-SM'43), was honored at the annual commencement exercises of Cornell College, Mount Vernon, Iowa, when he received the honorary degree of Doctor of Science.

Glenn H. Browning

Mr. Browning, who has been president of Browning Laboratories, Inc., Winchester, Mass., since 1937 when the company was formed, was born in Mount Vernon in 1897. He was graduated from Cornell College with the B.A. degree in 1921 and earned highest honors in the class. He was elected to membership in Phi Beta Kappa.

He was awarded the Lydia C. Perkins Scholarship to Cruft Laboratory graduate work at Harvard University. From 1923 to 1924 he was a research fellow in electrical engineering at Harvard. In conjunction with F. H. Drake, he developed a special mathematically designed tuned radio-frequency transformer which was incorporated in a circuit later known as the Browning-Drake Circuit.

From 1924 until 1937 he was a radio and research consulting engineer, as well as president of Browning-Drake Corp.

He is listed in "Who's Who in Engineering," "Who's Who In New England," and "Biographical Encyclopedia of the World."

❖

Lynn C. Holmes (M'44–SM'48–F'49), has been appointed associate director of research of the Stromberg-Carlson research laboratory, and will share with Director Benjamin Olney the guidance of the firm's research activities. He has been senior electrical engineer ever since he joined the company in 1943.

Lynn C. Holmes

Mr. Holmes, who was graduated from Rensselaer Polytechnic Institute in 1925, taught there for the following 18 years. He earned the degrees of E.E. and M.E.E.

Awarded the honor of Fellow grade in the IRE last year, Mr. Holmes is well known in the field of magnetic sound recording, where he has made important contributions to the theory and practice in the field. He is also affiliated with the AIEE, RMA, the American Standards Association, the Acoustical Society of America, and Sigma Xi. He is the alternate Stromberg-Carlson representative in the Industrial Research Institute.

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John M. Ide (A'47) has been promoted to one of the top 400 positions in the federal civil service. His post as chief scientist and director of research at Navy Underwater Sound laboratory has been allocated to civil service grade GS-16.

Dr. Ide, who has served in his present capacity since March 1, 1945, has devoted almost 20 years to teaching and research in communications engineering at Harvard University, industrial research in geophysics for the Shell Oil Co., and research and development for the Navy in the field of underwater sound at the Naval Research Laboratory, Washington, D. C., and the Sound Laboratory.

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R. C. Cheek (M'47), Central Station Engineer of the Westinghouse Electric Corp., is the 1949 winner of the award given by the Eta Kappa Nu Association, an electrical engineering honor society, to the most outstanding young electrical engineer of the year.

Given annually since 1936, the award has been won by many other IRE members including the following: **W. E. Kock** (SM'45) in 1938; **L. A. Meacham** (A'38–SM'45–F'48) in 1939; **J. E. Hobson** (M'45) in 1940; **Cledo Brunetti** (A'37–SM'46–F'49) in 1940; **J. R. Pierce** (S'35–A'38–SM'46–F'48) in 1942; **N. I. Hall** (SM'47) in 1943; **R. R. Hough** (S'40–A'43–SM'46) in 1947; and **A. M. Zarem** (S'42–A'46) in 1948.

Peter C. Goldmark (A'36–M'38–F'42) has been appointed vice-president in charge of engineering research and development at the Columbia Broadcasting System. Formerly, he was director of engineering and research development where he and his associates developed the CBS color television system and its long-playing records.

Peter C. Goldmark

Dr. Goldmark, who developed the long-microgroove phonograph record through nearly three years of intensive laboratory work, joined CBS in 1936 to take part in research and television activities. Within a short time he was named chief television engineer, and in September, 1940, demonstrated the CBS full-color television system which he developed for ultra-high frequency broadcasting. During the war Dr. Goldmark and his associates were engaged exclusively in electronic research for the Armed Services. Much of this work took him to the European and South Pacific theaters of war.

Born on December 2, 1906, in Budapest, Hungary, Dr. Goldmark is a graduate of Vienna Technical College, Vienna, Austria, from which institution he also holds the Ph.D. degree.

He was awarded the IRE Morris Liebmann Memorial Prize in 1945 for his "contributions to the development of television systems, particularly in the field of color." He has served on several IRE committees.

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Russel H. Varian (A'40) and Sigurd F. Varian have been awarded the John Price Wetherill Medals by the Franklin Institute, Philadelphia, Pa., for their development of the klystron, a new kind of radio tube which played a major role in World War II. The tube has made it possible to transmit telephone conversations over long distances without the use of wires, and has contributed to much peacetime exploratory work in the electromagnetic spectrum.

Dr. Russell Varian, who was born in Washington, D. C., in 1898, was graduated from Stanford University in 1925 and received the M.A. degree from the same institution two years later. He also holds an honorable D.Eng. from Brooklyn Polytechnic Institute. In 1929 he became a research physicist with the Humble Oil Company in Texas, and then went to Farnsworth Television Corporation in 1930.

He is a Stanford Research Associate, and also president of Varian Associates, organized in 1948. He holds membership in several scientific organizations.

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Walter F. Kram (A'49) has joined the engineering staff of the Ballantine Laboratories, Inc., at Boonton, N. J., as a senior engineer. Mr. Kram specialized for over 10 years in instrument development work for the Standard Telephone and Cable Company of London.

Industrial Engineering Notes[1]

RTMA Engineers Attack FM Interference Data

W. R. G. Baker, Director of the RTMA Engineering Department, has supplied each RTMA Board member with the latest information concerning FM radiation interference problems.

Dr. Baker noted that each company had been asked to prepare FM receivers with the latest interference reducing circuits, shields, etc., and to submit it for test. The sets should be sent to Stuart W. Seeley, of the RCA Laboratories, who will conduct the interference test.

It was also pointed out that a meeting with the FCC on both FM and TV interference has been scheduled for September 12 at the Carlton Hotel in Washington, D. C. Dr. Baker told the Directors: "This letter is being directed to you personally to call your specific attention to the importance of this matter and urge you to have your engineers pay particular attention to this important subject."

Mobilization

The Government has announced its general deferment policy for Army, Navy, and Air Force reserves, and National Guardsmen in key civilian jobs. The new policy does not apply to the draft. The Commerce Department has published a list of "essential" industries and the Department of Labor made public its list of "critical" occupations. Defense Department officials said the three armed services would use both lists in deciding whether a worker should be left at his civilian job, or called to active duty. Copies of the lists may be obtained by writing to either Department, Washington 25, D. C., and asking for "Tentative Lists of Essential Activities" from Commerce and the "List of Critical Occupations" from the Labor Department . . . . It appears that the military will give more weight to the "critical occupations" list than to the "essential activities" list. In the Labor Department list of "critical occupations" submitted to the Department of Defense, Secretary of Labor Maurice J. Tobin said: "In the preparation of the list of critical occupations, selection of specific occupations was made on the basis of three major considerations. These were: (a) The demand, in essential industries and activities for persons qualified to work in the occupations would exceed the total supply under conditions of full mobilization; (b) a minimum training time of 2 years (or the equivalent in work experience) is necessary to the satisfactory performance of all the major tasks found in the occupation; (c) the occupation is essential to the functioning of the industries or activities in which it occurs."

Key personnel of the radio and television manufacturing industry is in the Labor list under Engineer, Electrical. The definition submitted to the military is:

Engineer, Electrical (Electrical Engineer 0–17.01, DOT P. 447) Performs one or more the following functions in the field of electrical engineering requiring fulfillment of educational, experience or legal qualifications established by engineering colleges or licensing authorities: Plan and supervise construction and operation of electric power generating plants, transmission lines and distribution systems: plan and supervise construction and installation of illumination, wire communication, and electric transportation systems; design and develop radio, television, electronic and allied equipment and supervise manufacture of various types of electrical machinery and apparatus, such as motors and generators, converters and regulators, switch gear, and welding equipment. May also specialize in research, consulting, inspection, testing, teaching at the university level, specification and other technical writing, and sales and service of complex electrical equipment. This title includes all related titles with the same Dictionary of Occupational Titles code number.

If a key member of industry is a reservist or a National Guardsman, and is called up by the military, he can, if desired, apply for deferment. Such a request should be made only after a man is called to active duty.

Any deferment request should be handled as follows:

Army—If you are in the Army's organized reserves you should apply through your unit commander or unit instructor to the commanding general of the Army area in which you live. However, if your unit doesn't meet often, or you are an inactive reservist, it would be wise to save time by sending your request direct to the commanding general. He is the one who has the final decision on deferment in each individual case.

Navy—If you're a non-aviation reserve officer you should write The Chief, Bureau of Naval Personnel, through your naval district commandant. Aviation officers and enlisted men should apply to the Chief of Naval Air Reserve Training, Naval Air Station, Glenview, Ill. Non-aviation enlisted men write to their naval district commandant.

Marine Corps—You should apply through the usual "chain of command," starting with your local instructor.

Air Force—Write the headquarters of the numbered Air Force for the area in which you live.

National Guard (Army or Air)—Apply to the adjutant general of the state where your unit is organized.

Broader Industry Coverage Is Sought on United States List

Upon authorization of President Robert C. Sprague, Chairman John W. Craig of the RTMA Industrial Relations Committee has appointed a subcommittee with instructions to review the U. S. Labor Department's critical occupations list (RTMA Industry Report, Vol. 6, No. 34) and make recommendations for expanding it to give the radio-television industry adequate protection.

Harold W. Butler of the Philco Corp. was named Chairman of the subcommittee, and other members are E. M. Tuft of the RCA Victor Division, and Harvey T. Stephens of International Resistance Co.

Under the present critical occupations list, only engineers in the radio-television industry are clearly identified as subject to temporary deferments in calls for reserve officers. These are covered under the listing, "Engineers, Electrical."

The matter previously had been brought to the attention of the U. S. Labor Department, as well as manpower officials of the Munitions Board and the National Security Resources Board by General Manager James D. Secrest.

CCIR Study Group Reports Actions Taken at Geneva

The U. S. State Department has received a telegraphic report on actions taken by a subgroup of CCIR Study Group No. 11 which met in July at Geneva, Switzerland. RTMA was represented at this international television session by I. J. Kaar of General Electric Co., Chairman of the Committee on Television, Receiver Section of the RTMA Engineering Department.

The subcommittee under the chairmanship of W. Geber of Switzerland met in an effort to secure uniformity of characteristics of a standard using 625 lines. The group arrived at the following conclusions: (1) A 7-Mc channel; (2) 625 lines per picture interlaced 2 to 1; (3) operation independent of power supply frequency; (4) standard line scanning frequency of 15,625 with a tolerance of 1/10 per cent and a field frequency of 50 per second; (5) aspect ratio of 4 horizontally and 3 vertically; scanning left to right horizontally and top to bottom vertically; (7) amplitude modulation for video with specified asymmetric sideband characteristics; (8) negative video modulation; and (9) FM for sound with 50-kc deviation.

Air Matériel Command Surveys Requirements for Personnel

The Intelligence Department of the Air Matériel Command is completing a reanalysis of its functions and the determination of personnel requirements for the resultant organization. The survey has revealed an urgent need for certain types of highly qualified technical or scientific personnel in positions, most of which are established by Air Technical Intelligence Specialists.

The Air Technical Intelligence Specialists perform, direct, and control air technical intelligence research to determine the accomplishments of foreign nations in conducting aerial warfare. In assessing the capabilities of foreign countries in their various fields it is often necessary to use incomplete and fragmentary information. This type of work requires personnel capable of performing and supervising a high level of technical analysis work, so they must be well qualified as engineers or scientists in their specialized fields. In addition, they are required to be adaptable to intelligence research work, in that they must be capable of

[1] The data on which these NOTES are based were selected, by permission, from *Industry Reports*, issues of August 4, August 11, August 18, and August 25, published by the Radio-Television Manufacturers Association, whose helpful attitude is gladly acknowledged.

conducting studies utilizing fragmentary and incomplete information, employing to a considerable degree deductive reasoning based on their knowledge of sound application of scientific and engineering principles and good common sense. They must also be familiar with the significance of the technical capabilities of equipment with respect to air power in order to appreciate the intelligence significance of information with which they are dealing.

Starting salaries of positions for which personnel are most urgently needed at the present time range from $5400 to $8800 per annum. For further information write to Intelligence Dept., Air Matériel Command, Wright-Patterson Air Force Base, Dayton, Ohio.

Radio and Television News Abroad

Sales of radio receiving sets by Canadian producers in May totaled 51,616 units, valued at $3,805,166 at list prices, according to the U. S. Department of Commerce. Included are 686 television sets valued at $314,480. Producer sales for the first five months totaled 256,781 units valued at $20,920,969, including 4,248 television sets valued at $1,791,280. In the first five months of 1950, imports of radio receiving sets totaled 10,785 units and exports 13,956 units . . . .Radio and television receiving licenses in force June 30, 1950, in the United Kingdom were as follows: Radio, 11,871,312, of which 1,184,282 were in London; television, 400,375, of which 113,848 were in London.

RTMA-NSIA Committee Name Changed

The name of the National Electronics Mobilization Committee, formed August 8 by RTMA and the National Security Industrial Association, (RTMA Industry Report, Vol. 6, No. 33) has been changed to the Joint Electronics Industry Committee, Chairman F. R. Lack has announced.

He has also stated that the Committee has retained John L. Sullivan, former Secretary of Navy and now associated with the law firm of Sullivan, Bernard, and Shea, as special counsel.

H. G. Beauregard has been appointed Secretary of the Committee, and a headquarters has been established at 804 Ring Building, 1200 18 Street, N.W., Washington 6, D. C.

Books

Electron Tube Circuits by Samuel Seely

Published (1950) by McGraw-Hill Book Co., 330 W. 42 St., New York, N. Y. 488 pages+16-page index+23-page appendix+ix pages. 509 figures. 9×6½. $6.00.

With the ever-widening sphere of application of electron tubes there has come the need of a presentation of their properties on a somewhat more general level than is customary in texts devoted to the communications field. The nuclear physicist building an amplifier for detecting fast particles, the mathematician designing an electronic computer, or the process-control engineer making an electronic servomechanism are only a sample of the enlarged group now concerned with tubes and their application. The present text is an approach to this need, comprising a presentation of fundamentals with illustrations from both communications, and other fields. It is intended as a beginning text, presupposing a knowledge of basic ac circuit theory and some slight acquaintance with the physics of electron tubes. The first part of the book is devoted to a review of the fundamental properties of tubes and their circuits. Equivalent circuits are introduced on both current and voltage bases. The next section deals with untuned amplifiers including some of the fundamental computing circuits as well as switching and gating circuits. Then come sections on tuned amplifiers, oscillators, rectifiers, and filters (including a good discussion of stabilized power supplies) amplitude modulation and demodulation, frequency modulation and detection, and finally chapters devoted to special circuits originating largely in radar and television.

On the whole, the choice of topics seems to be excellent. To the reviewer it is a source of regret that no room is found for any mention of fluctuation noise as a limit to useful amplification, and of the factors in tube and circuit design that tend to minimize such noise. In addition, it seems that some topics are not treated as carefully as they should be: for example, on page 378 two circuits are shown labelled as pre-emphasis circuits. Actually one is and the other is not, as is borne out by the accompanying mathematical treatment, although the correct expression for a pre-emphasis circuit does not appear anywhere in the entire section. Some reservations must also be expressed concerning the treatment of feedback amplifiers in Chapter V. In spite of such points of criticism, the book should prove eminently useful, particularly after exposure to the eagle eyes of a few classes has caught the various errors that seem to be inevitable in a new book.

S. N. Van Voorhis
University of Rochester
Rochester, N. Y.

Practical Television Servicing and Trouble-Shooting Manual by The Technical Staff of Coyne Electrical and Radio-Television School

Published (1949) by Greenberg Publishers, 201 57 St., New York 22, N.Y. 392 pages+7-page index +iv pages. 265 figures. 5½×8½. $4.25.

The purpose of this new book is to bring the radio serviceman up-to-date in television. Presupposing a knowledge of elementary radio theory, an authoritative discussion of television servicing and trouble-shooting is presented.

The book is fairly complete and clearly written. No mathematics is used. Its chief attribute is that it tells the reader exactly what to do and how to go about servicing a modern television receiver.

It is the opinion of this reviewer that this book serves the above purpose well and that an excellent effort to explain TV circuits and circuit behavior has been made. Furthermore, the book abounds with illustrations. There are many reproductions of faulty kinescope test patterns, circuit diagrams, sketches of response curves and pulse shapes, and photographs of important TV set sections. Many of the circuit diagrams are taken from modern commercial receivers.

There are nine chapters cover TV servicing methods, service instruments, tuners, TV sound, IF amplifiers, video IF alignment, sound alignment, tuner alignment, the video detector, and amplifier. The next eight chapters cover kinescope input circuits, the sync section, sweep oscillators, afc circuits, sweep amplifier and deflection circuits, kinescope tube servicing and adjustment, TV power supplies, and TV antennas. (Built-in antennas, unfortunately, have been overlooked in the chapter on antennas.)

The eighteenth chapter is timely, covering color TV. Illustrative diagrams are actually printed in three colors and the rudiments of the simultaneous, field-sequential color systems are discussed. However, no mention is made of the RCA dot-interlace time-multiplex color system. A short discussion is also given on uhf and distributed-constant effects at these frequencies.

The book "Television Servicing and Trouble-Shooting" would make a useful addition to the library of anyone working in this field.

It should be kept in mind, however, that TV receiver design is undergoing a rapid development not only because of heavy consumer demand, but also because of improvements in transmitting methods, a possible shift in emphasis from vhf to uhf in view of new channel assignments, and other reasons. It should, therefore, very likely become necessary to revise this book (and most other books on TV servicing) within a few years.

Frank R. Arams
Special Development Group
Radio Corporation of America
Harrison, N. J.

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Abstracts and References

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NOTE: The Institute of Radio Engineers does not have available copies of the publications mentioned in these pages, nor does it have reprints of the articles abstracted. Correspondence regarding these articles and requests for their procurement should be addressed to the individual publications, and not to the IRE.

The number in heavy type at the upper left of each Abstract is its Universal Decimal Classification number and is not to be confused with the Decimal Classification used by the United States National Bureau of Standards. The number in heavy type at the top right is the serial number of the Abstract. DC numbers marked with a dagger (†) must be regarded as provisional.

The Annual Index to these Abstracts and References, covering those published in the PROC. I.R.E. from February, 1949, through January, 1950, may be obtained for 2s. 8d postage included from the *Wireless Engineer*, Dorset House, Stamford St., London S. E., England. This index includes a list of the journals abstracted together with the addresses of their publishers.

ACOUSTICS AND AUDIO FREQUENCIES

016:534 **2395**
References to Contemporary Papers on Acoustics—A. Taber Jones. (*Jour. Acous. Soc. Amer.*, vol. 22, pp. 395–399; May, 1950.) Continuation of 1838 of September.

534.012:[532.111+534.22 **2396**
The Mean Pressure and Velocity in a Plane Acoustic Wave in a Gas—P. J. Westervelt. (*Jour. Acous. Soc. Amer.*, vol. 22, pp. 319–327; May, 1950.) The one-dimensional equation for a travelling wave is discussed to the second order of approximation, using Airy's solution to the Lagrangean form of the equation as a basis; second-order effects are reviewed and correlated. Forces on a particle in the wave have a time-average component which is opposed to the radiation pressure and whose magnitude may exceed that of the radiation pressure; this may result in a particle drift velocity, in a direction opposed to wave propagation, of about 1 cm per second for a sound pressure level of 151 db in air. Standing waves of large amplitude are also considered.

534.321.9:061.3 **2397**
International Convention on Ultrasonics—G. Bradfield. (*Nature* (London), vol. 166, pp. 143–144; July 22, 1950.) A short report of the proceedings at the convention held in Rome, June, 1950, with an outline of some of the subjects discussed. Slightly more than 50 per cent of all the contributions were concerned with biology and medicine.

534.321.9:615 **2398**
Ultrasonic Apparatus for Therapeutic Applications—H. Thiede. (*Elektrotechnik* (Berlin), vol. 4, pp. 219–223; June, 1950.) Discussion of the physical characteristics of ultrasonic waves and description of piezoelectric and magnetostriction generators for frequencies up to about 3 Mc, including some giving focused beams.

534.322.3:534.6:621.396.822 **2399**
Background Noise and the Use of White Noise in Acoustics—P. Chavasse and R. Lehmann. (*Ann. Télécommun.*, vol. 5, pp. 229–236; June, 1950.) The theory of background noise is reviewed. A neon tube is preferred to a pentagrid tube as a practical generator of white noise. The circuit of the artificial voice (3404 of 1947) is shown. Architectural, physiological, and electroacoustic applications of noise sources are indicated. Some 50 references are given.

534.415 **2400**
Stroboscopic Audio-Frequency Spectrometer—F. A. Fischer. (*Fernmeldetech. Z.*, vol. 3, pp. 174–180; May, 1950.) The stroboscopic method of measurement is discussed and two instruments are described. A light source is modulated with the signal to be measured and viewed through a revolving stroboscopic screen characteristically sectioned. Alternatively, a fixed screen suitably sectioned, e.g., with a hyperbolic pattern for a linear frequency scale, is viewed in a revolving mirror. Frequency range is 60 to 6,000 cps.

534.613 **2401**
On the Radiation Pressure of Spherical Waves—R. Lucas. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 2004–2006; June 5, 1950. An expression is derived for the acoustic pressure due to the pulsation of a sphere of dimensions small compared with wavelength. Near the surface of the sphere the value is twice the density of the kinetic energy; this greatly exceeds the total energy density, and agrees with the value deduced from Brillouin's pressure tensor. Consideration of the pressure exerted by a stationary spherical wave on a reflecting boundary surface leads to an expression involving the factor 3, characteristic of spherically symmetrical systems. The limiting case of a stationary plane wave acting on a finite plane parallel reflector is considered, and the formula here developed is reconciled with that previously given by Brillouin.

534.7 **2402**
Auditory Sensation Produced by Rectangular Waves—R. Chocholle. (*Ann. Télécommun.*, vol. 5, pp. 237–242; June, 1950.) Discussion of the results of an experimental study. The wave form of the sound waves resulting from application of square waves to an acoustic transducer depends largely on the ratio between the fundamental frequency of the applied square waves and the resonance frequencies of the transducer. The character of the sensation produced is distinct in the very low-, low-, medium-, and high-frequency ranges. In certain narrow frequency bands, slight variation of frequency produces a marked change in sensation, i.e., intensity and general timbre. The ear is found to respond rather to the wave envelope and total effective amplitude than to the individual components of the sound spectrum.

534.78:621.395 **2403**
The Perception of Speech and Its Relation to Telephony—H. Fletcher and R. H. Galt. (*Jour. Acous. Soc. Amer.*, vol. 22, p. 327; May, 1950.) Correction to paper abstracted in 1844 of September.

534.833.4 **2404**
A Review of the Absorption Coefficient Problem—H. J. Sabine. (*Jour. Acous. Soc. Amer.*, vol. 22, pp. 387–392; May, 1950.) The problem of discrepancies between measurements of acoustic absorption coefficient obtained by different workers is discussed from the viewpoint of the materials manufacturer. Suggestions are made for further theoretical and experimental investigations.

534.843 **2405**
Transient Sounds in Rooms—D. Mintzer. (*Jour. Acous. Soc. Amer.*, vol. 22, pp. 341–352; May, 1950.) Laplace-transform methods are used for investigating the propagation of transient sound waves. In the one-dimensional case propagation of a plane wave in a rigid-walled tube is considered. The velocity potential for an arbitrary particle-displacement input is found as a series, each term of which represents the effect of a reflection from an end of the tube. In the three-dimensional case a spherical wave is considered in unbounded and variously bounded regions, and the case of a point source in a rectangular room is solved. An image method is used. Examples are calculated for both one- and three-dimensional systems.

534.844.1:621.3.015.33 **2406**
Pulse Statistics Analysis of Room Acoustics—R. H. Bolt, P. E. Doak, and P. J. Westervelt. (*Jour. Acous. Soc. Amer.*, vol. 22, pp. 328–340; May, 1950.) Reverberation time is not by itself an adequate index of the acoustic quality of a room, since it does not give information about the response to transients. To take account of the pulse-like nature of common sounds, an analytical technique has been developed in which the walls of the room are replaced by an infinite array of image sources; a statistical consideration of this array gives the long-term average response to transients. To illustrate the technique a description is given of experiments carried out in a rectangular room with hard plastered walls, using a sound source emitting 2-ms pulses of 3,600-cps damped waves.

534.846:621.396.712.3 2407
Rooms with Reverberation Time Adjustable over a Wide Frequency Band—P. Arni. (*Jour. Acous. Soc. Amer.*, vol. 22, pp. 353–354; May, 1950.) Description of a broadcasting studio at Helsinki having wall elements with variable absorption characteristics.

621.395.623.7 2408
A Contribution to the Design of Horn Loudspeakers—J. Merhaut. (*Tesla Tech. Rep.* (Prague), pp. 41–43; December, 1949.) The high-frequency response may be improved by subdividing the throat of the exponential horn, since the phase differences between the sound waves emanating from different parts of the diaphragm may thereby be reduced. A new design described gives a 4.5-db improvement at 8 kc.

621.395.623.7 2409
A New Loudspeaker Combination—H. Schmidt. (*Funk. und Ton*, vol. 4, pp. 226–232; May, 1950.) A high-efficiency loudspeaker with a strong electromagnet for transient damping has a "spherical-wave" modified exponential horn. In front of the light metal spherical membrane is a slotted acoustic cone for equalizing sound-path lengths in the neck of the horn, giving a further increase of the electroacoustic efficiency. With the associated filter circuit it is mounted above a bass-response loudspeaker which has a conical diaphragm of diameter 35 cm. The response curve of the combination is shown. It is nearly level between 50 cps and 10 kc. Crossover occurs at 500 cps.

621.395.623.8:621.398 2410
Remote Control of the Tesla RU Rack and Panel Public Address System—L. Pravenec. (*Tesla Tech. Rep.* (Prague), pp. 33–40; December, 1949.) Panel units are described, (a) for use at a main station, making remote control of up to ten substations possible, and (b) for use at a substation, enabling it to accept remote control from a main station while retaining the possibility of independent operation.

621.395.625.3+681.84/.85]:061.4 2411
Sound Reproduction—(*Wireless World*, vol. 56, pp. 255–258; July, 1950.) Short descriptions of equipment at the exhibition arranged by the British Sound-Recording Associations in London, May, 1950. In addition to disk, magnetic wire and tape, and film recorders, many examples of amplifiers, loudspeakers, pickups, microphones, and auxiliary and test equipment were shown, as well as BBC and Radio Luxembourg sound-recording vans.

681.85.001.4 2412
A Feedback-Controlled Calibrator for Phonograph Pickups—J. G. Woodward. (*RCA Rev.*, vol. 11, pp. 301–309; June, 1950.) The phonograph-pickup calibrator is an electromechanical device for imparting known and controllable lateral motions to the stylus of a pickup under test. The calibrator employs a dynamic driving system, and electromechanical feedback is used to secure a uniform response between 20 cps and 20 kc. The absolute as well as the relative response of a pickup can be conveniently and quickly measured within this frequency range.

681.85.004.62 2413
Record and Stylus Wear—G. H. H. Wood. (*Wireless World*, vol. 56, pp. 245–248; July, 1950.) Neither low stylus pressure nor sapphire points which are dimensionally correct can, of themselves, provide a satisfactory solution to the problem of record wear. It is shown, with the aid of photomicrographs, that greatly prolonged stylus life and low record wear result from the use of a crystal pickup head with a correctly designed sapphire stylus so mounted that the coupling is comparatively flexible in the vertical direction. Vertical movement of the stylus is not transmitted to the pickup head and the system reduces the effective mass of the stylus to a very low value compared with that of conventional stylus systems.

621.395.625+534.862.4 2414
The Recording and Reproduction of Sound [Book Review]—O. Read. Publishers: H. W. Sams & Co., Indianapolis, Ind., 1949, 364 pp., $5.00. (*Electronics*, vol. 23, pp. 136, 138; July, 1950.) The book brings together information which until now has been scattered in periodicals and instruction manuals; it is of interest more to the technician and the phonograph enthusiast than to the engineer. See also 632 of 1949 and back references.

ANTENNAS AND TRANSMISSION LINES

621.315.212 2415
Coaxial Cable and its Optimum Design—P. Ya. Shiniberov. (*Priroda*, No. 1, pp. 69–74; January, 1949. In Russian.) Cable theory is discussed and methods are indicated for determining the capacitance and attenuation for different types of cable. The ratio of the radii of the conductors for maximum dielectric strength is also determined. Multicore cables with several coaxial pairs are briefly considered.

621.315.34.011.4 2416
Capacity of a Pair of Insulated Wires—W. H. Wise. (*Quart. Appl. Math.*, vol. 7, pp. 432–436; January, 1950.) A method of calculating capacitance is given which is related to but involves less numerical computation than that noted in 660 and 1446 of 1946 (Craggs and Tranter). Certain analytical difficulties which arise when the insulating jackets are in contact are indicated.

621.392 2417
Unbalanced Terminations on a Shielded-Pair Line—K. Tomiyasu. (*Jour. Appl. Phys.*, vol. 21, pp. 552–556; June, 1950.) The simultaneous propagation of balanced and unbalanced modes along a shielded-pair line results in an unbalanced component of current. This can be determined by comparing the different standing-wave distributions on the two conductors. Measurements of the components of waves reflected from unbalanced radiative terminations (e.g., end-coupled antennas) on a slotted shielded-pair line are described.

621.392.1 2418
The Effect of a Bend and Other Discontinuities on a Two-Wire Transmission Line—K. Tomiyasu. (PROC. I.R.E., vol. 38, pp. 679–682; June, 1950.) The effect is analyzed by the vector-potential method, the equivalent-circuit elements being obtained by comparing the variable-line parameters near the bend with the conventional-line parameters found on an infinite line. Good agreement is found between theoretical and experimental values of the circuit elements for the bend and also for open-end and bridged-end lines. See also 28 of February (King and Tomiyasu).

621.392.26†:621.3.09 2419
Propagation of the TM_{01} Mode in a Metal Tube Containing an Imperfect Dielectric—D. L. Hetrick. (*Jour. Appl. Phys.*, vol. 21, pp. 561–564; June, 1950.) Losses in both the metal wall and the dielectric are taken into account in this analysis, the subject being treated as a boundary-value problem. General expressions are derived for the attenuation and phase constant, and the percentage error introduced by (a) neglecting wall losses and (b) assuming total attenuation to be the sum of the dielectric and wall-loss attenuations is computed for a particular case.

621.392.26†:621.392.52 2420
Ring Mode Filter for Type H_{11} Waves in Circular Waveguides—Z. Szepesi. (*Onde Élec.*, vol. 30, pp. 230–234 & 293–298; May and June, 1950.) Experimental investigation of ring mode filters of different dimensions in a waveguide of 8 cm diameter, using a wavelength of 11 cm. Results are tabulated. Good reflection occurs when the circumference of the ring is slightly greater than the free-space wavelength. The characteristics of the filter are practically independent of its conductivity. Impedance of the ring varies linearly with its circumference; when the circumference is less than that required for resonance the impedance is capacitive; when greater, it is inductive. Bandwidth increases with the cross section of the ring. Measured bandwidth is considerably greater than the value given by the theory of Feuer and Akeley (640 of 1949). Using a combination of two filters, $3\lambda_g/4$ spacing is the most effective where λ_g is the wavelength in the waveguide. The characteristics of a filter may be determined from its distance from a filter of known characteristics when the transmission coefficient of the combination has its maximum value.

621.392.43 2421
A Tapered Line Termination at Microwaves—G. J. Clemens. (*Quart. Appl. Math.*, vol. 7, pp. 425–432; January, 1950.) The termination considered is a short-circuited section of coaxial line with a uniformly tapered metallic outer conductor and an inner conductor consisting of a glass tube coated with a thin resistive metal film. The distribution of voltage and current is investigated theoretically, and the measured values of voltage SWR for the termination over the range 990–3,968 Mc are compared with those calculated for an exponential line, which is considered in the analysis as a convenient approximation.

621.396.67 2422
The Effect of an Obstacle in the Fresnel Field on the Distant Field of a Linear Radiator—G. A. Woonton. (*Jour. Appl. Phys.*, vol. 21, pp. 577–580. June, 1950.) An analytical method is developed for determining the errors due to diffraction which arise when measurements on microwaves are made by optical methods. The analysis is applied to the particular problem of determining the distant field of a horn radiator with the aperture of a large screen in front of it at an arbitrary angle. Satisfactory agreement is obtained between calculated and measured values (2133 of October).

621.396.67 2423
Effect of a Finite Groundplane on Antenna Radiation—A. Leitner and R. D. Spence. (*Phys. Rev.*, vol. 79, p. 199; July 1, 1950.) Summary of American Physical Society paper. The field of a $\lambda/4$ dipole over a circular ground plane is calculated exactly. Both the current in the ground plane and the radiation at great distances are found. As the radius of the ground plane is increased, the radiation resistance and the surface current oscillate about the values which characterize a $\lambda/4$ antenna above an infinite ground plane, but the radiation pattern is entirely different. The results are in good quantitative agreement with the experimental results of Meier and Summers (2440 of 1949).

621.396.67 2424
The Directive Properties of Receiving Aerials—É. Roubine. (*Onde Élec.*, vol. 30, pp. 259–266; June, 1950.) Full paper of which an abridged version was given in *Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1590–1592; May 3, 1950.) Distinction is made between the case of rectilinear polarization and the general case of elliptical polarization. A simple fundamental formula is derived expressing the current in a receiving antenna as a function of the incident field, the load, and the radiating properties of the antenna in the direction from

which the waves are received. The question of the identity of the directive properties of antennas for transmission and reception is in general not well defined. The directivity for emission is, in fact, controlled by a single characteristic, while for reception this characteristic is altered by the intensity and polarization of the incident field, so that all comparisons of the directive properties necessitate an arbitrary convention in respect of the incident field. When considering gain, this convention is modified. The effect of polarization is discussed by means of an optical analogy. Current formulas for the absorption surface and the effective length of a receiving antenna are generalized.

621.396.67:621.397.6 2425
Ultra-High-Frequency Antenna and System for Television Transmission—O. O. Fiet. (*RCA Rev.*, vol. 11, pp. 212–227; June, 1950.) The construction and performance are described of an omnidirectional, horizontal-polarization antenna consisting of two coaxial tubes. The inner one acts as a transmission line, feeding groups of slot antennas in the outer tube by means of radial probes. A power gain of 17.3 db is obtained at 530 Mc.

An account is also given of the design and performance of the waveguide feed between the transmitter and the antenna, the vestigial-sideband filter and a notch diplexer which enables the sound and vision signals to be fed along the common transmission line.

The antenna system was developed for operation at Bridgeport, Conn., [1795 of August (Guy, Seibert, and Smith)].

621.396.67.029.62/.63 2426
Experimental Investigations on Wide-Band Metre- and Decimetre-Wave Aerials—K. Lamberts and L. Pungs. (*Fernmeldetech. Z.*, vol. 3, pp. 165–173; May, 1950.) Study made in 1942 of the characteristics of single dipoles with reflectors. A greater bandwidth can be obtained by increasing the diameter when tubes are used as the dipole elements, or by the use of specially shaped flat elements. Circle diagrams show the influence of tube diameter, antenna length, and reflector distance on the frequency dependence of the antenna input impedance.

621.396.671:621.3.011.2 2427
Input Impedance of Two Crossed Dipoles—L. G. Chambers. (*Wireless Eng.*, vol. 27, pp. 209–211; July, 1950.) The mutual impedance is calculated by applying Carter's method; the input impedance is then found from a simple formula. These impedances are, to a good degree of approximation, expressible as simple functions of the angle between the dipoles.

621.396.671.029.63/.64 2428
Relations Concerning Wave Fronts and Reflectors—K. S. Kelleher. (*Jour. Appl. Phys.*, vol. 21, pp. 573–576; June, 1950.) Vector formulas are derived relating the incident and reflected wave fronts and the reflector surface. The analysis is applied to particular microwave problems, including the determination of the reflector surface which will transform an arbitrary incident wave front into a plane wave front.

621.396.677 2429
Rotatable Directive Antenna—J. Břiza. (*Tesla Tech. Rep.* (Prague), pp. 10–32; December, 1949.) The apex of a rhombic antenna is attached by means of insulators to the rotatable head of a wooden mast, the other three junctions being attached by sectioned cables to carriages running on a circular track round the mast. The positions of the carriages, which form a symmetrical system, are controlled to alter the direction of transmission or reception and also to adjust the apex angle to suit different frequencies. The antenna is fed at its lowest point by twin cable which is carried round with it. Computed polar diagrams are compared with experimental results for scale models.

621.396.677:621.397.828 2430
TV Antenna Phase Control—Carmichael. (*See* 2655.)

621.396.677.2:621.396.97 2431
Nine-Tower Broadcast Array—C. W. Winkler and M. Brasseur. (*Electronics*, vol. 23, pp. 102–104; July, 1950.) Brief illustrated account, with details of phasing and antenna-tuning equipment, of the WDGY station at Minneapolis, which produces a radiation pattern with suppression over an angle of 247°.

621.396.67.029.64 2432
Aerials for Centimetre Wavelengths [Book Review]—D. W. Fry and F. K. Howard. Publishers: Cambridge University Press, 1950, 172 pp., 18s. (*Jour. Brit. IRE*, vol. 10, p. vi; July, 1950.) "This book deals specifically with radiating systems for radar applications, but design principles are outlined in sufficient detail to enable the engineer to design a radiator for other applications."

CIRCUITS AND CIRCUIT ELEMENTS

517.41:621.3.016.352 2433
On the Representation of the Stability Region for Oscillation Phenomena by use of Hurwitz Determinants—E. Sponder. (*Schweiz. Arch. Angew. Wiss. Tech.*, vol. 16, pp. 93–96; March, 1950.)

621.3.012.2 2434
Classes of Circle Diagrams—C. E. Moorhouse. (*J. Inst. Eng.*, vol. 22, pp. 69–74; March, 1950.) Consideration of the properties of a general 4-pole network under specified terminal conditions leads to a general method of establishing the conditions for the existence of circle diagrams. Two classes of diagrams are discussed; variants of these include many of the circle diagrams used in electrical engineering.

621.316.8.029.55/.62 2435
Behavior of Resistors at High Frequencies—G. R. Arthur and S. E. Church. (*TV Eng.*, vol. 1, pp. 4–7; June, 1950.) The variation of resistance with frequency is discussed for various types of resistors by consideration of circuit equivalents which include a capacitive element. Theoretical results agree with measured values for commercial resistors in the frequency range 5–60 Mc to within 10 per cent. Except possibly for special high-frequency resistors, the effective resistance of resistors decreases more rapidly with frequency the higher the value of the dc resistance. For the same dc resistance, the smaller the physical dimensions the better are the high-frequency characteristics. Resistors using a carbon coating on insulating material were found better for high-frequency work than either carbon-block or composition resistors.

621.318.371 2436
Calculation of the Inductance of Circular Conductors and Single-Layer Close- or Open-Wound Solenoids—W. Keller. (*Bull. Schweiz. Elektrotech. Ver.*, vol. 41, pp. 442–450; May 27, 1950. In German.) A method of calculation is developed and a series of equations derived. Curves are plotted from which the value of inductance is readily determined when length, cross section, etc., are known.

621.318.5 2437
Magnetic Triggers—An Wang. (Proc. I. R. E., vol. 38, pp. 626–629; June, 1950.) Magnetic cores with fairly rectangular hysteresis loop are used in a trigger device in which magnetic fluxes are used instead of electrical currents to indicate the two stable states. The magnetic-flux polarity can be detected without mechanical motion. The construction and operation of several types of magnetic triggers are described. See also 1183 of June (An Wang and Way Dong Woo).

621.319.4 2438
The Theory of a Three-Terminal Capacitor—R. E. Corby. (Proc. I. R. E., vol. 38, pp. 635–636; June, 1950.) Equations are derived for the insertion loss of a 3-terminal capacitor and their solution gives results in agreement with experiment. The capacitor is assumed to function like a transmission line and the constants of the equivalent line are determined as a function of frequency. Skin effect and proximity effect are taken into consideration, and curves are plotted to facilitate computation.

621.389:681.142 2439
A Fast Multiplying Circuit—Chance, Busser, and Williams. (*See* 2558.)

621.392:512.831 2440
An Introduction to the Matrix Method of Solving Electrical-Network Problems—R. Guertler. (*Jour. Inst. Eng.*, vol. 22, pp. 46–52; March, 1950.) The fundamental principles of matrix algebra are outlined and simple working rules are given. The transformation matrices of series-impedance and shunt-admittance 4-pole networks are derived and these matrices, together with the rules of matrix algebra, are applied to discussion of networks in cascade, of L, T and π networks, and to calculation of the frequency response of a class-B modulator, this calculation being considerably simpler than that by ordinary methods, as given in Appendix 2. A simple application of matrix methods is given in Appendix 1, where the conditions are derived for the current or voltage in a circuit to remain constant under varying load.

621.392:517.512.2 2441
Application of Fourier Transforms to Variable-Frequency Circuit Analysis—A. G. Clavier. (*Elec. Commun.*, vol. 27, pp. 159–163; June, 1950.) Reprint. See 307 of March.

621.392:517.512.6 2442
A Note on the Solution of Certain Approximation Problems in Network Synthesis—R. M. Fano. (*Jour. Frank. Inst.*, vol. 249, pp. 189–205; March, 1950.) A method is given for the determination of appropriate functions to define the amplitude characteristics of networks when certain ideal responses are to be approached within specified limits. The method is based on simplification of the geometry of the problem by transformation in the complex plane.

621.392:621.3.016.35]:681.142 2443
Mathematical Problems of Feedback—H. Freudenthal. (*Ned. Tijdschr. Natuurk.*, vol. 15, pp. 275–281; November, 1949.) Paper given at the Symposium on Modern Calculating Machines, Amsterdam, May, 1949. From both the mathematical and the physical viewpoints, it is preferable to treat feedback circuit problems by the method of continuous rather than discrete iteration. Analysis establishes the necessary and sufficient conditions for convergence and hence for circuit stability.

621.392.011.2 2444
Reciprocity between Generalized Mutual Impedances for Closed or Open Circuits—A. G. Clavier. (*Elec. Commun.*, vol. 27, pp. 152–158; June, 1950.) Reprint. See 1358 of July.

621.392.43 2445
Wideband Series-Parallel Transformer Design—V. C. Rideout. (*Electronics*, vol. 23, pp. 122, 162; July, 1950.) Formulas for obtaining maximum flatness of the response curve are derived from filter-theory considerations. The circuit constants of a matching transformer to connect a coaxial line to the first tube of an amplifier are calculated numerically.

621.392.5 2446
The Bridged-T Network and its Properties—W. Taeger. (*Funk. und Ton*, vol. 4, pp. 253–259; May, 1950.) Mathematical analysis deriving expressions for attenuation and phase constant.

621.396.611.1 2447
Oscillation Phenomena in a Circuit with a Discontinuous Linear Characteristic—G. J. Elias and S. Duinker. (*Tijdschr. ned. Radiogenoot.*, vol. 15, pp. 79–91; May, 1950. In Dutch, with English summary.) A series circuit of negligible resistance is considered which comprises a capacitor and an inductor with a core material such as numetal or permalloy having a sharp knee in its magnetization curve. To facilitate analysis, an approximation to the curve is used which consists of three straight lines. In such a circuit, forced subharmonic oscillations can be maintained, but transition phenomena occur when the amplitude of the applied electromotive force is great enough to extend the operation beyond the knee of the curve. When operation is largely in the saturation region, the initial amplitude of the free oscillations is related to the magnitude of the corresponding transition; the oscillations are damped, owing to hysteresis losses. Each transition adds a fresh inner loop to the main hysteresis loop, corresponding to a pair of new real branch points in the hyperelliptic expression for the oscillations. See also 582 of April.

621.396.611.4:538.56 2448
Forced Oscillations in a Spherical Cavity Resonator—A. A. Piotrovski. (*Zh. Tekh. Fiz.*, vol. 20, pp. 282–294; 1950. In Russian.) Mathematical discussion of the excitation of a spherical cavity resonator by a single turn of wire at its center. An equivalent oscillatory circuit is derived and methods are indicated for determining its impedance and main parameters such as Q, damping factor, and loss resistance.

621.396.611.4:621.396.615.142.2 2449
Wide-Range Tunable Waveguide Resonators—W. W. Harman. (Proc. I. R. E., vol. 38, pp. 671–679; June, 1950.) See 2476 of 1949.

621.396.615.12 2450
***LC* Oscillators and their Frequency Stability**—J. Vackar. (*Tesla Tech. Rep.* (Prague), pp. 1–9; December, 1949.) Factors affecting stability are considered and their specific effects tabulated; the mechanical and electrical design and construction of highly stable circuits are discussed. A general analysis is given of the oscillator circuit and a formula for change of frequency with change of tube capacitance is derived. Known variable-frequency oscillators are reviewed and new circuits used in Tesla broadcasting transmitters and having a tuning range of 1.5:1 are described.

621.396.615.14:621.385.3 2451
Feedback in Very-High-Frequency and Ultra-High-Frequency Oscillators—F. J. Kamphoefner. (Proc. I.R.E., vol. 38, pp. 630–632; June, 1950.) A study of feedback considerations in low-power negative-grid triode oscillators for the frequency range 100–1,000 Mc. Discussion is mainly confined to oscillators using a single tuned circuit between grid and anode in the modified Colpitts circuit in which feedback is provided by the tube interelectrode capacitances. The optimum feedback conditions are deduced and the analysis is applied to several typical oscillators.

621.396.615.14:621.385.3 2452
Ultra-High-Frequency Triode Oscillator using a Series-Tuned Circuit—J. M. Pettit. (Proc. I. R. E., vol. 38, pp. 633–635; June, 1950.) Analysis of a modified Colpitts circuit for a particular triode gives, as a function of frequency, the series resistive and reactive components of the two-terminal impedance looking at the grid-anode terminals. Use of a series-tuned external circuit instead of the usual parallel-tuned circuit permits oscillation above the self-resonance frequency of the triode, provided transit-time effects are unimportant.

621.396.615.17 2453
Linear Sweep Generation—D. Sayre. (*Electronics*, vol. 23, pp. 171–175; July, 1950.) Linear positive or negative sweeps are produced by charging an electronically switched capacitor through a constant-current triode. The linearity obtained is between that of the bootstrap circuit and that of the Miller feedback circuit.

621.396.615.17 2454
Cathode-Coupled Multivibrator Operation—K. Glegg. (Proc. I. R. E., vol. 38, pp. 655–656; June, 1950.) An approximate analysis results in an expression for the period of the output pulse in terms of the values of the circuit elements. A series expansion shows that the output period is nearly a linear function of one of the circuit voltages.

621.396.615.17:621.397.62 2455
A Hard-Valve Time-Base—C. H. Banthorpe. (*Electronic Eng.* (London), vol. 22, p. 339; August, 1950.) Full circuit details and description of the action of a linear frame timebase in which the start of the scan is triggered and which is unaffected by line pulses which may be introduced owing to imperfect line/frame synchronization separation.

621.396.645 2456
Design of Cathode-Coupled Amplifiers—S. G. F. Ross. (*Wireless Eng.*, vol. 27, No. 322, pp. 212–215; July, 1950.) Previously published theoretical work on the subject is reviewed, and a new rigorous analysis is presented. Experimental results indicate that this analysis can be relied on within the accuracy of published tube data.

621.396.645 2457
Modern Methods of Power Amplification—M. Strutt. (*Bull. Schweiz. Elektrotech. Ver.*, vol. 41, pp. 479–484; June 10, 1950. In German.) Definition of power gain and discussion of its application in tube, semiconductor, magnetic, and amplidyne amplifiers.

621.396.645:621.316.722.078.3 2458
Stabilization of Wide-Band Direct-Current Amplifiers for Zero and Gain—E. A. Goldberg. (*RCA Rev.*, vol. 11, pp. 296–300; June, 1950.) Stabilization is obtained through the application of a mechanical chopper to detect any zero offset error voltage. The circuit is such that the stabilization device does not alter the high-frequency response characteristics of the amplifier. Primary application has been in the field of analogue electronic computers.

621.396.645:621.385.3:621.315.592† 2459
High-Frequency Operation of Transistors—C. B. Brown. (*Electronics*, vol. 23, pp. 81–83; July, 1950.) Dispersion in transit-time values, resulting from differences in lengths of flow paths between emitted and collector, is the primary cause of loss of high-frequency response. Magnetic bias of appropriate sign applied at right angles to the plane containing the axes of the collector and emitter electrodes reduces transit time and dispersion, with consequent increase of frequency range. The circuit of a 23-Mc amplifier for testing the high-frequency response is described, the transistor being biased by a field of strength 16,000 lines per square inch. A gain of 8 is obtained between 1,000-Ω input and output. Time and temperature stability of the transistors are discussed briefly. See also 2089 of September.

621.396.645:621.396.61 2460
Maximum Tank Voltage in Class-C Amplifiers—L. E. Dwork. (Proc. I. R. E., vol. 38, pp. 637–644; June, 1950.) Theoretical considerations are presented to justify the frequent appearance in class-C amplifiers of radio-frequency anode voltages which are greater than the dc anode voltage. A method is developed for predicting the magnitude of this radio-frequency voltage under any given set of conditions. The method is verified experimentally for a particular case.

621.396.645.37 2461
Intermediate-Frequency Gain Stabilization with Inverse Feedback—G. F. Montgomery. (Proc. I. R. E., vol. 38, pp. 662–667; June, 1950.) "Improvement in gain stability is related to the number of cascaded stages, the stage gain, and the magnitude of the feedback. A circuit is described which uses feedback over a pair of cascaded stages. Generalized selectivity curves for this feedback couple are shown, and the design procedure is outlined."

621.396.645.37 2462
General Formulae for Feedback Amplifiers—F. Job. (*Ann. Télécommun.*, vol. 3, pp. 436–444; December, 1948.) The general theory of linear networks including amplifying tubes, has been given in a very complete manner by Bode (3381 of 1948). A simple demonstration is here given of the most important formulas of Bode's theory. Examples and applications illustrate the extreme generality of the formulas.

621.396.645.37 2463
More about Positive Feedback—T. Roddam. (*Wireless World*, vol. 56, pp. 242–244; July, 1950.) When applied to those stages of an amplifier in which distortion is not inherently large, positive feedback permits a considerable increase in over-all gain with little increase in over-all distortion. The negative feedback applied to the whole amplifier may then be increased, reducing the over-all gain to the original level with a considerable reduction in the over-all distortion. Positive feedback increases phase shift, making it more difficult to keep the over-all system stable. Its use should therefore be confined to the frequency range over which it is particularly needed.

621.396.645.371 2464
Negative-Feedback Amplifiers—J. E. Flood. (*Wireless Eng.*, vol. 27, pp. 201–209; July, 1950.) Negative-feedback amplifiers designed to obtain a flat response curve may give an oscillatory response to transients if the feedback exceeds a certain amount. The present analysis shows that by suitably adjusting the time constants for the various stages and by modifying the feedback path, critical damping with resulting absence of overshoot for transients can be attained together with a flat response curve throughout the frequency band for both 2-stage and 3-stage amplifiers. Experimental results are in fair agreement with calculated values.

519.242:621.3 2465
The Extrapolation, Interpolation and Smoothing of Stationary Time Series with Engineering Applications [Book Review]—N. Wiener. Publishers: John Wiley & Sons, Inc. New York, N. Y. 1949, 163 pp., $4.00. (*Jour. Frank. Inst.*, vol. 249, p. 259; March, 1950.) This was issued as a classified report during the war. "The theory set forth is widely used and has guided modern thinking in filter and predictor theory."

621.314.3 2466
The Magnetic Amplifier [Book Review]—J. H. Reyner. Publishers: Stuart & Richards, London, England. 119 pp., 15s. (*Wireless Eng.*, vol. 27, p. 216; July, 1950.) The presentation is mainly descriptive; where mathematical reasoning is used it is expressed as far as possible in terms of elementary ac theory.

GENERAL PHYSICS

53.081+621.3.081 2467
The Introduction of the Giorgi System of Units—H. König, N. Krondl, and M. Landolt. (*Tech. Mitt. Schweiz. Telegr.—Telephverw.*, vol. 28, pp. 207–208; May 1, 1950. In French and German.) Corrections to paper noted in 1125 of June, for which the UDC number should be as above.

53.081.5 2468
On the Theory of the Dimensions of Physical Quantities—M. Landolt. (*Bull. Schweiz. Elektrotech. Ver.*, vol. 41, pp. 473–479; June 10, 1950. In German.) Review of different definitions of dimension and proposal of a new variant.

535.34-1:[546.28+546.289 2469
Theory of Infra-Red Absorption in Silicon and Germanium—J. Bardeen. (*Phys. Rev.*, vol. 79, p. 216; July 1, 1950.) Summary of American Physical Society paper.

535.37 2470
Scattering and Absorption of Luminescence Light in Polycrystalline Luminescent Layers under Excitation by High-Energy Quantic and Corpuscular Rays—I. Broser. (*Ann. Phys.* (Lpz.), vol. 5, pp. 401–416; January 16, 1950.) Experiments with a ZnS-Cu phosphor, using α particles, γ- and X rays for excitation, indicate that the scattering and absorption factors for the luminescence light are independent of the type of excitation. Measurements of the effect of grain size in the phosphor layer on the scattering and absorption are described and discussed.

535.42 2471
Diffraction by a Plane Screen—E. T. Copson. (*Proc. Roy. Soc. A*, vol. 202, pp. 277–284; July 7, 1950.) Boundary conditions involved when solving the problem of diffraction by a perfectly conducting plane screen using the integral-equation method are analyzed. Theorems given in a previous paper (*ibid.* vol. 186, pp. 100–118; 1946) are recast in a form which meets the criticism of Bouwkamp (2772 of 1948), and the integral-equation method is applied to the problem of diffraction by a half-plane. The form of the arbitrary function of integration is adjusted to limit the order of the singularity at the boundary.

535.42 2472
The Edge Condition in the Theory of the Diffraction of Electromagnetic Waves at Perfectly Conducting Plane Screens—J. Meixner. (*Ann. Phys.* (Lpz.), vol. 6, pp. 2–9; September 19, 1949.) The edge condition is based on the physically plausible requirement that the electromagnetic energy density in the neighborhood of the edge be integrable, i.e., that the field energy in each finite volume be finite. A particularly simple mathematical expression of the edge condition is given by the Debye electromagnetic field potentials. In the diffraction of sound waves at the edge of a screen the electromagnetic edge condition corresponds to the requirement that at the edge the sound pressure must remain finite.

535.42 2473
Energy Flow in the Near Field of a Diffracting Edge—W. Braunbek. (*Ann. Phys.* (Lpz.), vol. 6, pp. 53–58; September 19, 1949.) For the region near the edge (distance λ) the field can be found very simply by application of Sommerfeld's solution for diffraction at a semi-infinite plane. For the special case of perpendicular incidence on an infinitely thin, perfectly conducting screen, the electric vector being parallel to the diffracting edge, the phase surfaces of the field near the edge are calculated and also the surfaces of energy flow, which to a first approximation are confocal parabolic cylinders with the edge as focal line.

537.226.001.11:546.431.82-3 2474
Electronic Theory of Ferroelectrics—E. T. Jaynes and E. P. Wigner. (*Phys. Rev.*, vol. 79, pp. 213–214; July 1, 1950.) Summary of American Physical Society paper.

537.311.1:537.311.32/.33 2475
Conduction Electrons in Non-Metallic Solids—H. Fröhlich. (*Research* (London), vol. 3, pp. 202–207; May, 1950.) A review of the action of electrons in substances which in their normal state are nonconductors, including consideration of various processes where the conduction-electron concentration varies with time. The problems of polarization, self-trapping, and mean free path in crystals are discussed, assuming the number of interelectron collisions to be negligible. The increase of conductivity produced by a strong field and the decay after excitation by an ionizing radiation should be governed by the same time constant.

537.311.1:621.315.592† 2476
Potential Fluctuations in Homogeneous Semiconductors—H. M. James and G. W. Lehman. (*Phys. Rev.*, vol. 79, p. 216; July 1, 1950.) Summary of American Physical Society paper.

537.311.1:621.315.592 2477
On the Theory of Noise in Semiconductors—R. L. Petritz and A. J. F. Siegert. (*Phys. Rev.*, vol. 79, pp. 215–216; July 1, 1950.) Summary of American Physical Society paper.

537.311.3+621.315.592† 2478
Electrical Conductivity—Roulaud. (*See* 2543.)

537.312.8 2479
Effect of Magnetic Fields on Conduction: "Tube Integrals"—W. Shockley. (*Phys. Rev.*, vol. 79, pp. 191–192; July 1, 1950.) The mathematical expression for the effect of a magnetic field on conduction in a material can be reduced to integrals by using "tubes," which are specified regions in the Brillouin zone. The case of simple closed tubes, which are of interest for the case of semiconductors, is discussed. An expression for the total electron current is derived, evaluation of which may prove a means of determining the energy-surface parameters for Ge from magnetoresistance measurements on single crystals.

537.32+537.312.8 2480
Theory of Magnetic Resistance Effects in Metals—M. Kohler. (*Ann. Phys.* (Lpz.), vol. 6, pp. 18–38; September 19, 1949.) Approximate formulas are derived for the change of the electrical and the thermal resistance of a metal when a transverse magnetic field is applied. Theoretical results are compared with experimental values.

537.525.5:538.6 2481
Supersonic Wind at Low Pressures Produced by Arc in Magnetic Field—H. C. Early and W. G. Dow. (*Phys. Rev.*, vol. 79, p. 186; July 1, 1950.) Discussion of the effects of a transverse magnetic field on a low-pressure gas discharge. When the arc is in a large unconfined region, e.g., a vacuum chamber of 1 cubic meter, wind effects are observed. Power input and current density become many times larger; gas temperature is generally low because of the cooling effect of the wind; ion mobility is little reduced; equipotentials occur at skewed positions. In the case of an arc between a copper cylinder and surrounding ring, an axial magnetic field causes the arc to revolve at a speed of about 17,000 rps under particular conditions. The air inside the cylinder also revolves, but at a slower rate, its speed being estimated at 3,000–4,500 mph.

537.562 2482
Dispersive Power and Natural Oscillation of an Ionized Gas—G. Burkhardt. (*Ann. Phys.* (Lpz.), vol. 5, pp. 373–380; January 16, 1950.) The dispersion theory for an ionized gas can be derived from the Lorentz theory for a neutral gas if the natural frequency of the polarization electrons is assumed equal to a certain limiting frequency, which is the natural frequency of the oscillating plasma. This theory indicates unconditionally that the Lorentz polarization term should not appear in the dispersion formula for a plasma. See also 738 of 1944 (Darwin).

538.114 2483
Time Decrease of Magnetic Permeability in Alnico—R. Street and J. C. Woolley. (*Proc. Phys. Soc.*, vol. 63, pp. 509–519; July 1, 1950.)

538.221:538.569.4.029.64 2484
Ferromagnetic Resonance in Manganese Ferrite and the Theory of the Ferrites—C. Guillaud, W. A. Yager, F. R. Merritt, and C. Kittel. (*Phys. Rev.*, vol. 79, p. 181; July 1, 1950.) Discussion of observations of resonance absorption at 24,164 Mc in polycrystalline $Fe(MnFe)O_4$ at room temperature.

538.221:538.569.4.029.64 2485
Ferromagnetic Resonance in Single Crystals of Nickel Ferrite—W. A. Yager, J. K. Galt, F. R. Merritt, E. A. Wood, and B. T. Matthias. (*Phys. Rev.*, vol. 79, p. 214; July 1, 1950.) Summary of American Physical Society paper.

538.221:539.23 2486
Thin Ferromagnetic Films—M. J. Klein and R. S. Smith. (*Phys. Rev.*, vol. 79, p. 214; July 1, 1950.) Summary of American Physical Society paper.

538.221.001.11 2487
Theory of Magnetic Dispersion in Ferrites—C. Kittel. (*Phys. Rev.*, vol. 79, p. 214; July 1, 1950.) Summary of American Physical Society paper.

538.521:517.948.32 2488
The Induction of Electric Currents in a Uniform Circular Disk—A. A. Ashour. (*Quart. Jour. Mech. Appl. Math.*, vol. 3, Pt. 1, pp. 119–128; March, 1950). By regarding a uniform disk, or any symmetrically conducting surface of revolution, as composed of an infinite number of coaxial annular circuits, the determination of the electric currents induced by an external field is reduced to the solution of a Fredholm integral equation. Two methods of solving this equation are described.

538.566+[537.226.2:546.217 2489
The Velocity of Electromagnetic Waves and the Dielectric Constant of Dry Air—J. V. Hughes. (*Phys. Rev.*, vol. 79, p. 222; July 1, 1950.) Summary of American Physical Society paper. To obtain agreement between the commonly accepted value of the velocity of light (299,775 km) and the mean (299,790 km) of six recent determinations of the velocity of radio waves, it would be necessary to use a value of 1.00048 for the dielectric constant k of air when correcting measurements in air to vacuo. Values of k obtained by various experimenters, using frequencies from those of light waves down to 1 Mc, range from 1.000572 to 1.00060. The source of the discrepancy

must consequently be sought elsewhere. See also 1751 of August (Essen).

538.566:537.562 2490
Electro-Magneto-Ionic Optics—V. A. Bailey. (*Jour. Roy. Soc. NSW*, vol. 82, Pt. 2, pp. 107–113; 1948.) Theoretical study of the modes of propagation of electric waves in a medium composed of electrons, positive ions and molecules (or atoms) in the presence of static electric and magnetic fields. The solution for plane waves is found in a general form which specifies the refractive indices and attenuation coefficients (positive or negative) as a function of frequency. The theory has application to the ionosphere, to the solar atmosphere and to discharge tubes. A deduction from it is that certain waves will grow as they progress in space or with passage of time; this suggests a possible explanation of the origin of stellar, solar, and ionospheric noise. See also 2785 of 1949.

538.632:621.315.592† 2491
Theoretical Hall-Coefficient Expressions for Impurity Semiconductors—V. A. Johnson and K. Lark-Horovitz. (*Phys. Rev.*, vol. 79, pp. 176–177; July 1, 1950.)

538.652:621.317.4 2492
A New Method of Measuring Magnetostriction. Application to the Ferrite of Cobalt—L. Weil, M. Gallay, and P. Poensin. (*Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 224–226; July 17, 1950.) For measurements under widely different conditions of temperature or material, a method using a simple strain gauge of the resistance type has particular advantages. Accuracy of measurement is within about 1 per cent, but the method can be applied to material of any shape and its simplicity recommends it for investigations in connection with industrial equipment. Results obtained for Co ferrite, made highly magnetostrictive by cooling for 18 hours after heat treatment at 1,200° C, are presented.

GEOPHYSICAL AND EXTRATERRESTRIAL PHENOMENA

061.3:[55+621.396.11 2493
Summary of Proceedings of Australian National Committee of Radio Science, URSI, Sydney, January 16–20, 1950.—(*Jour. Geophys. Res.*, vol. 55, pp. 191–210; June, 1950.) Summaries are given of 33 papers presented at the conference.

523.746 2494
Final Relative Sunspot-Numbers for 1949—M. Waldmeier. (*Jour. Geophys. Res.*, vol. 55, pp. 211–213; June, 1950.)

523.746 2495
Provisional Sunspot-Numbers for January to March, 1950—M. Waldmeier. (*Jour. Geophys. Res.*, vol. 55, p. 217; June, 1950.)

537.562:538.566 2496
Electro-Magneto-Ionic Optics.—Bailey. (*See* 2490.)

538.711 2497
Measurement of the Earth's Magnetic Field at High Altitudes at White Sands, New Mexico—E. Maple, W. A. Bowen and S. F. Singer. (*Jour. Geophys. Res.*, vol. 55, pp. 115–126; June, 1950.) Measurements by means of a total-field magnetometer fitted in an Aerobee rocket showed that at a height of 368,000 ft the decrease in field strength was 28 milligauss. The results for the whole flight agree to within 2 milligauss with calculations based on the dipole theory. No evidence of the existence of magnetic fields due to current sheets was obtained.

550.384 2498
International Data on Magnetic Disturbances, Fourth Quarter, 1949—J. Bartels and J. Veldkamp. (*Jour. Geophys. Res.*, vol. 55, pp. 214–216; June, 1950.)

550.384 2499
Cheltenham Three-Hour-Range Indices K for January to March, 1950—R. R. Bodle. (*Jour. Geophys. Res.*, vol. 55, p. 217; June, 1950.)

550.385 2500
Principal Magnetic Storms [Oct. 1949–March 1950]—(*Jour. Geophys. Res.*, vol. 55, pp. 218–220; June, 1950.)

550.385:551.594.52 2501
Development of a Magnetic Storm: The Southward Shifting of the Auroral Zone—T. Nagata. (*Jour. Geophys. Res.*, vol. 55, pp. 127–142; June, 1950.) "The southward extension with the increased intensity of disturbance in the northern auroral zone is derived hour by hour for the geomagnetic storm of April 30, 1933."

551.510.535 2502
An Approach to the Approximate Solution of the Ionosphere Absorption Problem—J. E. Hacke, Jr. (PROC. I. R. E., vol. 38, pp. 683–684; June, 1950.) Discussion on 3115 of 1948.

551.510.535 2503
The Diurnal Variation of the Vertical-Incidence Ionospheric Absorption at 150 kc/s—A. H. Benner. (PROC. I. R. E., vol. 38, p. 685; June, 1950.) Measurements of the virtual height of the layer and the relative amplitudes of ground pulse and first and second echoes enable the diurnal variation of $|\log \rho|$ to be plotted. Marked absorption-curve transitions occur at ground sunrise and sunset. When $\log |\log \rho|$ is plotted against $\log \cos \chi$ (where χ is the sun's zenith angle), approximately straight lines are obtained for the morning and afternoon periods, the slopes being 0.675 and 0.76, respectively.

551.510.535 2504
Travelling Disturbances in the Ionosphere—G. H. Munro. (*Proc. Roy. Soc. A*, vol. 202, pp. 208–223; July 7, 1950.) The motion of disturbances of the vertical distribution of F region ionization density has been investigated by means of synchronized signals from three spaced common-frequency pulse transmitters. $P'f$ data from Sydney, Brisbane, and Canberra show that the larger disturbances can travel 900 km without major change in type or in velocity, which ranges from 5 to 10 km per minute. Quasicyclical variations of ionization density with periods of 10–60 minute are found which show a progressive phase lag with decrease in height. The magnitude of the disturbance appears, in general, to decrease with decrease of the period. The mean direction of horizontal motion is about 20° E of N in winter and about 110° E of N in summer, the changes in direction of occurring rather abruptly near the equinoxes. Some of the effects observed by Wells, Watts, and George (3279 of 1946) may be due to disturbances of the type here considered.

551.510.535:[537.568+533.15 2505
Diffusion in the Ionosphere—M. H. Johnson and E. O. Hulburt. (*Phys. Rev.*, vol. 79, p. 222; July 1, 1950.) Summary of American Physical Society paper. The nonlinear differential equations relating the ionic density in the atmosphere to the rate of ionization by solar radiation and the loss of ions by recombination and diffusion, have been integrated on the Naval Research Laboratory computer and ionic-density curves, have been obtained. Diffusion broadens the curve, thereby changing the relations between real and virtual heights, and shifts its maximum downwards, modifying the dependence of the height of the ionized region on the solar zenith angle.

551.510.535:621.396.11 2506
Ionosphere Observations at 50 kc/s—J. N. Brown and J. M. Watts. (*Jour. Geophys. Res.*, vol. 55, pp. 179–181; June, 1950.) A high-power pulse transmitter giving an output of about 200 kw into a large loop antenna has been used at Sterling, Va., by the CRPL for measurements of the height of the reflecting layer for signals of frequency about 50 kc. The record for one day is reproduced, the virtual height of the reflecting layer being about 80 km. Height changes are not great, and morning and evening peaks do not seem to occur every day. Double echoes are sometimes observed.

551.594.21 2507
The Free Electrical Charge on Precipitation Inside an Active Thunderstorm—R. Gunn. (*Jour. Geophys. Res.*, vol. 55, pp. 171–178; June, 1950.) An induction method was used in an aircraft to measure the charges on individual droplets. Results are given for seven different levels in the cloud. With the freezing level at 14,000 ft, the maximum electrification occurred at about 7,500 ft and at a temperature of 10° C, the electric field strength at the surface of the droplets being a large fraction of the dielectric strength of air.

551.594.6 2508
The Fine Structure of Atmospherics: Ionospheric and Meteorological Applications of Type 4—R. Rivault. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1846–1847; May 22, 1950.) Daily recordings made at Poitiers during October, 1949, enable the investigation of type 4 atmospherics to be carried further (see 2519, 2520, and 2521 of 1948). Oscillograms corresponding to sources in moving air masses exhibit rounded peaks, whereas those corresponding to sources in stationary air masses show a true short pulse clearly separated from its echoes. The normal value of h, the height of the reflecting layer, is found to be 75 km, with variations up to 12 km during the period 1700–2100. Uniformity of observations of atmospherics from different directions leads to the view that the D region is homogeneous and parallel to the ground. On days when large time variations were observed magnetic storms were also recorded.

521.03 2509
Some Recent Researches in Solar Physics [Book Review]—F. Hoyle. Publishers: Cambridge University Press, London, England, and New York, N. Y., 1949, 134 pp., 12s. 6d. or $3.00. (*Science*, vol. 111, p. 414; April 21, 1950. *Quart. J. R. Met. Soc.*, vol. 76, p. 112; January, 1950.) "This book in the new series of Cambridge Monographs on Physics deals mainly with Hoyle's own recent work on the corona and chromosphere . . ." Terrestrial phenomena directly influenced by the sun, and the emission of radio waves from the sun, are dealt with.

LOCATION AND AIDS TO NAVIGATION

527.5:518.12 2510
The Computation of Great-Circle Bearings and Distances—G. Millington. (*Marconi Rev.*, vol. 13, pp. 89–101; 3rd quarter, 1950.) To avoid the necessity of using seven-figure logarithms to obtain four-figure accuracy when using the standard formulas, trigonometrical transformations are used, a simplified rule of signs is adopted and the resulting computation is presented in a standard tabular form.

621.396.9:551.578.1 2511
The Effect of Rain on Marine Radar Echoes—S. E. Barden. (*Marconi Rev.*, vol.

13, pp. 102–109; 3rd quarter, 1950.) The failure of PPI radar gear to detect targets when heavy rain is falling is analyzed and curves are given to assist operators to determine the approximate ranges within which target echoes should be obtainable under different conditions.

621.396.933 2512
A Simple Localizer—J. W. Alexander. (*Tijdschr. ned. Radiogenoot.*, vol. 14, pp. 119–133; July, 1949. Discussion, p. 134.) Critical discussion of the SCS51 instrument landing system, and description of the Schiphol installation, in which an electrical method of modulation is used. It is suggested that PICAO specifications be redrafted to give a course angle of 2.5° on either side of the true direction.

621.396.933 2513
ILS-2 Instrument Landing Equipment—R. A. Hampshire and B. V. Thompson. (*Elec. Commun.*, vol. 27, pp. 112–122; June, 1950.) A description of equipment which complies with PICAO requirements. The localizer, operating in the 108–112 Mc band, provides a beam along the runway and extending for at least 25 nautical miles, the two halves of the beam being modulated at 90 cps and 150 cps, respectively. The glide path, adjustable from 2° to 4°, operates in the 329–335 Mc band, modulating frequencies of 90 cps and 150 cps being again used. Three marker beacons, all on 75 Mc but with different modulation frequencies, give distance indication. Speech transmission from the localizer can be made without interrupting its course modulation. Monitor and control circuits have been specially designed to ensure reliability of operation.

621.396.933 2514
Characteristics and Adjustment of 335-Mc/s Equisignal Glide Slopes—S. Pickles. (*Elec. Commun.*, vol. 27, pp. 140–151; June, 1950.) Different factors influencing the radiated signals are examined with a view to improved performance and increased safety of operation.

621.396.933 2515
A Source of Error in Radio Phase-Measuring Systems—R. Bateman, E. F. Florman, and A. Tait. (Proc. I. R. E., vol. 38, pp. 612–614; June, 1950.) When a mobile transmitter was moved between two points over different particular paths around reradiating structures, the measured total phase changes differed by $2\pi n$ radians, where n is an integer. If reradiation from a reflector is of the same order of magnitude as the radiation from an antenna, analysis of the resultant field shows that singularities occur and each traverse of a closed path around a point of singularity gives a total phase change of 360°.

621.396.933 2516
An Analysis of Some Anomalous Properties of Equiphase Contours—G. A. Hufford. (Proc. I. R. E., vol. 38, pp. 614–618; June, 1950.) Further investigation of cases, of importance in radio surveying or navigation systems, where the phase at certain points may be multivalued (see 2515 above).

621.396.9 2517
British Standard 204:1943. Supplement No. 4, Glossary of Terms used in Radar. [Book Notice]—Publishers: British Standards Institution, London, England, 1950, 8 pp., 2s. (*Brit. Stand. Inst. Mon. Inform. Sheet*, p. 1; June, 1950.)

MATERIALS AND SUBSIDIARY TECHNIQUES

531.788 2518
A Philips-Type Ionization Gauge for Measuring of Vacuum from 10^{-7} to 10^{-1} mm of Mercury—E. C. Evans and K. E. Burmaster. (Proc. I. R. E., vol. 38, pp. 651–654; June, 1950.) Description and detailed diagrams showing the construction of a modified gauge. For an account of an instrument giving similar performance see 1423 of July (Penning & Nienhuis).

535.215:546.23 2519
Photoconductivity in Amorphous Selenium—P. K. Weimer. (*Phys. Rev.*, vol. 79, p. 171; July 1, 1950.) Experimental results are given which indicate that the red amorphous form of Se is photoconductive and possesses properties very different from those of the common metallic form and of the red monoclinic-crystal form.

535.215:546.23 2520
Electron-Bombardment-Induced Conductivity in Selenium—L. Pensak. (*Phys. Rev.*, vol. 79, pp. 171–172; July 1, 1950.) An account of measurements on evaporated films of red amorphous Se.

535.37:546.42.221 2521
Two Infra-Red-Sensitive SrS Phosphors with Zn Dominant Activator—D. S. Bersis (*Jour. Opt. Soc. Amer.*, vol. 40, p. 335; May, 1950.)

535.37:546.46.45.284 2522
Variation of Emission Spectrum of Manganese-Activated Zinc Beryllium Silicate with Decay Time—J. H. Schulman, C. C. Klick, and R. J. Ginther. (*Jour. Opt. Soc. Amer.*, vol. 40, pp. 337–338; May, 1950.)

535.37:546.47-31 2523
Sulphide in Zinc-Oxide Luminophors—S. M. Thomsen. (*Jour. Chem. Phys.*, vol. 18, p. 770; May, 1950.) The presence of a small amount of ZnS, rather than free Zn, is responsible for the green luminescence of ZnO phosphors fired in hydrogen.

535.37:546.472.21 2524
Comparison of the Luminescence-Extinction Effects of Nickel and Cobalt on Zinc Sulphide—J. Saddy and N. Arpiarian. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1948–1950; May 31, 1950.)

537.228.1:621.39 2525
Piezoelectric Elements in Telecommunications Technique—J. J. Vormer. (*Tijdschr. ned. Radiogenoot.*, vol. 15, pp. 93–127; May, 1950. In Dutch, with English summary.) Review of the physical and electrical properties of crystals of different cuts commonly used, with a tabular summary.

537.533.8 2526
Secondary-Electron Emission from Metal Mixtures—H. Salow. (*Ann. Phys.* (Lpz.), vol. 5, pp. 417–428; January 16, 1950.) Mixtures produced by simultaneous evaporation of either Ag or Cu and either Mg, Al, or Be were found to have secondary-emission properties similar to those of alloys of the two components formed by melting. After a forming process in the presence of small quantities of O_2, secondary-emission factors of about 10 were obtained with 500-v primary electrons. The Cu/Mg metal mixture is easy to form, electrically and thermally stable, insensitive to dry CO_2-free gases, and very suitable for coating secondary-emission cathodes.

538.221 2527
The Magnetic Properties of Stainless Steels—W. A. Stein. (*Trans. AIEE*, vol. 67, Part II, pp. 1534–1537; 1948.) A comprehensive account of the magnetic properties of five stainless steels. Steel may contain up to 3 per cent Ni without serious detrimental effect on its magnetic properties; the inclusion of over 8 per cent Ni renders the steel nonmagnetic.

538.221 2528
Magnetic Properties of Ferrites—C. Guilland. (*J. Rech. Centre Nat. Rech. Sci.*, No. 12, pp. 113–122; 1950.) Experimental study and discussion of the saturation magnetization of simple and mixed ferrites. See also 1166 and 1171 of June (Néel).

538.221 2529
Properties of Single Crystals of Nickel Ferrite—J. K. Galt, B. T. Matthias, and J. P. Remeika. (*Phys. Rev.*, vol. 79, p. 214; July 1, 1950.) Summary of American Physical Society paper.

538.221 2530
Relation between the Crystalline Structure and the Magnetic Properties of Mixed Ferrites of Nickel and Zinc—M. Sage and C. Guillaud. (*Compt. Rend. Acad. Sci.*, (Paris), vol. 230, pp. 1751–1753; May 15, 1950.)

538.221 2531
Thermal Variation of the Spontaneous Magnetization of Ferrites of Nickel, Cobalt, Iron and Manganese—R. Pauthenet. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1842–1843; May 22, 1950.)

538.221 2532
A New Series of Ferromagnetic Substances: Ferrites of Rare Earths—H. Forestier and G. Guiot-Guillain. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1844–1845; May 22, 1950.)

538.221:538.65:536.413 2533
Length Anomaly in Ferrites—L. Weil. (*Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 122–124; July 10, 1950.) Experimental study of the expansion coefficients of magnetized ferrites when slowly heated and then cooled.

538.221:538.653.11 2534
Effect of Tension on Magnetic Properties in Iron-Cobalt—H. H. Plotkin and J. E. Goldman. (*Phys. Rev.*, vol. 79, p. 215; July 1, 1950.) Summary of American Physical Society paper.

539.23:537.311.31 2535
On the Law of Variation of the Electrical Resistance of Very Thin Deposited Metal Films as a Function of the Applied Potential—B. Vodar and N. Mostovetch. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 2008–2010; June 5, 1950.) Previous experimental results (1701 of August) indicate that log (R_v/R_o) is a linear function of the applied potential except at very low potentials, R_v being the resistance at potential V. The change in conductivity is attributed to the lowering of the potential barrier between adjacent particles of the layer by the applied field. This effect is discussed in relation to a formula established by Schottky for the case of thermionic emission.

539.23:546.57:621.314.6 2536
On Certain Detector Properties of Thin Silver Films—A. Blanc-Lapierre and M. Perrot. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1749–1751; May 15, 1950.) Measurements were made of the variation of current with applied direct and alternating voltages and with frequency. Appreciable detection effects were observed when the total power dissipated in the film was $\not> 1$ μw. See also 2242 of October.

539.23:546.81 2537
The Structure of Lead Sulfide Films—J. Doughty, K. Lark-Horovitz, L. M. Roth, and B. Shapiro. (*Phys. Rev.*, vol. 79, p. 203; July 1, 1950.) Summary of American Physical Society paper. Investigation of films of PbS, PbSe, and PbTe by X rays, electron diffraction, and electron microscope.

539.23:546.81:535.215 2538
Temperature Variation of Properties of Photo-Sensitive Lead Sulfide Films—R. H.

McFee. (*Phys. Rev.*, vol. 79, p. 203; July 1, 1950.) Summary of American Physical Society paper. Measurements made of dark current, photo current and noise as functions of temperature between 90° K and 300° K.

546.841-3:537.311.3 2539

Electrical Resistance of Thoria—W. E. Danforth and F. H. Morgan. (*Phys. Rev.*, vol. 79, pp. 142–144; July 1, 1950.) Measurements were made in vacuo at temperatures up to 2073° K. On activation, values of 1 Ω-cm at 1,900° and 10Ω-cm at 1,000° were obtained. Activation energies determined from pulse measurements were between 3.2 v and 0.58 v. The density of impurity centers was calculated as 10^{18} per cm^3 and was found independent of the degree of activation by current. This result does not agree with the hypothesis of the electrolytic origin of impurity centers.

546.92:537.323 2540

Effect of Heat Treatment on the Electrical Properties of Platinum—R. J. Corruccini. (*Phys. Rev.*, vol. 79, p. 202; July 1, 1950.) Summary of American Physical Society paper.

620.197:679.5 2541

Casting-Resin Techniques—J. Bayha. (*Electronics*, vol. 23, pp. 100–101; July, 1950.) Practical details are given about the materials and methods of preparation and use of NBS casting resin.

621.314.632 2542

Metallographic Study of Germanium Point-Contact Rectifiers—M. H. Dawson and B. H. Alexander. (*Phys. Rev.*, vol. 79, p. 217; July 1, 1950.) Summary of American Physical Society paper.

621.315.592†+537.311.3 2543

Electrical Conductivity—R. Roulaud. (*Rev. Gén. Élec.*, vol. 59, pp. 211–225; May, 1950.) In Part 1 different theories which have been proposed to account for the conductivity of solid bodies are discussed, particularly the contributions made by the quantum theory and wave mechanics. The classification of conductors according to electron activity is outlined and the potential barrier is discussed with reference to thermionic emission and the Schottky effect. Part 2 deals with semiconductors and discusses the effect of impurities on resistivity and also their physical characteristics and rectifying properties. Part 3 reviews the properties of Ge, particularly those made use of in the transistor.

621.315.592† 2544

Electrical Properties of Semiconductors with Macroscopic Discontinuities—J. C. M. Brentano and D. H. Davis. (*Phys. Rev.*, vol. 79, p. 216; July 1, 1950.) Summary of American Physical Society paper.

621.315.592†:546.28 2545

The Transition from Insulating to Metallic Behavior in Semiconducting Silicon—G. W. Castellan and F. Seitz. (*Phys. Rev.*, vol. 79, p. 216; July 1, 1950.) Summary of American Physical Society paper.

621.315.61.011.5:577.3 2546

The Determination of the Dielectric and Magnetic Properties of Inhomogeneous Dielectrics, Particularly Biological Substances, in the Decimetre-Wave Region—H. Schwan. (*Ann. Phys.* (Lpz.), vol. 5, pp. 253–310; January 16, 1950.) In three parts, dealing with (a) theory of resonance methods of determining complex resistance, (b) effect of the support at the end of the Lecher line used in such methods, (c) practical methods and appropriate formulas for determining the required constants of the materials tested.

621.315.61.011.5:621.317.3.029.63 2547

Decimetre-Wave Measurements of Temperature-Dependent Dielectric Properties of Insulating Materials—Kreft. (*See* 2559.)

621.315.612.4.011.5 2548

The Ferroelectric Properties of Certain Titanates and Zirconates of Bivalent Metals Having Perovskite Structure—G. A. Smolenski. (*Zh. Tekh. Fiz.*, vol. 20, pp. 137–148; 1950. In Russian.) An experimental as well as a theoretical investigation which shows that $CdTiO_3$, $PbTiO_3$, $PbZrO_3$, and the solid solutions (Ca, Pb)TiO_3 and (Sr,Pb)TiO_3 possess ferroelectric properties.

621.318.2:621.3.016.35 2549

Stabilized Permanent Magnets—P. P. Cioffi. (*Trans. AIEE*, vol. 67, Part II, pp. 1540–1543; 1948.) Permanent magnets are stabilized against forces tending to demagnetize them, by partial demagnetization. After stabilization the magnet operates on a secondary demagnetization curve. The derivation of this curve and its applications to magnet design problems are discussed.

621.396.822:539.23:621.315.616.9 2550

Random Noise in Dielectric Materials—R. F. Boyer. (*Jour. Appl. Phys.*, vol. 21, pp. 469–477; June, 1950.) An account is given of observations on fluctuating currents resulting from the application of direct-voltage gradients of 10–300 volts per mil to thin films of polar dielectrics containing moisture. The fluctuations have frequencies of 60–1,000 cps and last for some minutes, even persisting for a few seconds after removal of the voltage; after dying out they can usually be restored by reversing the voltage. The corresponding noise level is about 1,000 times higher than that of the circuit noise. The noise is the greater the more polar the polymer and the higher the moisture content of the material. Groups of ions rather than single ions are believed to act as the random charge carriers.

669.15:538.652 2551

Magnetostriction in Magnetic Alloys with Preferred Crystal Orientation—J. E. Goldman. (*Phys. Rev.*, vol. 79, p. 215; July 1, 1950.) Summary of American Pnysical Society paper.

MATHEMATICS

517.392 2552

Numerical Evaluation of Integrals of the Form $I=\int_{x_1}^{x_2} f(x)e^{i\phi(x)}dx$ and the Tabulation of the Function $Gi(z)=(1/\pi)\int_0^\infty Sin\ (uz+\frac{1}{3}u^3)du$—R. S. Scorer. (*Quart. Jour. Mech. Appl. Math.*, vol. 3, Part 1, pp. 107–112; March, 1950.)

517.564 2553

On the Fourier and Mellin Transforms of Inverse Bessel Functions—P. Barrucand. (*Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 102–104; July 10, 1950.) The transforms are applied to derive a formula by means of which many integrals can be developed in the form of series.

517.942:621.3.09 2554

The B.W.K. Approximation and Hill's Equation: Part 2—L. Brillouin. (*Quart. Appl. Math.*, vol. 7, pp. 363–380; January, 1950.) The BWK procedure, developed in connection with wave mechanics, is shown capable of yielding a good approximate solution for problems in many other fields, including the propagation of em waves along a transmission line whose properties vary from point to point. The validity of various approximations is examined.

517.942.82:517.522 2555

The Laplace Transformation and Summation Formulae—P. Barrucand. (*Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 20–22; July 3, 1950.) The Poisson, Hardy, and other summation formulas are derived by use of the Laplace transformation.

517.942.932 1556

Notes on the Solution of the Equation $y''-xy=f(x)$—J. C. P. Miller and Z. Mursi. (*Quart. Jour. Mech. Appl. Math.*, vol. 3, Part 1, pp. 113–118; March, 1950.)

681.142 2557

A Differential Analyzer for the Schrödinger Equation—R. L. Garwin. (*Rev. Sci. Instr.*, vol. 21, pp. 411–416; May, 1950.) A simple and rapid analyzer is discussed having an accuracy within about 0.5 per cent. Possible extensions of its application are mentioned.

681.142:621.389 2558

A Fast Multiplying Circuit—B. Chance, J. Busser, and F. C. Williams. (*Phys. Rev.*, vol. 79, p. 244; July 1, 1950.) Summary of American Physical Society paper. A development on the principle of the "quarter-square" multiplication method. A single parabolic characteristic is used to square the amplitudes of alternate half-cycles of an 82-kc square wave which represent respectively the sum and difference of the two input voltages A and B. The difference of the squares gives the desired product $4AB$. The circuit gives the product of the inputs every 12 μ seconds to within 1 per cent.

MEASUREMENTS AND TEST GEAR

621.317.3.029.63:621.315.61.011.5 2559

Decimetre-Wave Measurements of Temperature-Dependent Dielectric Properties of Insulating Materials—W. Kreft. (*Fernmeldetech. Z.*, vol. 3, pp. 203–211; June, 1950.) A resonance method is described for measuring loss angle and permittivity, using two tubular concentric lines short-circuited at one end and capacitively coupled. One line, with a coupled current-meter, serves as indicator of the changes in the other line when a test sample is inserted which completely fills a certain length of the space between inner and outer conductor. Measurements in the wavelength range 10–75 cm were made on various insulating materials at temperatures from 20° to 80°C. Results are given graphically and show that in general the loss angle increases very considerably with temperature, but the permittivity only slightly.

621.317.31 2560

Measurement of Current Intensity with an Ammeter of Apparent Resistance Zero—M. Matschinski. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1937–1939; May 31, 1950.) The principle is analogous to that of the potentiometer method of voltage measurement. An auxiliary voltage source and variable resistor in series are connected across the ammeter and adjusted so as to compensate for the voltage drop due to the resistance of the ammeter. Applications of the method in the measurement of very weak currents are indicated.

621.317.32 2561

Negative-Feedback D.C. Amplifier with Grid-Controlled Valve Converter—W. Geyger. (*Arch. Tech.* (Messen), No. 172, p. T57; May, 1950.) The magnetic amplifier of the instrument described in 1450 of July is replaced by a differential double-triode circuit which acts as an anode-modulated converter. Voltage range becomes approximately 0.1–1 v with input resistance 1–100 MΩ.

621.317.332 2562

Resistance Measurement at High Frequencies by means of Lossless Quadripoles—A. Egger and H. H. Meinke. (*Funk. und Ton*, vol. 4, pp. 233–238; May, 1950.) Two equal, lossless, and symmetrical 4-terminal networks

are connected in series and terminated with the resistor R_1 to be measured. Measurements are made of the voltages across the two ends of the whole network and across the junction of the two networks. From these voltages and the known characteristic impedances and phase constants of the networks, R_1 may be determined. The method is suitable for the measurement of resistors of 20–300 Ω in the frequency range 30–100 Mc, but its applications are not restricted to this range.

621.317.335.3†+621.317.374 2563

Measurement of the Specific Inductive Capacity and Loss Angle of Dielectrics between 50 kc/s and 100 Mc/s—J. Jourdan. (*Onde Élec.*, vol. 30, pp. 285–292; June, 1950.) Description of a Marconi high-precision instrument, using the reactance-variation method, for industrial use. A Hartley oscillator is loosely coupled to the measurement circuit, which comprises a coupling inductor and two capacitors C_1 and C_2 in parallel across it. The dielectric material is placed between the plates of C_1 which has a micrometer adjustment; C_2 is tuned for resonance, as indicated on the 50-cm scale of a mirror galvanometer in the circuit of a square-law tube voltmeter; C_2 is then adjusted to give equal deflections on either side of resonance; the procedure is repeated without the dielectric. Loss angle and dielectric constant (and also resistance and capacitance) may be simply calculated from the formula given. The theory of the method of measurement and accuracy of the instrument are discussed.

621.317.335.3.029.64† 2564

Recording Microwave Refractometer—(*Electronics*, vol. 23, pp. 120, 122; July, 1950.) The difference in resonance frequency produced by introducing a sample of a substance under test into one of two otherwise identical cavity resonators is a measure of the dielectric constant of the sample. The equipment described, which was developed at the National Bureau of Standards, is sensitive to Q changes in the test cavity, hence its use is restricted to testing samples of low-loss materials. Possible application to the measurement of atmospheric refractive index at frequencies above 30 Mc is considered.

621.317.335.3.029.64†:546.217 2565

Apparatus for Recording Fluctuations in the Refractive Index of the Atmosphere at 3.2 Centimeters Wave-Length—C. M. Crain. (*Rev. Sci. Instr.*, vol. 21, pp. 456–457; May, 1950.) A modified version of apparatus previously used for measuring dielectric constants of gases. Two microwave oscillators have their frequencies controlled respectively by two invar cavity resonators, one of which is sealed off while the other has air drawn through it. By appropriate circuit arrangements, changes in the resonance frequency of this second cavity, due to fluctuations in the dielectric constant of the air, produce proportional deflections in a recording milliammeter.

621.317.36:621.396.611.21.001.4 2566

Checking Crystals—P. O. Farnham. (*Electronics*, vol. 23, pp. 150–154; July, 1950.) The method and apparatus described are for crystals whose fundamental frequencies lie between 6 and 13 Mc. Modifications may be made to suit other applications.

621.317.7:621.392.26† 2567

An Automatic Standing-Wave Indicator—P. J. Allen. (*Trans. AIEE*, vol. 67, Part II, pp. 1299–1302; 1948.) The probe of a conventional standing-wave indicator is moved over a distance exceeding one wavelength (3 cm) by a reciprocating device operated by a 100-rpm motor. The output from the probe is amplified and applied to the vertical-deflection plates of a cro. A voltage derived from a potentiometer attached to the probe carrier is simultaneously applied to the horizontal-deflection plates to give a deflection corresponding to the position of the probe, so that the waveform is displayed on the long-persistence screen. A graticule on the face of the cathode-ray tube permits the SWR to be read directly. A cursor is provided for measuring the position of the standing-wave pattern.

621.317.7.029.62 2568

V.H.F. Testing and Measuring Equipment—(*Electronic Eng.*, vol. 22, pp. 349–350; August, 1950.) A brief review of British very-high-frequency electronic equipment, with technical details supplied by the manufacturers, who will give further details on request.

621.317.7.029.64 2569

Measurement Apparatus for Ultra-High Frequencies—M. Bouix. (*Ann. Télécommun.*, vol. 5, pp. 210–218; June, 1950.) Descriptions and illustrations of cavity wavemeters, slotted-line and coaxial-line sections, attenuators and matched terminations for wave-guides, power meters, and the like, developed in the Centre National d'Études des Télécommunications for measurements at wavelengths of about 3 cm and 10 cm.

621.317.725 2570

The Diotron—An Aid to R.M.S. Instrumentation—R. D. Campbell. (*Electronics*, vol. 23, pp. 93–95; July, 1950.) A circuit is described comprising basically a temperature-limited diode and a dc amplifier connected in a feedback arrangement. It can be used as a voltmeter giving true rms readings. The voltage to be measured is applied to the diode filament terminals together with the dc heating voltage, the effect of feedback then being to maintain the filament heating power constant. A linear power scale is obtained for small inputs. Details are given of an instrument covering the frequency range 40 cps–10 Mc and having a response time of 15 ms for frequencies above 1 kc; full-scale deflection on the highest voltage range corresponds to 10w across a 600-Ω load. Possible applications to computing circuits are outlined.

621.317.725:621.3.018.78† 2571

Distortion-Measurement Apparatus—H. Boucke and H. Lennartz. (*Funk. und Ton*, vol. 4, pp. 217–225; May, 1950.) Descriptions are given of (a) a simple instrument for harmonic measurement for input frequencies of 10, 20, 40, 160 kc; (b) an improved instrument for the same frequencies; (c) a distortion meter for 800 cps. Circuits are shown; selective feedback and multiple harmonic filters are used. Harmonic voltages down to 0.5 per cent of that of the fundamental can be measured with negligible error.

621.317.74:621.315.212 2572

Test Set for Impedance Frequency Measurement on Coaxial Cables—A. F. Boff. (*Elec. Commun.*, vol. 27, pp. 123–137; June, 1950.) Determination of the characteristic impedance, attenuation constant, phase constant, and velocity ratio of long lengths of coaxial cable is discussed. By avoiding frequency-dependent parameters in the measuring circuits, precise measurements may be made with a rapidity impossible with previous methods. A description is given of a portable test set covering the range from 5 to 30 Mc.

621.317.757 2573

Simple Wave Analyser—D. M. Tombs. (*Wireless Eng.*, vol. 27, pp. 197–200; July, 1950.) The selective property of a simple tuned circuit is used to isolate a particular frequency existing in a complex wave, the isolated harmonic and the wave to be analyzed being both displayed on a double-beam cro. The Q value of the tuned circuit governs the amplitude of the selected harmonic component and is adjusted over a wide range by means of a negative-resistance element.

The ratio of two harmonic amplitudes may be found by adjusting the Q values to make the magnified voltages the same for both in succession. The ratio is then given by a simple relation involving the frequency of the harmonics and the incremental adjustments of the tuning capacitor to obtain the half-power points, both these quantities being easily measured.

621.317.772 2574

High-Frequency Phase Measurement with Direct Indication: Part 1—With a Cathode-Ray Tube as Indicator—A. Ruhrmann. (*Arch. Tech.* (Messen), No. 172, pp. T52–T53; May, 1950.)

621.317.799†:[621.396.813+621.396.822 2575

A Set for Noise and Distortion Tests on Carrier and Broadcast Systems—A. F. Boff. (*Marconi Rev.*, vol. 13, pp. 110–118; 3rd Quarter, 1950.) The use of two simultaneous test tones enables distortion measurements to be made at frequencies up to the limit of the pass band and overcomes the limitations of conventional harmonic testing methods. Description and circuit and performance details are given of a compact test unit suitable for noise and distortion measurements on very-high-frequency links.

621.385.012:621.317.79 2576

The Application of Direct-Current Resonant-Line-Type Pulsers to the Measurement of Vacuum-Tube Static Characteristics—J. Leferson (Proc. I.R.E., vol. 38, pp. 668–670; June, 1950.) A method is described for obtaining the static characteristics of a tube in the positive-grid region by applying to the grid 4-μs pulses obtained from a conventional line-type pulse generator using a resonance method of charging. Grid and anode currents are observed on a synchroscope by means of voltages developed across noninductive resistors.

621.392.3:621.316.849:621.396.67.029.62 2577

High-Load H.F. Resistors of the Transmission-Line Type with Uniform Damping—A. Kraus. (*Fernmeldetech. Z.*, vol. 3, pp. 157–160; May, 1950.) Expressions are derived for the dimensions of a 60-Ω air-cooled dummy antenna in which a length of resistance wire is wound spirally on a grooved metal cylinder which constitutes the low-loss return line. Three such units illustrated are for use in the 30–200 Mc band for powers of 250 w, 1 kw and 10 kw; the last is cooled by air blast. Resistance is within 10 per cent of 60Ω over the whole frequency range.

621.396.615:621.316.726.078.3 2578

A Method of Locking Oscillators in Integral and Non-Integral Frequency Ratios—E. A. G. Shaw. (*Brit. Jour. Appl. Phys.*, vol. 1, pp. 154–157; June, 1950.) The output voltages of the two audio-frequency oscillators to be locked are applied to a nonlinear mixer. A selected beat provides a suitable correction voltage to operate a reactance tube controlling the frequency of one oscillator. A practical circuit is described, giving locking with a 1,000-cps signal over quite a large range of frequency ratios. Further developments are discussed. The method appears to be applicable to any frequencies.

621.396.615.17 2579

Various Applications of the Square-Wave Generator—R. de L. Ortueta and J. M. H. Botas. (*Rev. Telecomun.* (Madrid), vol. 5, pp. 40–52; September, 1949.)

621.396.615.17:621.318.572]:621.392.018.424.001.4 2580

An Impulse-Generator Electronic-Switch for Visual Testing of Wide-Band Networks—T. R. Finch. (Proc. I.R.E. vol. 38, pp. 657–661;

June, 1950.) The instrument may be used to test any network that can be arranged to store a dc charge, the discharge characteristics produced by the network under test and by a reference network being simultaneously displayed for comparison. Representative applications are illustrated with the aid of cro traces and the circuit functions are described in detail.

621.396.616.029.64.001.4 2581
Kilomegacycle Buzzer Test Oscillator—G. L. Davies, C. B. Pear, Jr., and P. E. P. White. (*Electronics*, vol. 23, pp. 96–99; July, 1950.) Voltage pulses with repetition frequency 800 per second derived from a battery-driven buzzer are applied to a cavity resonator tunable in the range 3–11 kMc. Outputs up to 200 μv across a 50-Ω load are obtained by adjustment of a simple piston attenuator.

OTHER APPLICATIONS OF RADIO AND ELECTRONICS

534.321.9:061.3 2582
International Convention on Ultrasonics—Bradfield. (*See* 2397.)

534.321.9:542.952.6 2583
Emulsion Polymerization with Ultrasonic Vibration—A. S. Ostroski and R. B. Stambaugh. (*Jour. Appl. Phys.*, vol. 21, pp. 478–482; June, 1950.) Magnetostriction and piezoelectric oscillators, operating at 15 kc and 500 kc, respectively, were used to study the effect of ultrasonic vibrations on the formation of such products as polystyrene and synthetic rubber. Above the threshold intensity required to produce cavitation, the time taken to reach a given yield of polymer is approximately inversely proportional to the power used.

534.415:621.396.615.141.2 2584
Stroboscopic Mapping of Time-Variable Fields—L. Marton and D. L. Reverdin. (*Jour. Appl. Phys.*, vol. 21, pp. 617–618; June, 1950.) During investigations of the static field of a cutoff magnetron it was found that the space-charge field was completely masked by the magnetic field due to the filament heating current. To avoid this a half-wave rectifier was used to feed the filament and observation of the space-charge distribution was restricted to the half cycle when no heating current was flowing. Pulse operation of the electron beam used for observation of the space charge was effected by square-wave modulation of the electron-gun bias voltage. The stroboscopic method is applicable in principle to the observation of recurrent, time-variable fields up to very high frequencies, a practical upper frequency limit being estimated at about 300 Mc.

621-57:621.318 2585
The Magnetic-Fluid Clutch—J. Rabinow. (*Trans. AIEE*, vol. 67, Part II, pp. 1308–1315; 1948.) A detailed discussion of principles and applications. See also 969 of May and back references.

621.316.86:536.581 2586
Thermistor Thermostats—A. H. Taylor. (*Electronics*, vol. 23, pp. 154–159; July, 1950.) Two simple low-cost circuits suitable for controlling refrigerators and using Western Electric thermistors Type V-514 are described.

621.318.572:621.387.422† 2587
High-Speed Electronic Scaler—W. M. Sessler and A. V. Masket. (*Rev. Sci. Instr.*, vol. 21, p. 494; May, 1950. Ge diodes are used in a circuit developed from one described in vol. 19 of the MIT Radiation Laboratory series (2006 of 1949).

621.38.001.8:616–07 2588
Electronic Instruments in Diagnostic Medicine—S. N. Pocock, W. Grey Walter, H. A. Hughes. (*Electronic Eng.*, vol. 22, pp. 355–356; August, 1950.) Comments on 1207 of June (Hughes) and author's replies.

621.384.6 2589
200-kV Accelerator with Gas-Recovery System—E. Almqvist, K. W. Allen, J. T. DeWan, T. P. Pepper, and J. H. Sanders. (*Phys. Rev.*, vol. 79, p. 209; July 1, 1950.) Summary of American Physical Society paper.

621.384.6 2590
Optimum Parameters for Particle Acceleration by TM_{010} Cylindrical Cavities—B. L. Miller. (*Phys. Rev.*, vol. 79, p. 209; July 1, 1950.) Summary of American Physical Society paper.

621.384.6 2591
Production of High-Current Electron Pulses by a Resonant-Cavity Accelerator—G. W. Clark and L. B. Snoddy. (*Phys. Rev.*, vol. 79, p. 232; July 1, 1950.) Summary of American Physical Society paper. Pulses of duration $<10^{-7}$ second with electron energies >40 kev are obtained by use of a doubly reentrant cylindrical cavity operating at 400 Mc in conjunction with a pulsed electron source.

621.384.6 2592
Theory of the Radial Oscillations of the Electrons in an Electron Accelerator—H. Jahn and H. Kopfermann. (*Ann. Phys.*, (Lpz.), vol. 6, pp. 305–320; September 19, 1949.) Approximate formulas for the oscillation amplitude of the electrons about their instantaneous orbits, and for the damping of the oscillations, are derived and applied to the field of the 6-Mev electron accelerator of the Siemens-Reiniger Werke.

621.384.611.1† 2593
An 80-MeV Model of a 300-MeV Betatron—D. W. Kerst, G. D. Adams, H. W. Koch, and C. S. Robinson. (*Rev. Sci. Instr.*, vol. 21, pp. 462–480; May, 1950.)

621.384.611.1† 2594
Performance of 300-MeV Betatron—G. D. Adams, D. W. Kerst and C. S. Robinson. (*Phys. Rev.*, vol. 79, p. 208; July 1, 1950.) Summary of American Physical Society paper.

621.384.611.2† 2595
Operation of the M.I.T. 350-MeV Electron Synchrotron—I. A. Getting, J. S. Clark, J. E. Thomas, Jr., I. G. Swope, and M. L. Sands. (*Phys. Rev.*, vol. 79, p. 208; July 1, 1950.) Summary of American Physical Society paper.

621.384.611.2† 2596
Electron Injection Gun for the M.I.T. 350-MeV Synchrotron—O. Stone (*Phys. Rev.*, vol. 79, p. 209; July 1, 1950.) Summary of American Physical Society paper.

621.384.611.2† 2597
Design of Magnet Ends and Straight Sections for a Racetrack Synchrotron—G. B. Beard, J. L. Levy, W. A. Nierenberg, and R. W. Pidd. (*Phys. Rev.*, vol. 79, p. 209; July 1, 1950.) Summary of American Physical Society paper.

621.384.611.2† 2598
Synchrotron Radiofrequency System—M. H. Dazey, J. V. Franck, A. C. Helmholz, C. S. Nunan, and J. M. Peterson. (*Rev. Sci. Instr.*, vol. 21, pp. 436–439; May, 1950.) Description of the radio-frequency system of the Berkeley synchrotron.

621.384.611.2† 2599
Theory of the Capture Process in a Betatron-Injected Synchrotron—D. C. de Packh and M. Birnbaum. (*Rev. Sci. Instr.*, vol. 21, pp. 451–456; May, 1950.)

621.384.612.1† 2600
A Focusing Device for the External 350-MeV Proton Beam of the 184-Inch Cyclotron at Berkeley—W. K. H. Panofsky and W. R. Baker. (*Rev. Sci. Instr.*, vol. 21, pp. 445–447; May, 1950.)

621.385.833+535.822 2601
Electrons vs Photons: A Comparison of Microscopes—L. Marton. (*Jour. Opt. Soc. Amer.*, vol. 40, pp. 269–274; May, 1950.) Comparison of electron and light microscopes with discussion of the discrepancy between the theoretical resolving power of the electron microscope and the values obtained in practice.

621.387.4†:549.211 2602
Some Phenomena in Diamond Gamma-Ray Counters— E. Pearlstein and R. B. Sutton. (*Phys. Rev.*, vol. 79, p. 217; July 1, 1950.) Summary of American Physical Society paper.

621.387.424 2603
Geiger Counter for Lectures—R. L. Ives. (*Electronics*, vol. 23, pp. 105–107; July, 1950.) Circuit details are given of a portable instrument capable of delivering 50 w amplitude frequency output at 1,000 counts per second, and provided with strobotron flasher and thyratron-driven rate meter as auxiliary indicators.

778.37:537.523.4 2604
A Barium Titanate Coaxial Cable for the Production of a Short-Duration Spark—J. A. Fitzpatrick and W. J. Thaler. (*Phys. Rev.*, vol. 79, p. 231; July 1, 1950.) Summary of American Physical Society paper. A coaxial capacitor of overall length 16.5 cm, with $BaTiO_3$ dielectric of permittivity 1,575 giving a capacitance of 0.02 μF, was used in spark photography of ultrasonic waves in water at frequencies up to 7 Mc.

621.387.4† 2605
Ionization Chambers and Counters, Experimental Techniques [Book Review]—B. B. Rossi and H. H. Staub. Publishers: McGraw-Hill Book Co., New York, N. Y. 1st ed. 1949, 243 pp., 19s.6d. (in Great Britain). (*Electronic Eng.*, vol. 22, pp. 352–353; August, 1950.)

PROPAGATION OF WAVES

061.3:[621.396.11+55 2606
Summary of Proceedings of Australian National Committee of Radio Science, U.R.S.I., Sydney, January 16–20, 1950— (*Jour. Geophys. Res.*, vol. 55, pp. 191–210; June, 1950.) Summaries are given of 33 papers presented at the conference.

538.566+[537.226.2:546.217 2607
The Velocity of Electromagnetic Waves and the Dielectric Constant of Dry Air—Hughes. (*See* 2489.)

538.566 2608
Higher-Order Approximations in Ionospheric Wave-Propagation—J. Feinstein. (*Jour Geophys. Res.*, vol. 55, pp. 161–170; June, 1950.) The ionosphere, even when assumed homogeneous, can be considered as a linear medium for the propagation of em waves only as a first approximation. Second-order terms give rise to harmonics, and to sum and difference frequencies when two independent waves traverse the same physical region. These new frequencies are, in general, of the nature of forced vibrations, except in the case where their propagation characteristics are those of a natural mode capable of existing in the region. A resonance effect then occurs, the new wave increasing its energy at the expense of the interacting waves, and assuming an independent existence. While these effects couple energy from the primary wave, they do not affect its propagation characteristics.

As a result of the degeneracy of the determinantal equation for the propagation constant, the introduction of any disturbing physical

effects, such as a layer drift velocity, raises the degree of the equation, resulting not merely in changes in the values of the propagation constants of the usual modes, but, in addition, introducing new ones. For the usual disturbing effects encountered in the ionosphere, the energy content of these new modes is negligibly small.

538.566:537.562 2609

Electromagnetic Waves in Bounded Magneto-Ionic Media—B. Lax. (*Phys. Rev.*, vol. 79, p. 222; July 1, 1950.) Summary of American Physical Society paper. Discussion and theoretical study of high-frequency waves in ionized gases in the presence of an external magnetic field, particularly the determination of the complex frequency when the value of the propagation vector is fixed or limited, as might be the case in a finite cavity resonator or waveguide.

621.396.11+535.222 2610

Can the Velocity of Propagation of Radio Waves be Measured by Shoran?—C. I. Aslakson. (*Trans. Amer. Geophys. Union*, vol. 30, pp. 475–487; August, 1949.) An account is given of radar methods used to measure 47 lines of a shoran system of lengths ranging from 67 to 367 miles. Six of these lines could be compared with distances determined by first-order triangulation. Coordination of the shoran results with the geodetic measuremenst gave a mean value of 299 792.3 km for the velocity of propagation of radio waves, after making various corrections. See also 2010 of September (Essen).

621.396.11:551.510.535 2611

Ionosphere Observations at 50 kc/s—Brown and Watts. (*See* 2506.)

621.396.11:551.57 2612

The Importance of Water Vapor in Microwave Propagation at Temperatures below Freezing—D. G. Yerg. (*Bull. Amer. Met. Soc.*, vol. 31, pp. 175–177; May, 1950.) Calculations based on experimental results indicate that the vapor-pressure term in the refractive-index equation cannot be neglected for temperatures above −35°C. For temperatures between −35°C and −20°C the vapor-pressure gradient may be quite significant and at temperatures above −20°C it may be the dominant factor.

621.396.81 2613

Propagation of Waves of Frequencies from 2.5 to 35 Mc/s between Washington and Madrid—(*Rev. Telecomun.* (Madrid) vol. 5, pp. 2–18; September, 1949.) An account of NBS prediction methods, and of an experimental investigation into quality of reception for these frequencies, with a view to obtaining data on which predictions might be based. Observations were made hourly from 0700 to 2200 for the period December, 1948 to March, 1949. Graphs of receiver output power against time of day, and of muf and optimum working frequency against time of day, display the results for each fortnight of the period. These are discussed and compared with NBS charts, and suggestions are made for extension of the WWV standard-frequency transmissions.

621.396.81 2614

"Probable Law" of Propagation of Short Waves for Ranges of 1,300 km from Europe—R. G. Sacasa. (*Rev. Telecomun.* (Madrid), vol. 5, pp. 19–35; September, 1949.) Observations were made in 1945 and 1947–1949 of the times at which reception in Madrid of various standing-wave signals from London commenced or ceased to be of practical value. From the results obtained a simple numerical relation is deduced between the optimum working frequency F Mc at sunrise (or sunset) and the optimum frequency f Mc at a time n hours later or earlier (n being respectively positive or negative). $f=F\pm 2n$, the sign depending on whether the interval is measured from sunrise or from sunset, for which the value of F is much higher than for sunrise. For E-W or W-E transmissions a different value of the factor of n is required. Graphs of the signal strength of transmissions from London around 1,300 GMT indicate some dependence on the length of day.

621.396.81 2615

Optimum Working Frequencies for Ranges from 0 to 2,500 km—R. G. Sacasa. (*Rev. Telecomun.* (Madrid), vol. 5, pp. 36–39; September, 1949.) An adaptation of a NBS abac is reproduced and explained; from this either the muf or the optimum working frequency can easily be determined for any range up to 2,500 km.

621.396.81.029.63/.64 2616

A Microwave Propagation Test—J. Z. Millar and L. A. Byam, Jr. (Proc I.R.E., vol. 38, pp. 619–626; June, 1950.) A description is given of a microwave propagation test which was conducted over a period of a year with simultaneous transmission on wavelengths of 16.2, 7.2, 4.7, and 3.1 cm over an unobstructed 42-mile overland path. Comparative charts depict variations in daily fading range, illustrate diurnal and seasonal characteristics of fading, and reveal the marked difference between winter and summer fading. Curves are presented showing relative field-strength distribution for both winter and summer periods, and also the distribution of hourly minima. These curves may be useful in considerations bearing on the continuity of service that may be expected for different wavelengths and times of day in winter or in summer.

621.396.812 2617

Effects of Radio Gyrointeraction and Their Interpretation—M. Cutolo. (*Nature* (London), vol. 166, pp. 98–100; July 15, 1950.) Experiments performed in Italy during June-July 1949 are described, using Taranto as the wanted station with Turin as the receiving station and Radio Florence II as the interfering station. The interfering carrier frequency was varied from 1092 kc to 1333.33 kc to pass through the gyrofrequency, estimated to be 1190 kc at 90 km above the earth. The results obtained were in agreement with the theoretical double-humped curve given by Bailey (9 of 1939) and are used to explain earlier results (1476, 1477 and 1767 of 1949). Minimum interaction was obtained on a frequency corresponding to the local gyrofrequency at Montefalco, midway between transmitter and receiver, the presumed reflection point of the wanted wave. Solar activity was also observed to influence gyrointeraction, the phenomenon being weak or absent for a Wolf's number up to 100.

RECEPTION

621.396.621 2618

Low-Noise F.M. Front End—J. Marshall. (*Radio and Electronics*, vol. 21, pp. 58–63; June, 1950.) Construction details are given of a ganged, low-noise radio-frequency amplifier and mixer circuit which converts signals in the FM band to an intermediate frequency of 10.7 Mc. A high-gain intermediate-frequency amplifier for use with this equipment will be described in a subsequent article.

621.396.621:621.396.822 2619

Note on Low-Noise-Figure Input Circuits—A. C. Hudson. (Proc. I.R.E., vol. 38, pp. 684–685; June, 1950.) Comment on 1504 of July (Lebenbaum).

621.396.823 2620

Measurement of Interference from Radio-Frequency Heating Equipment—G. H. Brown. (*Trans. AIEE*, vol. 67, Part II, pp. 1102–1106; 1948.) Discussion of medium-frequency and very-high-frequency radiation from dielectric- and induction-heating equipment, and a comparison of the theoretical radio-frequency field strengths with measurements.

621.394.3+621.396.3 2621

Military Teletypewriter Systems of World War II—F. J. Singer. (*Trans. AIEE*, vol. 67, Part II, pp. 1398–1408; 1948.) An illustrated review of equipment developed by the Bell Telephone Laboratories for line and radio links, with discussion and block diagrams of the various systems used.

621.395:06.053 2622

Fifteenth Plenary Assembly of the Comité Consultatif International Téléphonique, Paris, 1949—P. E. Erikson. (*Elec. Commun.*, vol. 27, pp. 87–100; June, 1950.) See also 455 of March and 994 of May.

621.396.13:621.396.619.2 2623

The Single-Sideband System of Radio-Communication—H.D.B. Kirby. (*Electronic Eng.*, (London) vol. 22, pp. 259-263; July, 1950.) The advantages of the ssb system over the normal dsb method of operating a radio link are discussed and a short description is given of ssb equipment in *SS Caronia*.

621.396.65.029.63 2624

Decimetre Waves in the German Telephone Service—E. Dietrich and P. Barkow. (*Fernmeldetech. Z.*, vol. 3, pp. 145–154; May, 1950.) Description of two multichannel FM systems in use and brief discussion of optimum operating conditions.

621.396.73:621.396.61 2625

V.H.F. Equipment for Sound Broadcasting—(*Electronic Eng.* (London), vol. 22, p. 309; August, 1950.) A pack-set transmitter-receiver has been designed for use by BBC commentators at golf matches, race meetings etc. It is carried on the commentator's back. The FM transmitter operates in the 90-Mc band and has an output of about 1 w, with a frequency response curve sensibly flat from 50 cps to 6 kc. A 250-v HgO anode battery and accumulator filament battery are adequate for three hours continuous working. The 70-Mc receiver enables the commentator to receive information from the engineer at the control point, where a small AM transmitter is installed. The portable transmitter, complete with batteries, $\lambda/4$ whip antenna and harness, weighs 17 lbs; the receiver brings the total weight to 26 lbs.

621.396.931/.932 2626

The Metropolitan Police Radio Communication System—E. C. Brown. (*Electronic Eng.* (London), vol. 22, pp. 316–322; August, 1950.) A review of developments from 1923 onwards, with a description of the present radio-telephone system which provides two-way communication between New Scotland Yard and some 200 vehicles, besides 13 launches of the River Thames Division. The present scheme uses two main frequency channels, but will eventually use four main and two subsidiary channels. A system of coaxial filters at the main station enables separate transmitters on different frequencies to feed into a common wide-band antenna. A similar system is used for the station receivers. Phase modulation is used and all transmitting equipment is crystal controlled. The output of the main-station transmitter is 250 w in the 95–100 Mc band, that of the mobile transmitters being 10 w in the 80–84 Mc band. The receivers are of the double-superheterodyne type. Further developments envisaged are the provision of radio-telephone equipment for motorcycles,

with selective calling arrangements and the use of similar equipment for cars.

621.396.931/.932 2627
Multi-Station V.H.F. Schemes—J. R. Brinkley. (*Electronic Eng.* (London), vol. 22, pp. 323–325; August, 1950.) Many very-high-frequency mobile-communication problems can be solved by the single-station method, but for large rural areas, estuaries, trunk roads and railways, multistation systems have decided advantages, though they are necessarily more complex. A 2-station scheme using A.M. has been in use by the Hertfordshire county police since 1947 with complete reliability. Ten further multicarrier schemes have been installed in various counties in England, seven being 2-station and the rest 3-station schemes. A 3-station scheme is also in use by the Lanarkshire Fire Service. For estuary and coastal problems the GPO authorities have decided to use multicarrier AM equipment.

621.396.931/.932 2628
Design Problems of V.H.F. Mobile Equipment—L. W. D. Sharp. (*Electronic Eng.* (London), vol. 22, pp. 331–337; August, 1950.) Operating requirements of very-high-frequency mobile systems and equipment are outlined and transmitter and receiver design problems are discussed with particular reference to transmitter power, receiver sensitivity and the type of modulation adopted. Block diagrams illustrate the essential differences between FM and AM transmitters and receivers. Tubes, components, power-supply units, and types of construction to make equipment suitable for use under the adverse conditions frequently met and to permit easy servicing are also discussed.

621.396.931/.932 2629
Planning V.H.F. Mobile Systems—E. R. Burroughes. (*Electronic Eng.* (London), vol. 22, pp. 298–304; August, 1950.) See also 1248 of June.

621.396.931/.932:621.396.6 2630
V.H.F. [mobile] Equipment—(*Electronic Eng.* (London), vol. 22, pp. 340–348; August, 1950.) A review, with illustrations and technical details, of a wide-range of British transmitting and receiving equipment available for very-high-frequency mobile radio services. The information has been supplied by the manufacturers, who will give further details on request. Both AM and FM systems are included.

621.396.931 2631
Mobile Radio—A. Bailey. (*Trans. AIEE* vol. 67, Part II, pp. 923–931, Discussion, pp. 932–933; 1948.) An account of the development of mobile communication systems for police, taxicab, and general telephone services from 1921 onwards, with details of the Bell System general mobile service for urban and highway use.

621.396.931:621.396.619.13 2632
Multi-Station V.H.F. Communication Systems Using Frequency Modulation—W. P. Cole and E. G. Hamer. (*Jour. Brit. I.R.E.*, vol. 10, pp. 244–258; July, 1950.) Reasons for the use of multistation systems for mobile communications are discussed and previous tests with FM equipment are reviewed. Standard equipment for a single-station scheme is examined and from this the performance requirements for multistation equipment are derived. Suitable frequencies for the main carriers and for a control link are discussed and equipment used in a multistation scheme is described in detail. Distortion due to multipath transmission is analysed and practical tests of multistation systems in the London area and in Scotland are described. Future trends in development of such systems are indicated.

621.396.932 2633
New Marine V.H.F. Radio System—(*Overseas Eng.*, vol. 23, pp. 449–450; July, 1950.) General description of the ship/shore telephony system for the port of Liverpool. Transmitters and receivers are crystal controlled and AM is used. Six channels are provided; the shore stations transmit on frequencies in the range 163.1–163.6 Mc, the frequencies for the mobile equipment being 4.5 Mc lower in each case. The weight of the portable sets, which are both rain- and waterproof, is <20 lb. Good communication is obtained within a range of 25 miles.

SUBSIDIARY APPARATUS

621-526 2634
Note on the Maximum Accuracy of Perfectly Stable Servomechanisms—G. Lehmann. (*Onde Elec.*, vol. 30, pp. 267–270; June, 1950.) If an unconditionally stable servomechanism includes an element which introduces a phase change not effectively zero, its accuracy at relatively low frequencies is necessarily limited. This limiting value is calculated for a simple case. Attention is drawn to the possible danger of exceeding this limit in a system only conditionally stable.

621.313.3:621.3.026.441† 2635
Flea-Power Industrial Synchronous Motors—A. B. Poole. (*Elec. Mfg.*, vol. 43, pp. 74–77 . . . 172; January, 1949.) Operational principles, design features and output characteristics of three basic types of clock motor: the hysteresis, shaded-pole induction, and inductor types.

621.314.622 2636
Rectifiers with Mechanical Contacts—A. Mongault. (*Rev. Gén. Élec.*, vol. 59, pp. 208–210; May, 1950.) Discussion of requirements for efficient commutation in a polyphase generator, i.e., minimum sparking and optimum inductance of the windings.

TELEVISION AND PHOTOTELEGRAPHY

621.397.5 2637
Distant Electric Vision—J. D. McGee. (PROC. I. R. E., vol. 38, pp. 596–608; June, 1950.) Reprint. See 761 of April.

621.397.5 2638
Television in Relief and in Colour—Y. Delbord. (*Ann. Télécommun.*, vol. 5, pp.219–228; June, 1950.) Paper presented at the Television Congress in Milan, September, 1949. Four methods are discussed and their advantages, disadvantages, and applicability to black-and-white, relief, or color television are tabulated. These are (a) fixed vertical-band scanning frame; (b) rotating disk; (c) multiple channel; (d) multiple image. The basic principles of the methods are described with regard to transmission and reception. For relief television, method (d) is regarded as affording the simplest satisfactory solution. For color television a combination of the best features of (b), (c) and (d) is desirable.

621.397.5 2639
Kell-Factor and Picture Definition in Television Transmissions with Constant Bandwidth—E. Schwartz. (*Fernmeldetech. Z.*, vol. 3, pp. 185–190; June, 1950.) Discussion of the optimum "K factor" (ratio of horizontal to vertical resolution) and comparison of different systems. See also 3942 of 1940 (Kell, Bedford, and Fredendall).

621.397.5 2640
An Analysis of the Sampling Principles of the Dot-Sequential Color-Television System—RCA Laboratories Division. (*RCA Rev.*, vol. 11, pp. 255–286; June, 1950.) A quantitative treatment of the influence of the width of the sampling pulse on color crosstalk, the response of standard monochrome-television receivers and color-television receivers to sinusoidal variations and to step functions, the way in which the method of "mixed highs" combines with the sampling procedure to produce high-frequency detail, and circuit methods of eliminating crosstalk.

621.397.5:535.88:532.62 2641
The Eidophor Method for Theater Television—E. Labin. (*Jour. Soc. Mot. Pic. Televis. Eng.*, vol. 54, pp. 393–406; April, 1950.) See also 3561 of 1949 (Thiemann).

621.397.5(083.74) 2642
Choice of Television Standards—(*Wireless World*, vol. 56, pp. 249–250; July, 1950.) Reasons are given for considering that the choice of the 405-line standard is fundamentally sound, both from an engineering and an economic viewpoint. When the capabilities of 405-line television have been fully exploited, any small improvement that might be obtained by increasing the number of lines will be offset by the difficulties which will be encountered in maintaining the larger bandwidth required and in ensuring that the degree of overall distortion does not exceed the more stringent limits of tolerance.

621.397.6:535.88 2643
Projection Television—J. Haantjes and J. J. P. Valeton. (*Tyjdschr. ned. Radiogenoot.*, vol. 14, pp. 99–117; July, 1949.) Description of the MW6-2 cr tube, Schmidt-type optical system and high-voltage supply unit. See also 2387 of 1948 (Rinia et al.)

621.397.6:621.396.67 2644
Ultra-High-Frequency Antenna and System for Television Transmission—Fiet. (*See* 2425.)

621.397.6.001.8 2645
Industrial Television System—R. W. Sanders. (*Elec. Commun.*, vol. 27, pp. 101–111; June, 1950.) Detailed description of equipment which uses a new type of image-dissector tube with a translucent instead of a solid photo cathode. See also *Electronics*, vol. 23, pp. 88–92; July, 1950.

621.397.61 2646
A New Ultra-High-Frequency Television Transmitter—Lappin and Bennett. (*See* 2659.)

621.397.61(47) 2647
Television in the Soviet Union—(*Fernmeldetech. Z.*, vol. 3, p. 218; June, 1950.) Brief note of what is known of the two transmitters in service at Moscow and Leningrad. Two other stations are under construction, one in Kiev to serve South Russia, the other in Sverdlovsk to serve eastern Russia and the Ural region.

621.397.62 2648
General Description of Receivers for the Dot-Sequential Color-Television System which Employ Direct-View Tri-Colour Kinescopes—RCA Laboratories Division and RCA Victor Division. (*RCA Rev.*, vol. 11, pp. 228–232; June, 1950.) See also 2363 of October.

621.397.62 2649
Sync-Separator Analysis—W. Heiser. (*Electronics*, vol. 23, pp. 108–111; July, 1950.) The response of a particular clipper circuit for separating synchronizing signals from the composite television signal is analyzed. Formulas are derived for use in designing such circuits, and calculated results are compared with measured results for test signals.

621.397.62:621.396.615.16 2650
A Hard-Valve Time-Base—Banthorpe. (*See* 2455.)

621.397.67:[621.317.733+621.392.52 2651
Television Antenna Diplexers—W. H. Sayer, Jr. and J. M. De Bell, Jr. (*Electronics*, vol. 23, pp. 74–77; July, 1950.) Methods are described which enable two or more closely spaced radio-frequency signals (e.g., vision and sound carriers) to be transmitted from one antenna, with a minimum of interaction between signal sources. Impedance bridges or "notch" filters formed of coaxial lines are used. Diplexers may also be used to add together the synchronized outputs of two generators so as to obtain high power; waveguide and coaxial-ring (rat-race) hybrids for this purpose are described. See also 1355 of July, No. 80.

621.397.81 2652
Midlands Television Area—(*Wireless World* vol. 56, pp. 266; July, 1950.) A map, based on a BBC survey, showing approximate field-strength contours for the vision signals from the Sutton Coldfield transmitter.

621.397.82 2653
A Study of Cochannel and Adjacent-Channel Interference of Television Signals: Part 2—Adjacent-Channel Studies—RCA Laboratories Division. (*RCA Rev.*, vol. 11, pp. 287–295; June, 1950.) In the observer tests, color signals characteristic of the field-sequential, line-sequential, and dot-sequential systems were included. A standard monochrome signal was paired with a monochrome signal and the color signals in some of the tests. In all instances, the interfering sound signal was present. From the standpoint of allocation, no substantial difference in the tolerable ratios was found for the various combinations of color and monochrome signals used. Part 1: 1798 of August.

621.397.828:621.396.61 2654
Tackling TVI at the Output End—Tapson. (*See* 2661.)

621.397.828:621.396.677 2655
TV Antenna Phase Control—G. N. Carmichael. (*Radio-Electronics*, vol. 21, pp. 54, 56; June, 1950.) A method of reducing co-channel interference and increasing wanted signal strength in fringe areas. Two Yagi arrays are used, spaced to give a phase difference of about 90° for angles of incidence of the downcoming wave commonly found in fringe areas. The outputs from the two arrays are fed separately to a variable phase control, which partially cancels unwanted, and adds wanted signal voltages. Depending on the angle between the directions of the incoming signals, the combination can give a rejection ratio of 35–40 db, and a forward gain of 15 db, compared with a dipole.

621.397 2656
Facsimile [Book Review]—C. R. Jones. Publishers: Murray Hill Books, New York, N. Y. 1949, 422 pp., $6.00. (*Jour. Franklin Inst.*, vol. 249, p. 255; March, 1950.) Apparently intended for those concerned with the use and maintenance of facsimile equipment rather than with its design.

TRANSMISSION

621.396.61:621.396.645 2657
Maximum Tank Voltage in Class-C Amplifiers—Dwork. (*See* 2460.)

621.396.61:621.396.97 2658
A New 150-kW Transmitter for Standard-Band Broadcasting—T. J. Boerner. (*Trans. AIEE*, vol. 67, Part II, pp. 943–951; 1948.) A detailed description of a 540–1600-kc transmitter with high-level modulation. A class-B modulator is used for AM of the class-C final radio-frequency amplifier, thus permitting the highest over-all efficiency with relative ease of adjustment.

621.397.61 2659
A New Ultra-High-Frequency Television Transmitter—L. S. Lappin and J. R. Bennett. (*RCA Rev.*, vol. 11, pp. 190–211; June, 1950.) Detailed description of the 1-kW Type-TTU-1A transmitter recently installed and now in operation at Bridgeport, Conn. The high-frequency tripler and power amplifier are operated as grounded-cathode grounded-screen amplifiers, each using eight Type-4X150A tubes mounted in a single cavity. A sectional diagram of the cavity is given. See also 1795 of August (Guy, Seiberg, and Smith).

621.397.67:[621.317.733+621.392.52 2660
Television Antenna Diplexers—Sayer and De Bell (*See* 2651.)

621.397.828:621.396.61 2661
Tackling TVI at the Output End—M. E. Tapson. (*Short Wave Mag.*, vol. 8, pp. 265–266; June, 1950.) Use of 3rd-harmonic anode traps in the power amplifier link coupling to the antenna system, and a folded dipole adjusted for low feeder SWR, minimized interference from a 100-w 14-Mc amateur transmitter, so that television reception was practicable within about 20 yards.

TUBES AND THERMIONICS

537.533.8 2662
Secondary—Electron Emission from Metal Mixtures—Salow. (*See* 2526.)

621.383.27† 2663
Calculation of the Elements of an Electron-Multiplier Tube and Its Realization—A. Lallemand. (*Le Vide* (Paris), vol. 4, pp. 618–624; May, 1949.) Various factors relevant to the design of photoelectric multiplier tubes are discussed, including type of cathode, form and coating of multiplier surfaces, and arrangements for obtaining suitable electron trajectories. The multiplier grids finally adopted consisted of parallel narrow strips inclined at about 45° to the mean grid plane, a grid of fine wires maintained at the same potential as the multiplier being fixed a little in front of each multiplier. Tubes using this type of construction have been produced with 7, 12, 17 and 19 stages, the multiplication factors for 120-v operation ranging from 1,215 to $3.7+10^6$.

621.383.27† 2664
Rise Times of Voltage Pulses from Photo-Multipliers—O. Martinson, P. Isaacs, H. Brown, and I. W. Ruderman. (*Phys. Rev.*, vol. 79, p. 178; July 1, 1950.) Includes a brief description of a method of measuring short rise-times without the use of a high-speed oscillograph.

621.385.012:621.317.79 2665
The Application of Direct-Current Resonant-Line-Type Pulsers to the Measurement of Vacuum-Tube Static Characteristics—Leferson. (*See* 2576.)

621.385.029.63/.64+621.396.615.142.2 2666
Recent Developments in Amplifying Valves for Centimetre Waves—G. Goudet. (*Ann. Télécommun.*, vol. 3, pp. 445–455; December 1948.) Limitations of klystron amplifiers are discussed and a description is given of the wide-band double-resonator klystron designed by engineers of the Laboratoire Central de Télécommunications. With anode voltage 8kv, electron current 100 ma, the output is 12 w, power gain 40 and pass-band 50 Mc, or 20 w under slightly different operating conditions, with gain 100 and pass band 25 Mc. A similar 2-stage tube (3 cavity resonators) has an output of 2 w, a power gain of 290 and pass band 20 Mc.

Travelling-wave tubes are also considered briefly and a simplified theory of their operation, with derivation of design and performance formulas, is given in an appendix. See also 1294 of June.

621.385.032.216 2667
Pulsed Operation of Oxide Cathodes—C. Biguenet. (*Le Vide* (Paris), vol. 4, pp. 661–668; July, September, 1949.) The theory of steady-state operation is reviewed briefly and a theory of pulsed operation is proposed which assumes that under the action of the electric field, a displacement of Ba ions in the thickness of the oxide layer occurs. These positive charges create a very intense field near the surface of the metal support and tear electrons out of it. Some of these electrons neutralize the positive charges, the rest play an important part in the electron emission. The time necessary for the neutralization of the positive charges appears to be of the order of 5 μ seconds, so that in pulsed operation the duration of application of the anode voltage should be less than this value. Experiments with the object of confirming the theory are described.

621.385.032.216 2668
Deterioration of Oxide-Coated Cathodes Under Low Duty-Factor Operation—J. F. Waymouth, Jr. (*Phys. Rev.*, vol. 79, p. 233; July 1, 1950.) Summary of American Physical Society paper. Investigation of the effect of emission-current duty-factor on cathode life.

621.385.032.216:621.3.011.2 2669
Electrical Conductivity of Oxide-Cathode Coatings—D. A. Wright. (*Brit. Jour. Appl. Phys.*, vol. 1, pp. 150–153; June, 1950.) See 2078 of September.

621.385.1:621.396.822 2670
Current Fluctuations in D.C. Gas-Discharge Plasma—P. Parzen and L. Goldstein. (*Phys. Rev.*, vol. 79, pp. 190–191; July 1, 1950.) Mathematical analysis deriving an expression for the effective noise power due to current fluctuations in a gas-discharge plasma in a rectangular waveguide transmitting in only its lowest mode. For ordinary gas tubes used as microwave noise sources the contribution of the frequency-dependent term in the expression for noise power is small.

621.385.2/.5].029.62 2671
A Survey of V.H.F. Valve Developments—(*Electronic Eng.* (London), vol. 22, pp. 310–315; August, 1950.) The factors limiting the upper working frequencies of ordinary tubes are discussed, together with the difficulties encountered in the development of very-high-frequency tubes; and a survey is made of the various types at present available. A list of the characteristics of a selected range of disk-seal triodes is included.

621.385.3:621.315.592†:546.815.221 2672
On the Frequency Response of PbS Transistors—P. C. Banbury and H. K. Henisch. (*Proc. Phys. Soc.*, vol. 63, pp. 540–541; July 1, 1950.) The response of PbS transistors is found to be comparable with that of Ge transistors. The high-frequency response can be extended somewhat by the use of higher collector voltages or by the application of a suitably oriented magnetic field. The variation produced by the magnetic field, though only small, shows by its sense that the charge carriers are electrons, as in the Ge *p*-type transistor. See also 2088 of September (Gebbie, Banbury, and Hogarth), 2089 of September (Brown) and 2459 above.

621.385.3:621.315.592†:621.396.645 2673
High-Frequency Operation of Transistors—Brown. (*See* 2459.)

621.385.3.029.62:621.396.822 2674
On the Space-Charge Smoothing of Shot Fluctuations in Triode Systems Responding to Very High Frequencies—I. A. Harris. (*Jour. Brit. IRE*, vol. 10, pp. 229–240; July, 1950.) Existing theoretical results are briefly

reviewed and a simplified theory of space-charge smoothing of shot fluctuations in a diode, based on the Benham-Llewellyn theory, is discussed in some detail. This theory is then applied to cases in the very-high-frequency range where electron transit times are appreciable, and it is concluded that the low-frequency smoothing factor is not appreciably changed up to transit angles of about 1 radian. Application of the theory to the noise of grounded-grid triode circuits gives results in better agreement with experiment than those based on earlier theories. The effect on noise of applying a nonuniform field at the cathode of a triode is also considered.

621.385.38 2675
Studies of Thyratron Behavior: Part 1—The Effect of Grid Resistance on the Recovery Time of Thyratrons—L. Malter and E. O. Johnson. (*RCA Rev.*, vol. 11, pp. 165–177; June, 1950.) Theoretical and practical investigations of the grid-voltage and current characteristics of a commercial thyratron, for different values of grid resistance and bias, during the period following the interruption of the discharge. During the recovery period electron current may flow to the grid if the voltage drop across the grid resistor due to positive-ion current exceeds the bias voltage. The increase of recovery time with increasing value of grid resistance is due to the delayed return of the grid voltage to its bias value and not to a decreased rate of deionization, this rate being essentially independent of grid resistance and voltage. The recovery time is also dependent upon the instantaneous value of the actual grid voltage. See also 2676 below.

621.385.38 2676
Studies of Thyratron Behavior: Part 2—A Study of the Effect of Grid Potential Variations during the Afterglow Period upon the Recovery Time of Thyratrons—E. O. Johnson and L. Malter. (*RCA Rev.*, vol. 11, pp. 178–189; June, 1950.) Experiments with commercial thyratrons indicate that the recovery time is a function of the instantaneous effective grid potential and is independent of previous grid-potential variations provided that no abrupt changes have occurred shortly before the grid regains control. Such changes set up an unstable charge distribution in the plasma and sheaths within the tube, this unstable state passing to a quasistationary one in a time dependent on diffusion.

621.396.615.14 2677
A New Wide-Range, High-Frequency Oscillator—O. Heil and J. J. Ebers. (PROC. I. R. E., vol. 38, pp. 645–650; June, 1950.) Wide-range tuning in a Barkhausen type of oscillator is achieved by concentrating electrons from a highly efficient electron gun and also concentrating the electric field of a cavity resonator at the point where the electron beam has its greatest density. Although efficiency is low, variation of circuit capacitance alone gives a 3:1 tuning range, since the resonator gap capacitance is small. The oscillator construction is described and operating characteristics are given, output powers of 0.1–1 w being obtained in the wavelength range 4.5–12 cm.

621.396.615.141.2 2678
Resonance Frequencies of the Space Charge in a Magnetron—P. Fechner. (*Compt. Rend. Acad. Sci.* (Paris), vol. 230, pp. 1848–1849; May 22, 1950.) Analysis is effected for values of anode voltage and magnetic field such that the magnetron cannot oscillate, high-frequency excitation being introduced from an auxiliary source. Expressions are derived relating the resonance frequency of the vibrating space-charge electrons to the value of the electric field, for both whole-anode and multi-cavity magnetrons.

621.396.615.141.2:534.415 2679
Stroboscopic Mapping of Time-Variable Fields—Marton and Reverdin. (*See* 2584.)

621.396.615.141.2†:621.317.361 2680
Measurement of the Resonance Frequency of the Space Charge in a Coaxial Magnetron—P. Fechner. (*Compt. Rend. Acad. Sci.* (Paris), vol. 231, pp. 124–125; July 10, 1950.) The formula connecting magnetic field H with interelectrode voltage V at resonance (see 2678 above) is verified by measurements made by the following method. The nonoscillating magnetron is energized from a klystron source coupled to it by a matched coaxial line. A probe inserted in this line serves to increase the variations of impedance as V is altered. The resonance condition is determined by means of a crystal detector at an electric-field antinode of the coupling line.

621.385 2681
Röhren Vade-Mecum 1950 [Book Review]—P. H. Brans. Publishers: Brans-Verlag, Antwerp, 8th ed. 1950, 508 pp., 15.40 Swiss francs. (*Bull. Scdweiz. Elektrotech. Ver.*, vol. 41, p. 463; May 27, 1950. In German.) Includes the data of former editions combined with particulars of the latest tubes: viz. 9-element tubes, accelerometers, projection tubes, phasitrons, planat triodes, crystal diodes, etc. Contents list and instructional notes are printed in eight languages.

Books

Aerials for Centimetre Wave-lengths by D. W. Fry and F. K. Goward

Published (1950) by Cambridge University Press, 51 Madison Ave., New York 10, N. Y. 164 pages+4-page index+x pages+75 figures. 5½×8½. $3.50.

This book is a welcome addition to that collection of books which have been published recently on the subject of antennas. The authors confine themselves mainly to a discussion of antennas used in scanning systems at wave lengths of 10 centimeters or less. This particular aspect of antenna theory has not been covered heretofore as completely as is done here.

Emphasis has been placed on design principles and on the advantages and disadvantages of particular types of antennas in meeting particular requirements, such as wide scanning range or accurate beam shape. The physics of antennas rather than their mathematical analysis is stressed, although applicable formulas and some simple derivations are given wherever possible.

Among the subjects covered thoroughly with reference to the scanning problem are point and line sources for use with reflectors, single and doubly curved reflectors, lenses, and horns. Optical aberrations as well as polar diagram theory are discussed briefly. Wartime developments both in the U. S. and England are reviewed.

The book is well suited to the needs of the radar engineer since it indicates clearly how to choose an antenna to meet a particular scanning requirement. A fairly complete set of references is given to the literature. The writing style is clear and readable.

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Questions and Answers in Television Engineering by Carter V. Rabinoff and Magdalena E. Walbrecht

Published (1950) by Whittlesey House, McGraw-Hill Book Co., Inc., 330 W. 42 St., New York, N. Y. 280 pages+15-page index+vii pages+3-page appendix. 178 figures. 6×9. $4.50.

This is the first edition of a "semi-textbook" written primarily for engineers, technicians, amateurs, service men, and students. With the exception of TV links, the book contains sections on all phases of the television broadcast field, including standards, laws, and regulations. The authors have attempted to arrange the questions and answers for maximum coherence and readability.

Section 12 describes the RCA 8-TS-30 and the GE model 802 receivers. By relating the questions to the circuit diagrams, the authors have adopted a classroom approach with excellent results. The reader interested in receivers might begin with this chapter and study other sections of the book as needed to expand the answers given.

This reader believes the book is thorough, carefully written, and to the point. It should have great appeal for servicemen and engineering managers alike who want concise up-to-date information without technical detail.

J. ERNEST SMITH
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Handbook H42, Safe Handling of Radioactive Isotopes, 29 pages, 15 cents a copy, is available from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Remittances from foreign countries should be in United States exchange, and should include an additional sum of one-third the publication price to cover the cost of mailing.

Recommendations for the safe handling of artificially produced radioactive isotopes in the typical laboratory or small industrial operation are concisely set forth in the booklet, which was prepared by the National Committee on Radiation Protection established by the National Bureau of Standards and drawing members from the medical organizations of the United States, radiation equipment companies, and other Government departments.